Gravitational wave data analysis

Badri Krishnan

Max Planck Institut für Gravitationsphysik
Albert Einstein Institut, Germany

Pasadena, June 2011

1. Introduction to gravitational waves
2. Gravitational wave detectors
3. Sources of gravitational waves
4. Searches for binary inspirals
5. Searches for continuous signals
6. Conclusions

Gravitational Waves

- GR can be formulated in terms of a spacetime Lorentzian metric g_{ab} satisfying the Einstein equations $R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi G T_{ab}$
- Lnearize the field equations around the Minkowski metric: $g_{ab} = \eta_{ab} + h_{ab}$
- Work in coordinates where components of h_{ab} are small
- We have the gauge freedom associated with infinitesimal coordinate transformations: $x^a \to x^a - \xi^a$
- h_{ab} transforms as $h_{ab} \to h_{ab} + \partial_a \xi_b + \partial_b \xi_a$
- Analogous to gauge freedom in electromagnetic theory $A_a \to A_a + \partial_a f$

Gravitational Waves

- After some algebra, we write the linearized Einstein equations as a wave equation

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \bar{h}_{ab} = -16\pi G T_{ab}$$

  where $\bar{h}_{ab} = h_{ab} - \frac{1}{2} h \eta_{ab}$ and $\partial^a \bar{h}_{ab} = 0$
- We still have the gauge freedom with $\partial^2 \xi_a = 0$
- It can be shown that we can use this additional gauge freedom in vacuum to set $h = 0$, $h_{00} = 0$, $h_{0i} = 0$
- Thus, only the spatial components h_{ij} are non-vanishing

Gravitational Waves

- The waves are also transverse, i.e. $h_{ij}k^j = 0$ where k^i is the direction of propagation
- For a wave propagating in the z direction, the metric perturbation is then

$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xx} & h_{xy} & 0 \\ 0 & h_{xy} & -h_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- We have two independent polarizations $h = h_+\mathbf{e}_+ + h_\times\mathbf{e}_\times$ where

$$\mathbf{e}_+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{e}_\times = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Gravitational Waves

We can make a rotation in the x-y plane

$$\mathbf{x}' = \mathbf{x}\cos\psi + \mathbf{y}\sin\psi$$
$$\mathbf{y}' = -\mathbf{x}\sin\psi + \mathbf{y}\cos\psi$$

Which leads to a rotation of $\mathbf{e}_{+,\times}$

$$\mathbf{e}'_{+} = \mathbf{e}_{+}\cos 2\psi + \mathbf{e}_{\times}\sin 2\psi$$
$$\mathbf{e}'_{\times} = -\mathbf{e}_{+}\sin 2\psi + \mathbf{e}_{\times}\cos 2\psi$$

For an elliptically polarized wave, we can find a frame such that

$$h_{+} = A_{+}\eta(t)\cos\phi(t)$$
$$h_{\times} = A_{\times}\eta(t)\sin\phi(t)$$

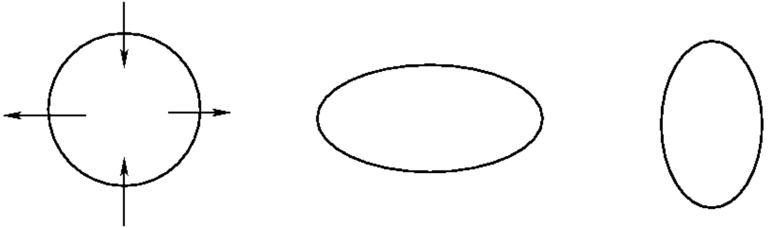
This will be sufficiently general for our purposes

Gravitational Waves

The effect of a GW on a ring of freely falling particles in the x-y plane in the TT gauge

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \ddot{h}_{xx} & \ddot{h}_{xy} \\ \ddot{h}_{xy} & -\ddot{h}_{xx} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}$$

For the $+$ polarization:



