

Heirarchical Resolution

John Rice
Lightning Talk

Nicolai Meinshausen, Peter Bickel, and John Rice. *Efficient Blind Search: Optimal Power of Detection under Computational Cost Constraints* [[ANNALS OF APPLIED STATISTICS, Vol. 3, No. 1, 38-60, 2009](#)]

Search Space

- Consider no drift. Good frequency resolution depends on matching phase of photons at beginning and end of the record. If true frequency is f_0 , the number of cycles in time T is T/f_0 , so if the hypothesized frequency is $f = f_0 + \delta f$, δf should be $o(T^{-1})$ in order for a photon at the end of the record to be in phase with one at the beginning. The phase error at the end of the record is $T\delta f$.
- 10 days = 864,000 sec, - $\delta f = T^{-1}$. If a 40 Hz range has to be searched, a minimum of $40 \times 864000 = 34,560,000$ possible frequencies must be examined.
- Similarly, drift must be resolved within $o(T^{-2})$. To search the interval of possible frequency derivatives at this resolution, about 400-500 values must be examined.
- Consequence is that a test statistic must be evaluated $\sim 10^9$ values of frequency and its derivative.

Power Versus Computational Cost: Blocking

$$\text{Power} \propto \theta^2 T$$

$$f \text{ resolution} \propto T^{-1}$$

$$\dot{f} \text{ resolution} \propto T^{-2}$$

Calculation of statistic for a single $(f, \dot{f}) \propto T$

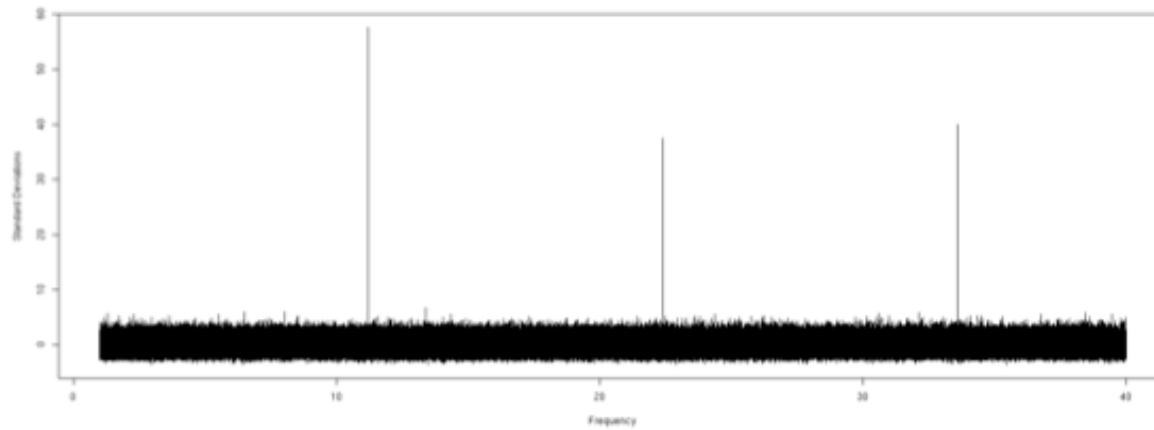
$$\text{FLOPS} \propto T^4$$

Partition T into B blocks, compute statistic in each block and average:

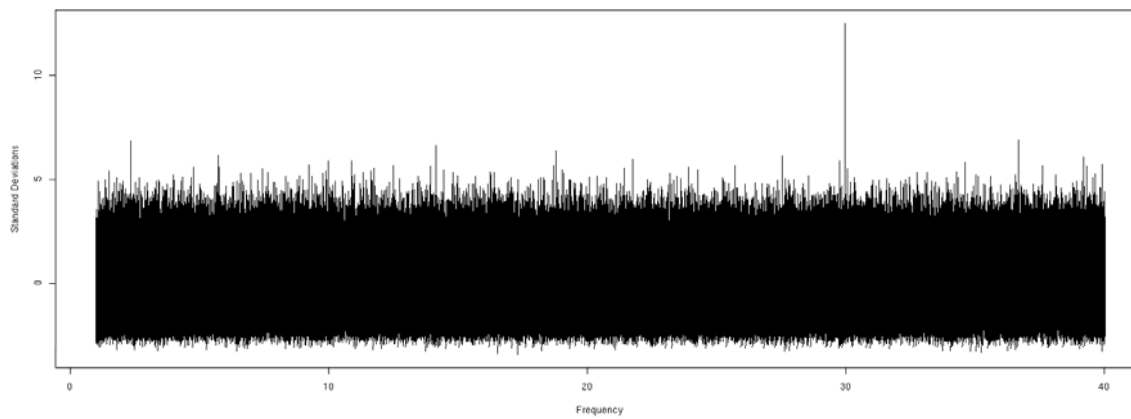
$$\text{Power} \propto \frac{\theta^2 T}{\sqrt{B}}$$

