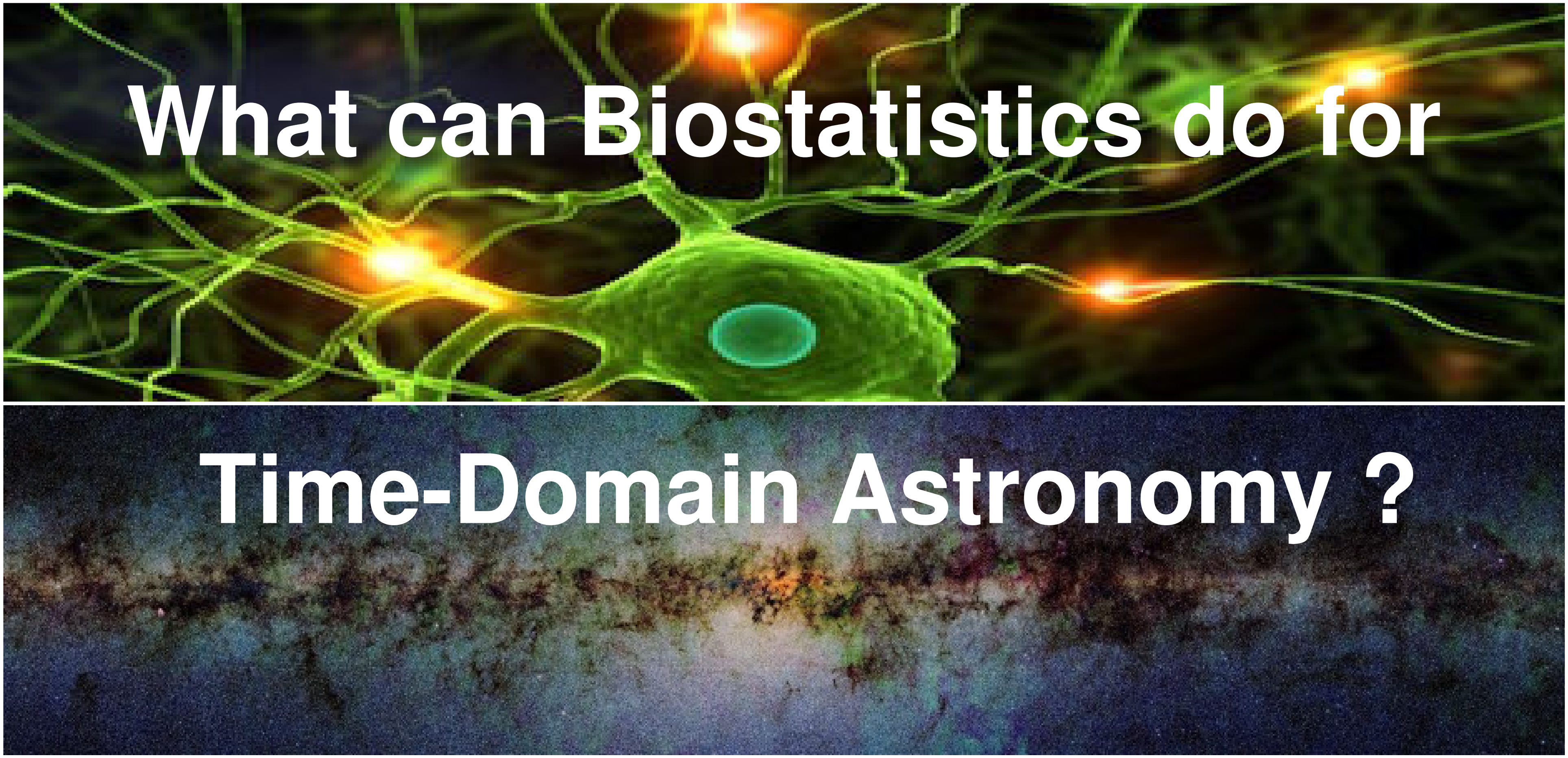

Baback Moghaddam

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Machine Learning Group

Classical (Bio)Statistics

Jerzy Neyman

(1894 – 1981)

- Polish / American Statistician
- Founder of **Hypothesis Testing**
- Co-Inventor of **Confidence Intervals**
- *Neyman-Pearson Lemma*

  (with his advisor Egon Pearson)

" The trouble is that what we [statisticians] call modern [orthodox] statistics was developed under strong pressure on the part of biologists. As a result, there is practically nothing done by us which is directly applicable to problems of astronomy." -- Jerzy Neyman (late in his life)

Modern Biostatistics

"In academia, the Bayesian revolution is on the verge of becoming the majority viewpoint, which would have been unthinkable 10 years ago."