GW Detectors

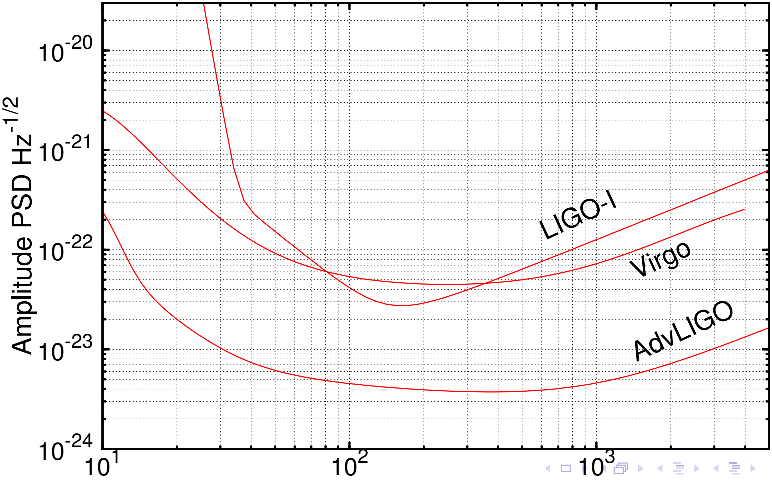
Some current and planned detectors:

- Ground based detectors: LIGO, Virgo, GEO: $\mathcal{O}(10^1) - \mathcal{O}(10^3)$ Hz
- The space based LISA detectors: $\mathcal{O}(10^{-3}) - \mathcal{O}(10^{-1})$ Hz
- Pulsar timing arrays: $\mathcal{O}(10^{-9}) - \mathcal{O}(10^{-7})$ Hz

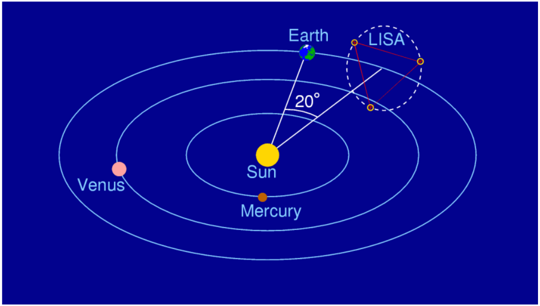
GW Detectors

- The 4km LIGO detectors in Hanford and Livingston
- The 600m GEO detector in Hannover, Germany
- The 3km Virgo detector in Pisa, Italy

GW Detectors

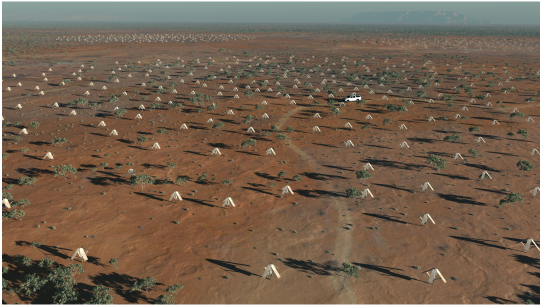


GW Detectors



- LISA consists of three spacecraft in a triangular configuration
- Each spacecraft contains two test masses in free fall

GW Detectors



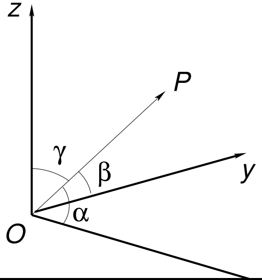
- Pulsar timing arrays are based on observing the highly regular millisecond pulsars
- GW signals are detected by correlating timing residuals from a number of pulsars

GW Detectors

Each of these detectors relies on, in one way or another, the effect of GWs on the path of a photon

- The path of a photon is not exactly a Euclidean straight line
- The GW lenses the path and varies the photon frequency

Consider a photon with frequency ν emitted in a particular direction



GW Detectors

In going from a point P to P':

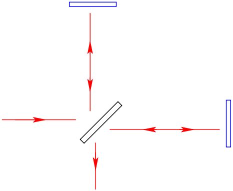
$$\frac{\delta\nu}{\nu} = \frac{1}{2}\frac{\alpha^2-\beta^2}{1-\gamma}(h_+ - h'_+) + \frac{\alpha\beta}{1-\gamma}(h_\times - h'_\times)$$

Remarkably, this depends only on the difference in h_{ab} at the two points. More geometrically:

$$\frac{\delta\nu}{\nu} = -\frac{1}{2}\frac{\mathbf{x}_i \Delta h_{ij} \mathbf{x}j}{1-\mathbf{k}\cdot\mathbf{x}}$$

GW Detectors

For the ground based detectors we have this basic configuration:



- We assume that the GW wavelength is much larger than the size of the detector

GW Detectors

We measure the difference in phase between the photons in the two arms which can be shown to be

$$\Delta\Phi = 2\omega_0 L h(t)$$

where

$$h(t) = \frac{1}{2}(\hat{y}^i\hat{y}^j - \hat{x}^i\hat{x}^j)h_{ij}$$

This is conveniently written as

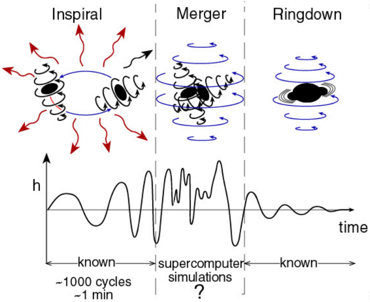
$$h(t) = F_+(t)h_+(t) + F_\times(t)h_\times(t)$$

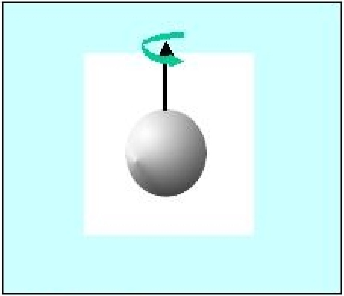
$F_{+,\times}$ depends on the sky-location of the source and the polarization angle ψ

Sources of GW signals

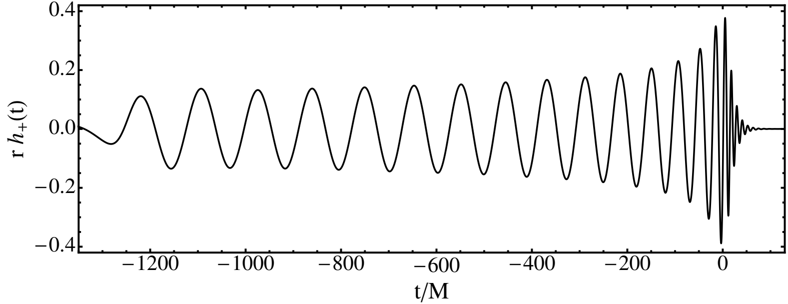
We shall consider only two sources here

- Inspiralling binaries of compact systems
- Rapidly rotating neutron stars





Binary Inspiral



- The signal from a system of inspiralling binaries is a "chirp"
- The full waveform is hard to describe analytically
- But we have good analytic descriptions for the inspiral and ringdown phases

Binary Inspiral

- Assume binary system evolves adiabatically through a sequence of circular orbits
- To leading order in v/c the flux of energy is given by the quadrupole formula
- The dominant component of the signal has twice the orbital frequency
- At leading order the waveform is elliptically polarized of the form

$$h(t) \sim (t_c - t)^{-1/4} \cos\left(2\Phi_0 - 2\left(\frac{t_c - t}{5\tau}\right)^{5/8}\right)$$

Binary Inspiral

- The inspiral signal is, in principle, of infinite length if the two masses start from infinitely far away
- However, the amount of time spent above some lower cutoff frequency f_{low} is finite
- For an equal mass system, at leading order

$$T_{chirp} = \frac{5M}{64} \frac{1}{(\pi M f_{low})^{8/3}}$$

- For two neutron stars $(1.4M_\odot, 1.4M_\odot)$ and $f_{low} = 40\,\text{Hz}$, $T_{chirp} \approx 140\,\text{sec}$
- For Advanced LIGO, $f_{low} = 10\,\text{Hz}$ and $T_{chirp} = 93\,\text{min}$

Binary Inspiral

- A useful (but very rough) measure of the ending frequency of the signal is
$$f_{cutoff} = \frac{1}{6\sqrt{6}\pi M}$$
- For the binary neutron star system this is 4420 Hz (actually much too high for LIGO)
- The merger is visible within the detector bandwidth for higher mass systems