from The New York Times, January 20th 2004

- Bradley P. Carlin

Mayo Professor of Public Health

Head of Division of Biostatistics

University of Minnesota

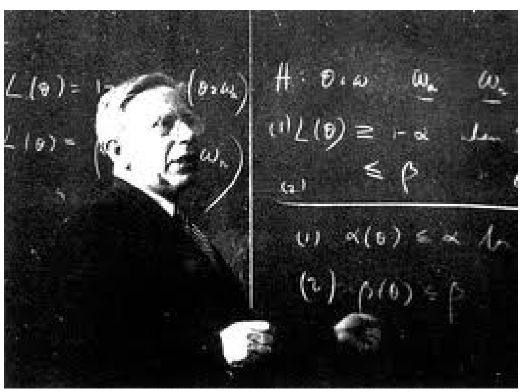
Modern Biostatistics

- Pathologies of Orthodox Statistics are more acknowledged nowadays
- Tests of ***statistical significance*** and design of ***clinical trials*** are now increasingly Bayesian
- Journals **banning** use of ***p-values***
  - The Lancet
  - Medical J. of Australia
  - The British Heart Journal
  - American J. of Public Health
  - Int’l Committee of Medical Journal Eds

Sequential Analysis

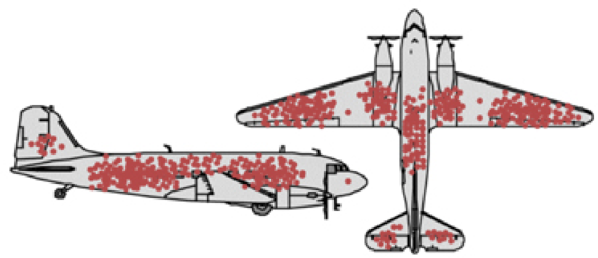
Abraham Wald

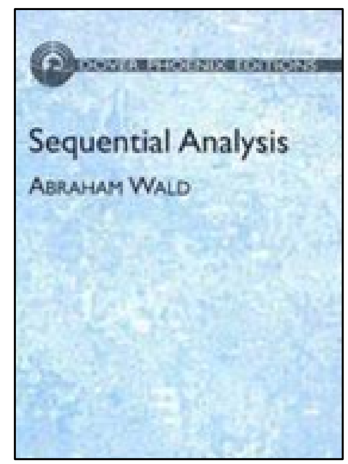
(1902 - 1950)



- Hungarian mathematician
- Fled Nazi Europe in late 1930s
- Invented Sequential Analysis

  doing QC for Allied bombers in WW2

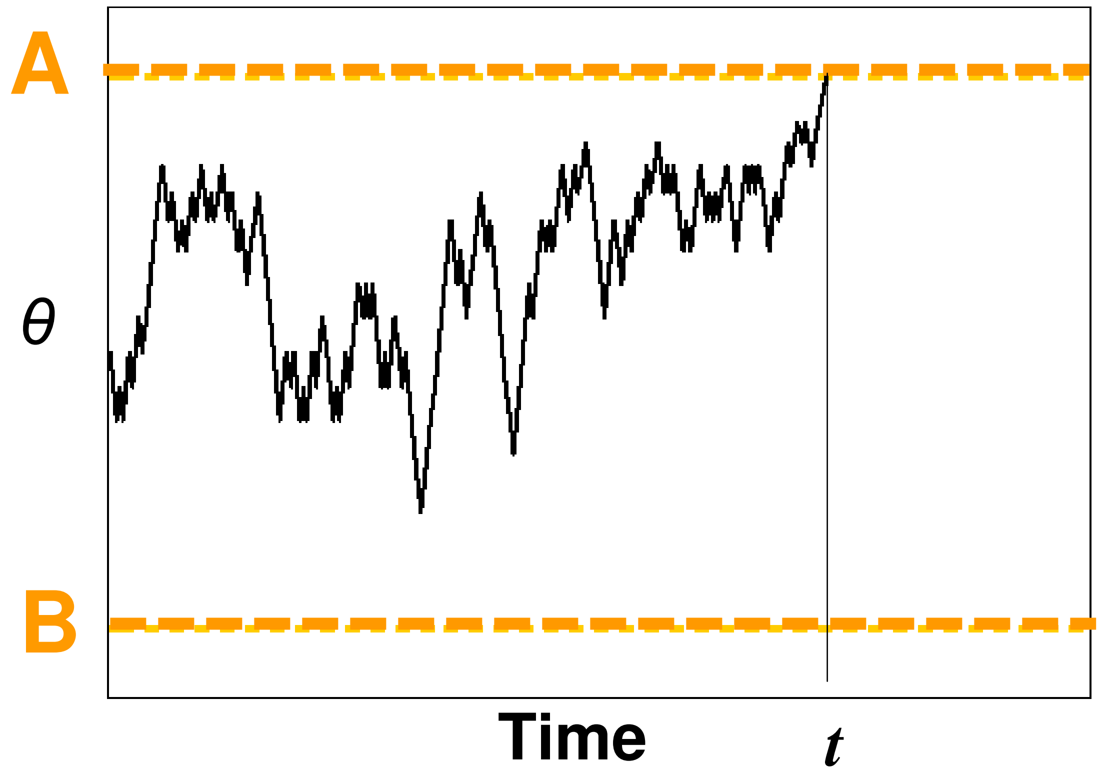




- Wrote classic text in 1947
- Sequential Probability Ratio Test (SPRT)

Optimal Decision Policy

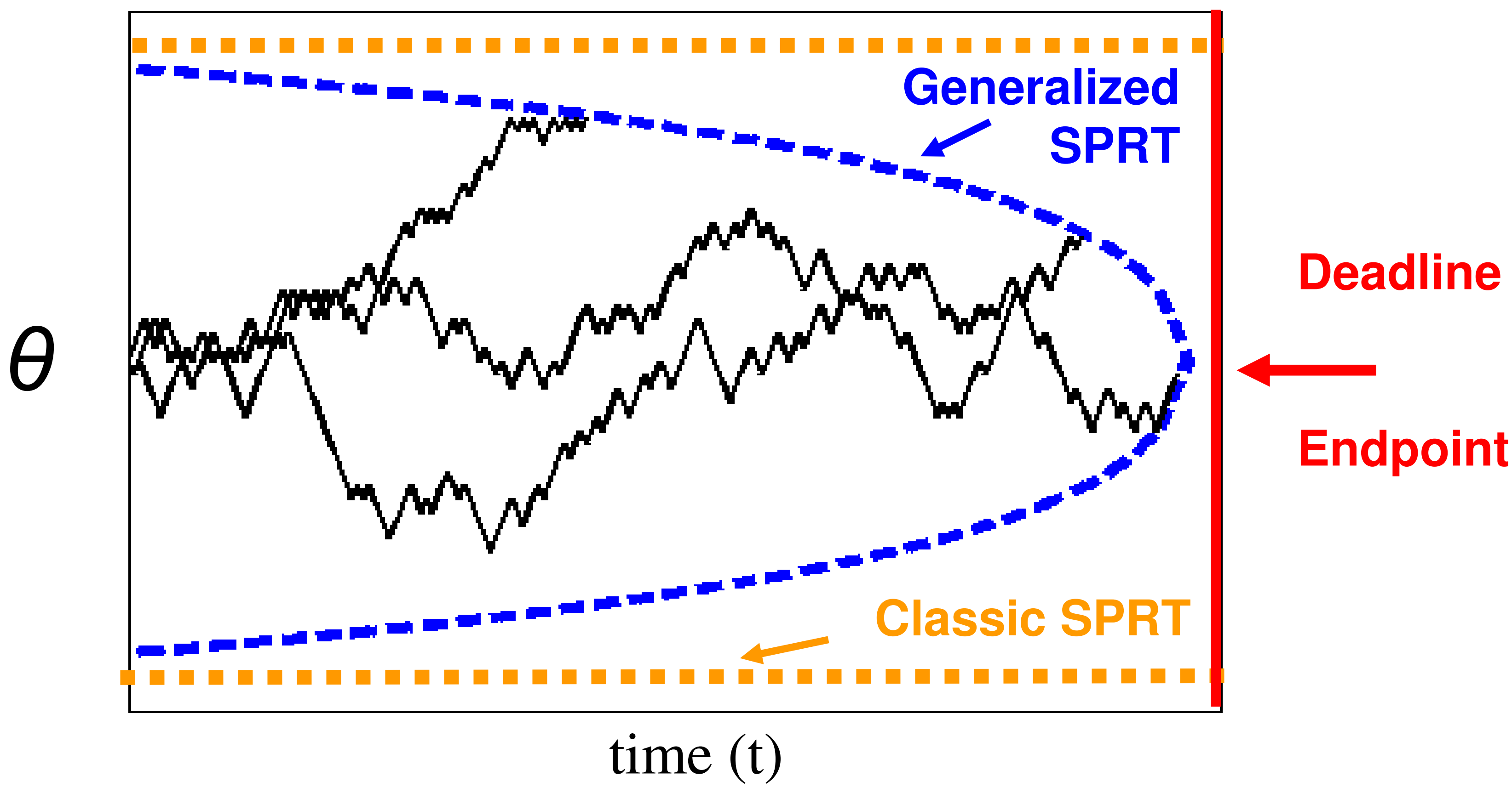
Wald & Wolfowitz (1948) showed that the optimal policy is to stop when θ exits an interval [A,B] and to choose the more likely hypothesis