Searching for binary inspirals

- The parameters of the signal are: the overall amplitude, the coalescence phase, time of coalescence, and (m_1, m_2)
- Other parameters such as spins and eccentricity might need to be taken into account if necessary in the future
- The basic tool here is matched filtering and the time of arrival is efficiently searched using the FFT

$$4\mathrm{Re}\int_0^\infty \frac{\tilde{x}(f)\tilde{h}^\star(f)}{S_n(f)} e^{2\pi i f t} df$$

Blind wide parameter space searches

The initial phase can be searched over easily by adding in quadrature

$$z(t) = x(t) + iy(t) = 4\int_0^\infty \frac{\tilde{x}(f)\tilde{h}^\star(f)}{S_n(f)} e^{2\pi i f t} df$$

The templates are normalized conveniently by

$$\sigma^2 = 4\int_0^\infty \frac{|\tilde{h}_0(f)|^2}{S_n(f)} df$$

where h_0 is computed for a source at some fiducial effective distance (say 1Mpc)

- The matched filter is to be computed over a suitable template bank in (m_1, m_2) space

Signal based vetoes

- The amplitude SNR is $\rho(t) = |z(t)|/\sigma(t)$
- In ideal Gaussian noise, the matched filter is all we need and simple thresholding on $\rho(t)$ suffices
- However, in practice this is not sufficient and the data has a number of glitches which can mimic a signal
- We need to consider properties of the signal which are not shared by typical glitches
- The most useful of such tests has turned out to be the frequency evolution of the signal

Signal based vetoes

- Consider the contribution to $\rho(t)$ from different frequency bands
- Based on just the signal and $S_n(f)$, we can define p frequency bands $\Delta f_1, \Delta f_2, \ldots, \Delta f_p$ such that the expected contribution are all equal to z/p
- We can calculate the contribution z_i ($i = 1, \ldots, p$) using the data
- The differences then form a χ^2 statistic

$$\chi^2 = p \sum_{i=1}^{p} \left(z_i - \frac{z}{p} \right)^2$$

which can be turned into a statistical test in an obvious way

Continuous Gravitational Waves

In the rest frame of the star, the signal is a slowly varying sinusoid with a quadrupole pattern:

$$h_+(\tau) = A_+ \cos\Phi(\tau) \qquad h_\times(\tau) = A_\times \sin\Phi(\tau)$$

$$A_+ = h_0 \frac{1+\cos^2\iota}{2} \qquad A_\times = h_0 \cos\iota$$

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz}\epsilon f_r^2}{d} \rightarrow \text{Model Dependent}$$

- ι: pulsar orientation w.r.t line of sight
- $\epsilon = (I_{xx} - I_{yy})/I_{zz}$: equatorial ellipticity
- f_r: rotation frequency
- d: distance to star

The waveform phase

The phase is very simple:

$$\Phi(\tau) = \Phi_0 + 2\pi \left[f(\tau - \tau_0) + \frac{1}{2}\dot{f}(\tau - \tau_0)^2 + \ldots \right]$$

Need to correct for the arrival times depending on the motion of the detector and also possibly the motion of the star

- Unlike the binary inspiral waveform, here the signal is narrow band and can last for months or years
- The frequency can shift because of intrinsic spindown and Doppler shifts
- The orbital velocity of Earth is $v_{orb}/c \sim 10^{-4}$. This leads to a (sky dependent) Doppler shift of $\Delta f \sim 10^{-4} f$.

The Crab Nebula

- One of the most famous pulsars: The Crab (b.1054 AD)

(NASA/CXC/SAO (Chandra X-ray observatory))

The Crab result

- The Crab is about 2 kpc away from us, it is observed to be rotating at $\nu \approx 29.78\,\mathrm{Hz}$
- It is spinning down at $\dot{\nu} \approx 3.7 \times 10^{-10}\,\mathrm{Hz/s}$
- This corresponds to a kinetic energy loss of $\approx 4.4 \times 10^{31}$ W (assuming $I_{zz} = 10^{38}\,\mathrm{kg\text{-}m}^2$)
- If all of this energy loss were due to emission of gravitational waves at 2ν, then they would have an amplitude

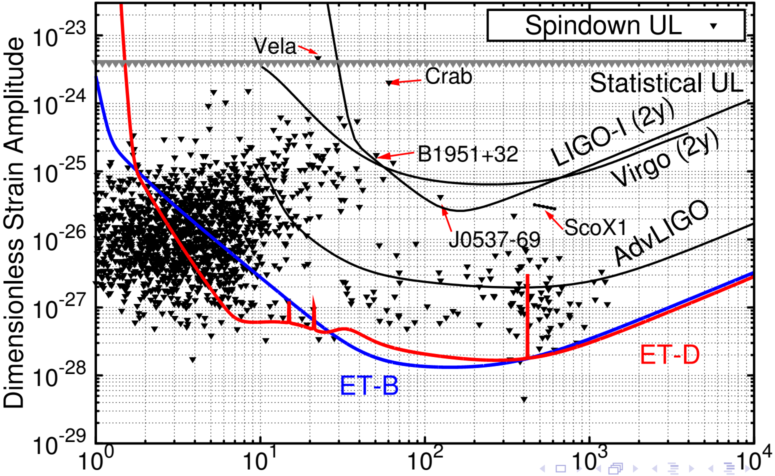
$$h_0^{sd} = 8.06 \times 10^{-19} \frac{I_{38}}{d_{kpc}} \sqrt{\frac{|\dot{\nu}|}{\nu}} = 1.4 \times 10^{-24}$$

- In reality, most of this spindown is due to electromagnetic braking, but we want to measure this directly

The Crab result

- Recall the amplitude parameters $(h_0, \Phi_0, \iota, \psi)$
- Use priors on ι and ψ from X-ray observations of the Nebula (Ng $ Romani (2004, 2008))
- The search did not result in a detection
- The 95% degree-of-belief upper limit on h_0 with these priors is 2.7×10^{-25}
- This corresponds to an upper limit of $\approx$ 4% of the spin-down energy loss
- With uniform priors, the corresponding number is 6%
- In terms of ellipticity, this corresponds to $\epsilon := (I_{xx} - I_{yy})/I_{zz} \leq 1.8 \times 10^{-4}$
- Also performed a search in a small frequency range which led to a 95% confidence upper limit of 1.7×10^{-24} (worse due to more statistical trials)

Known pulsar searches



Blind wide parameter space searches

- Expect to have nearby neutron stars not visible as pulsars
- Some of these might be visible in gravitational waves
- This implies a blind search in $(\nu, \dot{\nu}, \mathbf{n})$ (`gr-qc/0605028`)
- Sensitivity goes as $\sqrt{T_{obs}}$

$$h_0 \propto \sqrt{\frac{S_n(f)}{T_{obs}}}$$

- Number of templates increases rapidly with T_{obs}
- For short T_{obs} ($\ll 1$ year) we have approximately (for an all sky search including f and $\dot{f}$:

$$N_{templates} \propto T_{obs}^5$$

- And of course we need a large T_{obs} to get decent SNR

Blind wide parameter space searches

Fully coherent matched filter searches

- Feasible only for precisely known sources

Semi-coherent searches

- Break up data T_{obs} into N smaller segments T_{coh} and combine the segments semi-coherently
- This is forced upon us for targeted or blind searches by computational cost constraints
- Set up a common “coarse” template grid to analyze each segment using matched filtering
- Combining the results from each segment requires a “fine” grid refining in the sky and/or in spindown

Blind wide parameter space searches

- A semicoherent method is currently being used on our largest computing platform: `Einstein@Home`
- `Einstein@Home` is a public distributed computing project
- 270,000 users, 1.8 Million computers, 300Tflops of computing power 24/7
- E@H also looks for signals in electromagnetic data using very similar methods – has successfully found new pulsars in radio data from Arecibo

Conclusions

- This has been a very brief tour of GW data analysis
- We discussed basics of GWs and detectors, and some aspects of searches for inspirals and continuous waves
- Lot of topics not discussed: unmodeled burst and stochastic background searches, multi detector anlalyses, sky localization and parameter estimation, ...