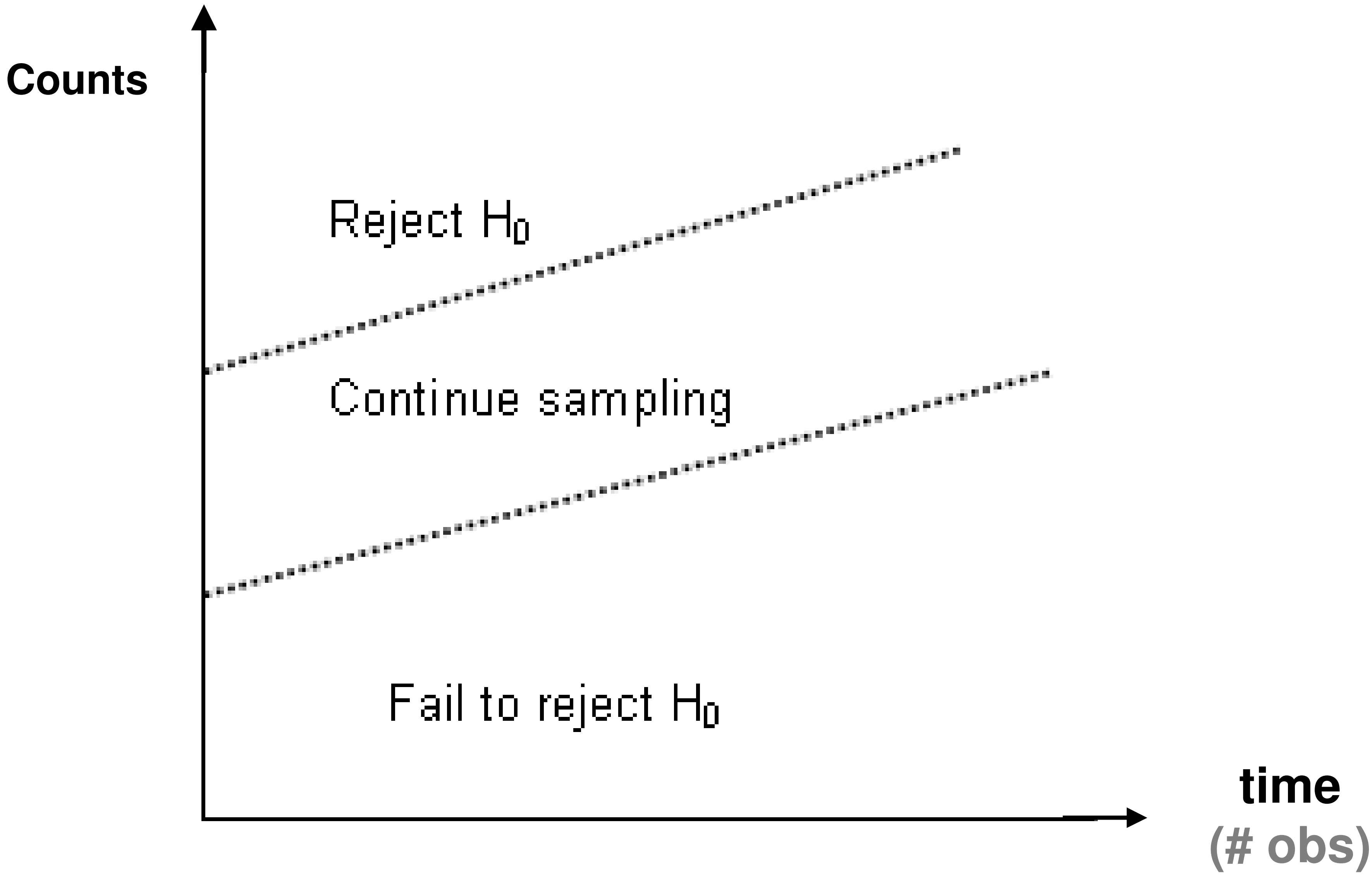
Sequential Probability Ratio Test (SPRT)

Optimal Decision Policy

The optimal policy with a **time-penalty** is to stop as soon as θ exits a region that narrows with time.

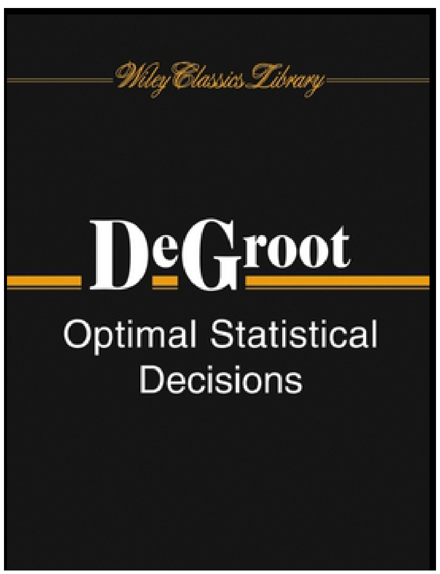


Sequential Probability Ratio Test



Optimal Decisions

Morris DeGroot
(1931 – 1989)



- American Statistician / CMU Professor
- Extended and formalized Wald's SPRT
- Bayesian Sequential Decision Making
- Wrote seminal text book in 1970
- Optimal decision algorithms

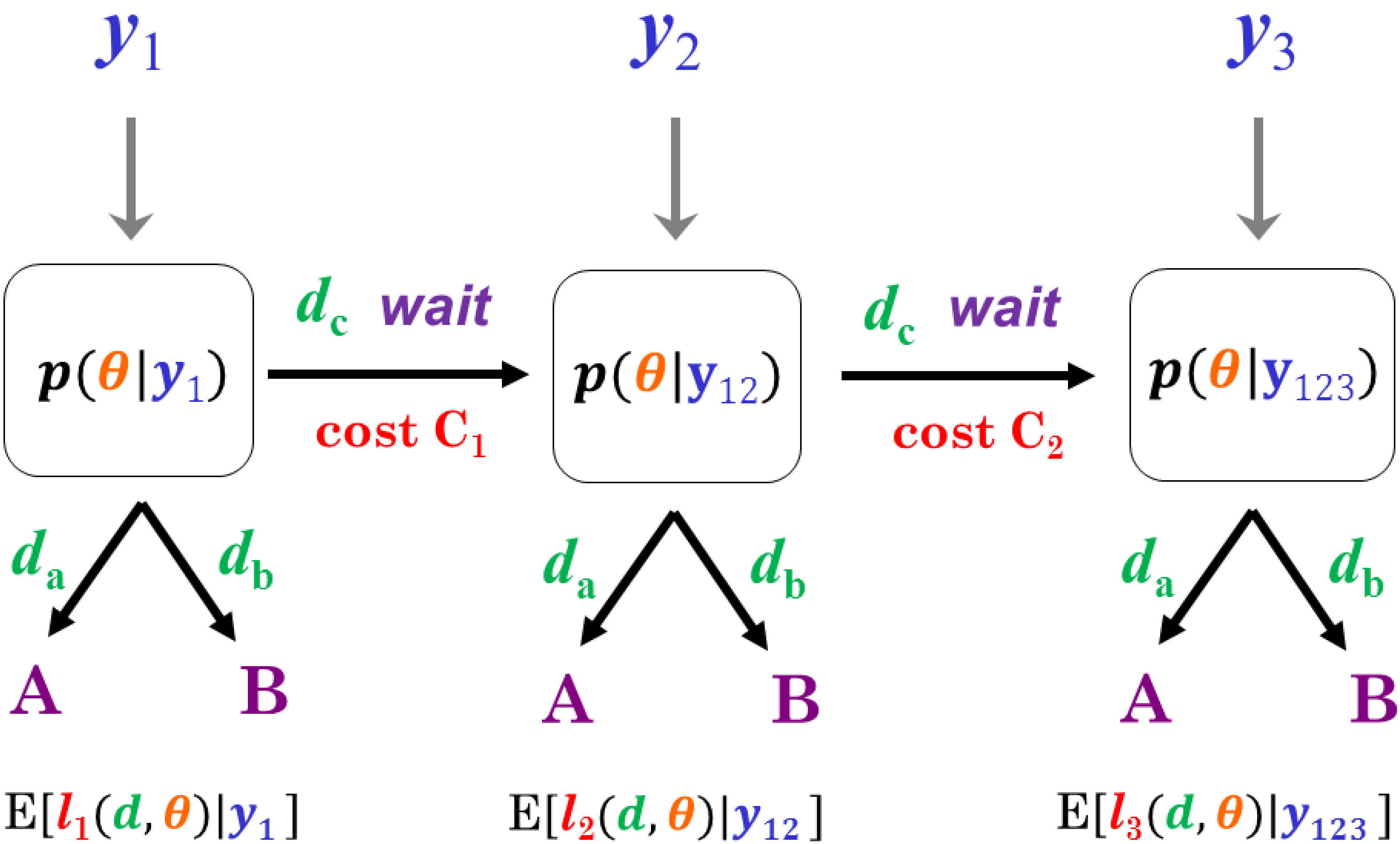
- For example, **Backward Induction** (BI)
  - solves the problem "backward in time"
  - also used in Dynamic Programming
  - unfortunately has **exponential complexity**

Sequential Decision Theory

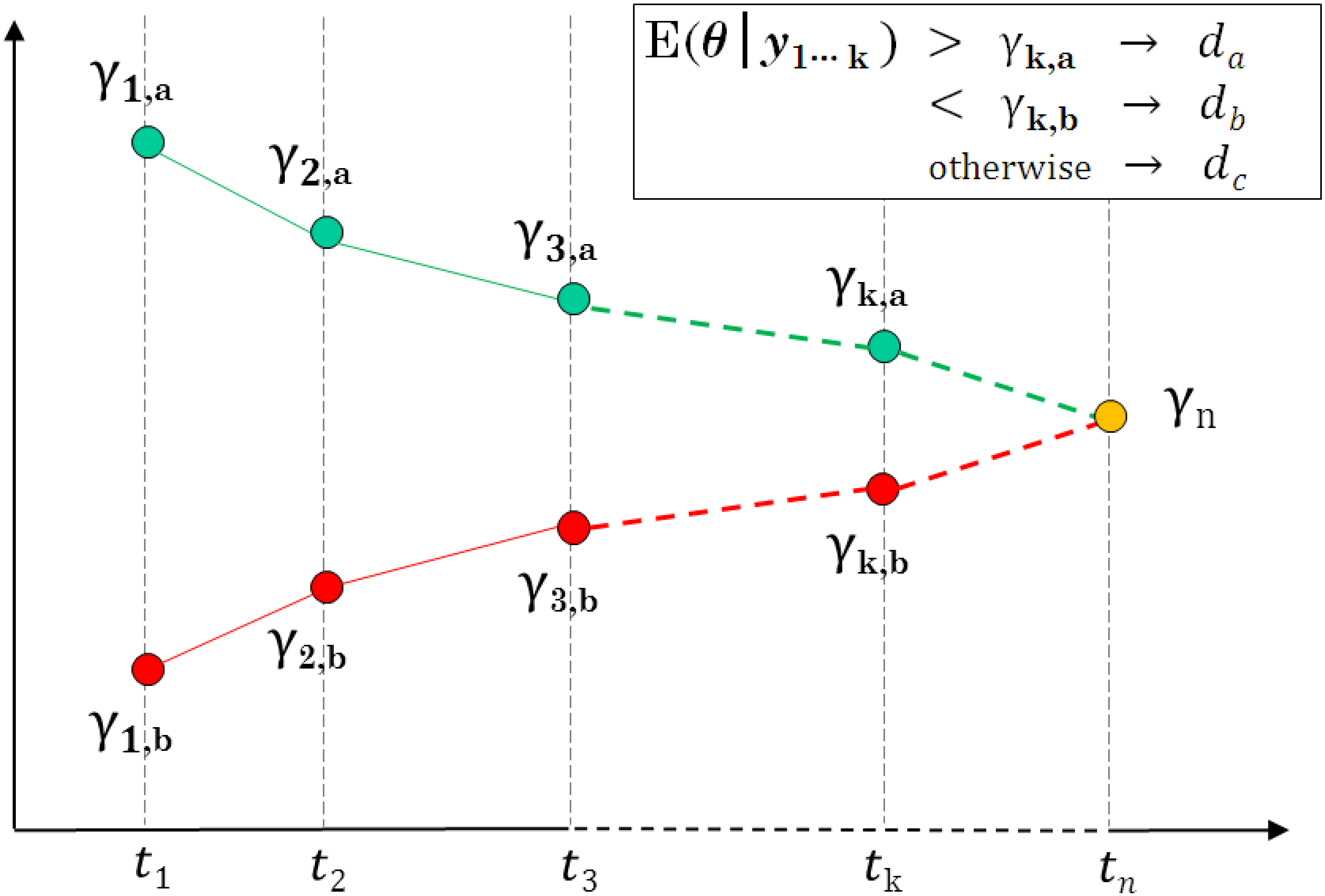
- Observations "trickle-in" $\{\, \boldsymbol{y}_i \,\} \quad i = 1 \ldots k$
- Update current beliefs $p(\boldsymbol{\theta} \mid \boldsymbol{y}_1, \ldots, \boldsymbol{y}_{\mathrm{k}})$
- Must "act" (make decisions) $\boldsymbol{d}_a,\ \boldsymbol{d}_b,\ \boldsymbol{d}_c$
- Which leads to losses $L_a,\ L_b,\ L_c$
  - timeliness (efficiency)
  - decision-induced costs
  - various domain-specific trade-offs

- Need an optimal policy (usually beforehand)
  - minimize posterior expected loss

Sequential Hypothesis Tests



Sequential Hypothesis Tests

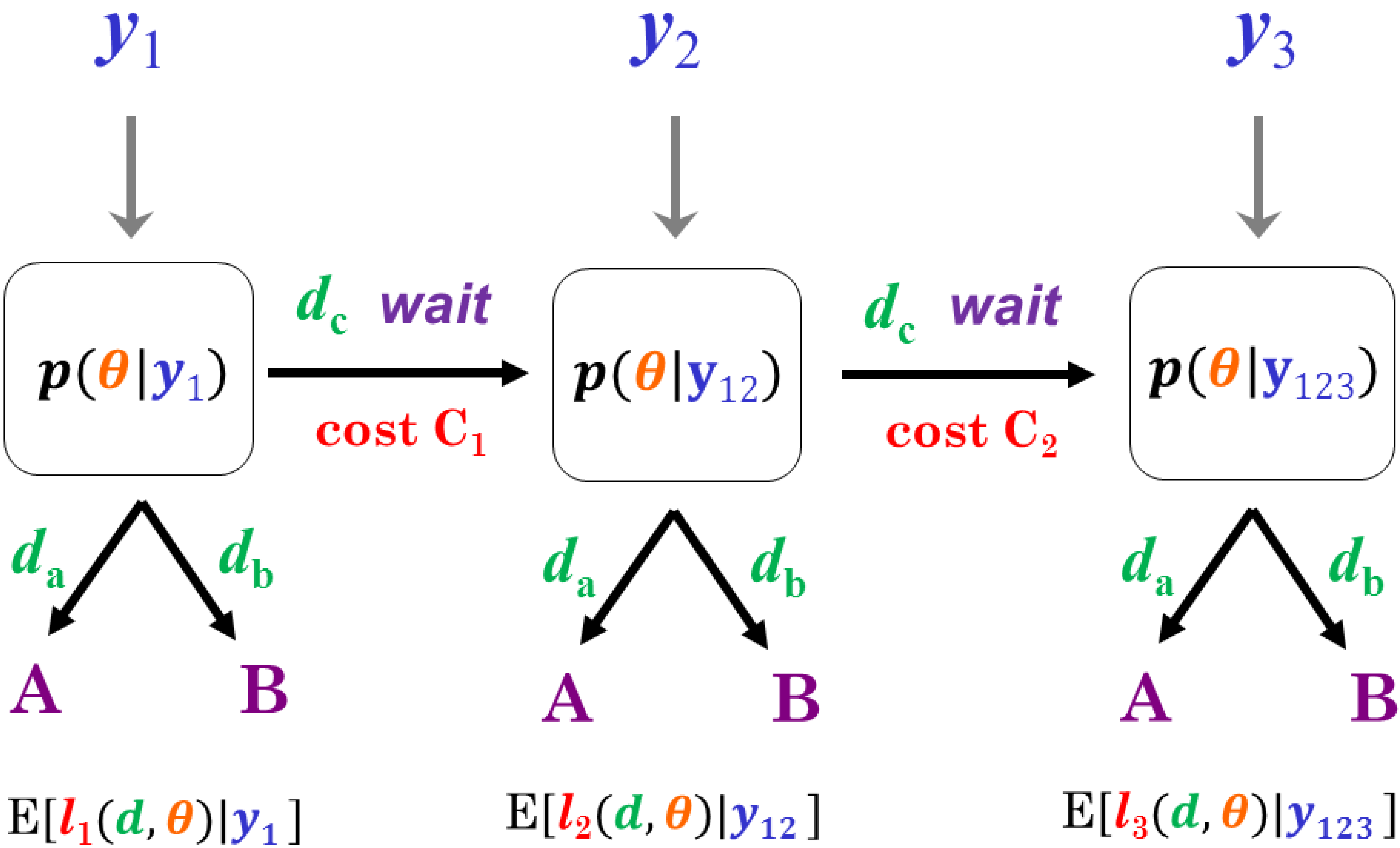


Alternative to BI

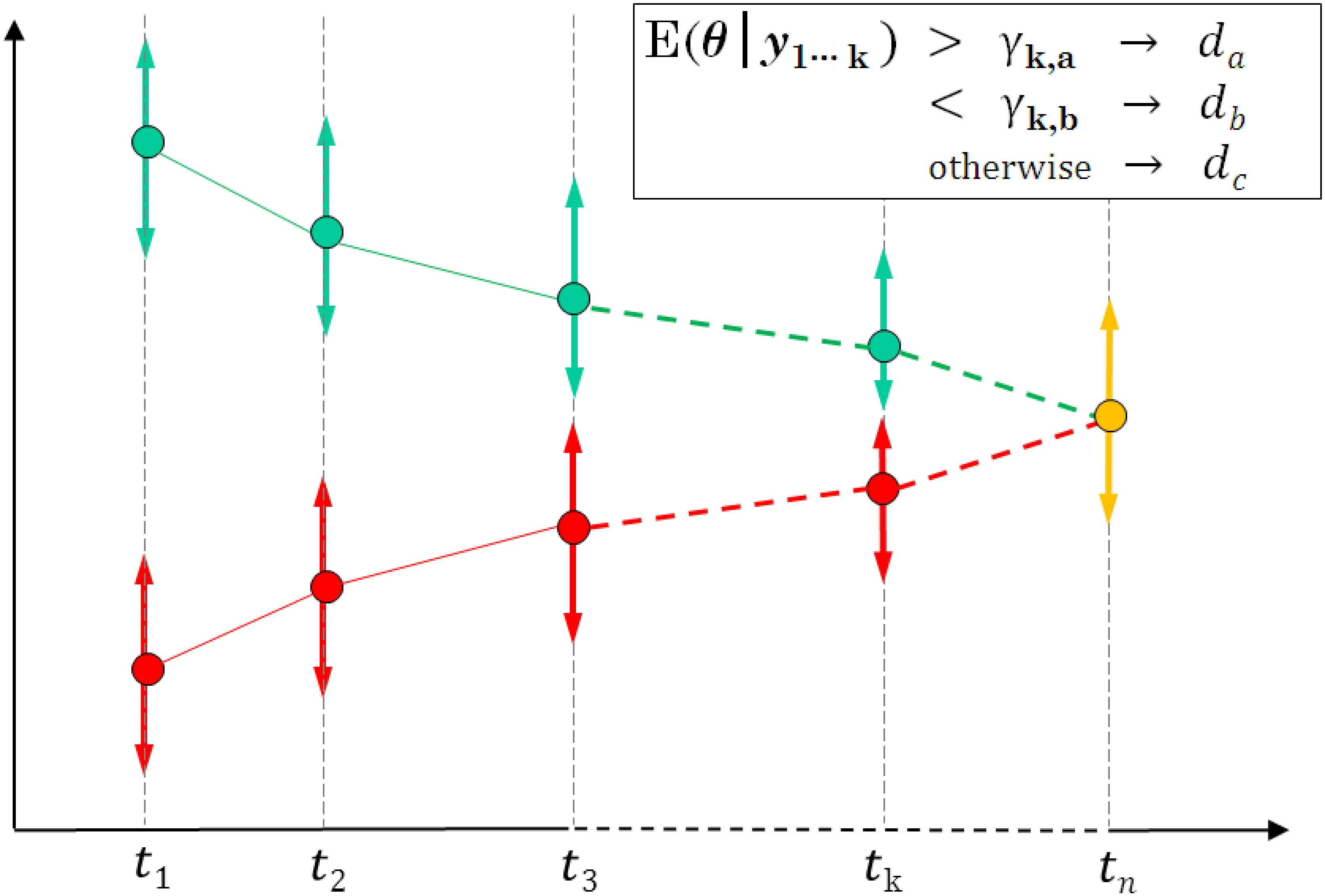
- For ~ 30 years **BI** was the best solution
- Then a new method: **Forward Sampling**
  - Carlin, Kadane and Gelfand, "Approaches to Optimal Sequential Decision Analysis in Clinical Trials," *Biometrics*, 54, 964-975, 1998

- Forward (generative) Monte Carlo sampling
- No complex integration required
- Handles arbitrary loss functions
- Complexity is **linear** in # of decision points

MC sample : $\{\theta, y_1, y_2, y_3\} \sim p(\theta, y_1, y_2, y_3)$



$$\min\ E[\text{total loss}]\ \text{wrt}\ \{\gamma_{\mathbf{k,d}}\}$$



Forward Sampling *vs.* BI

- Complexity is linear *vs*. exponential for BI
- FS finds the **optimal** policy (same as BI)
  - for single-parameter exp-family models, *etc*
- Unlike BI, FS is a (parallelizable) *continuous* optimization problem (grid search, ICM, *etc*)
- FS offers far greater flexibility in terms of loss functions & probability distributions

for further inspiration …