$$\text{FLOPS} \propto \frac{T^4}{B^3}$$

Blocking Vela and Crab



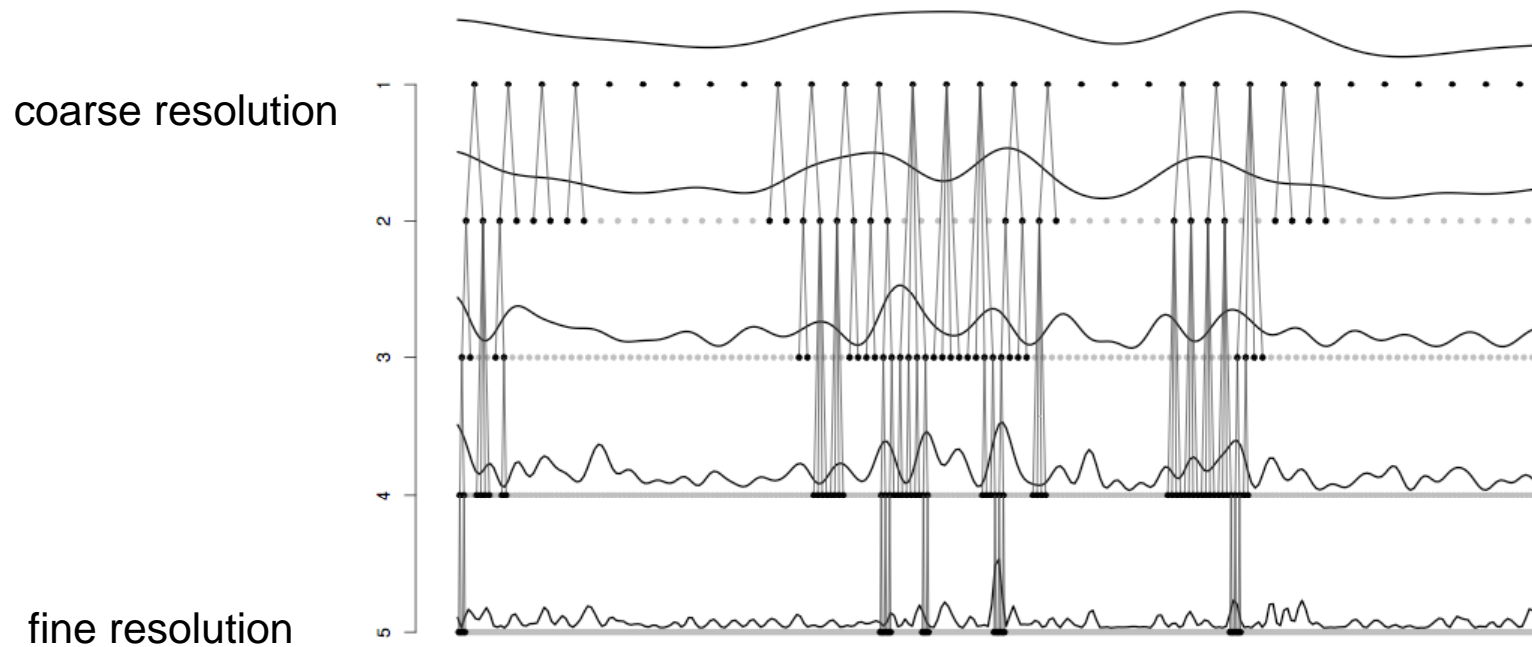
Vela: 318 blocks



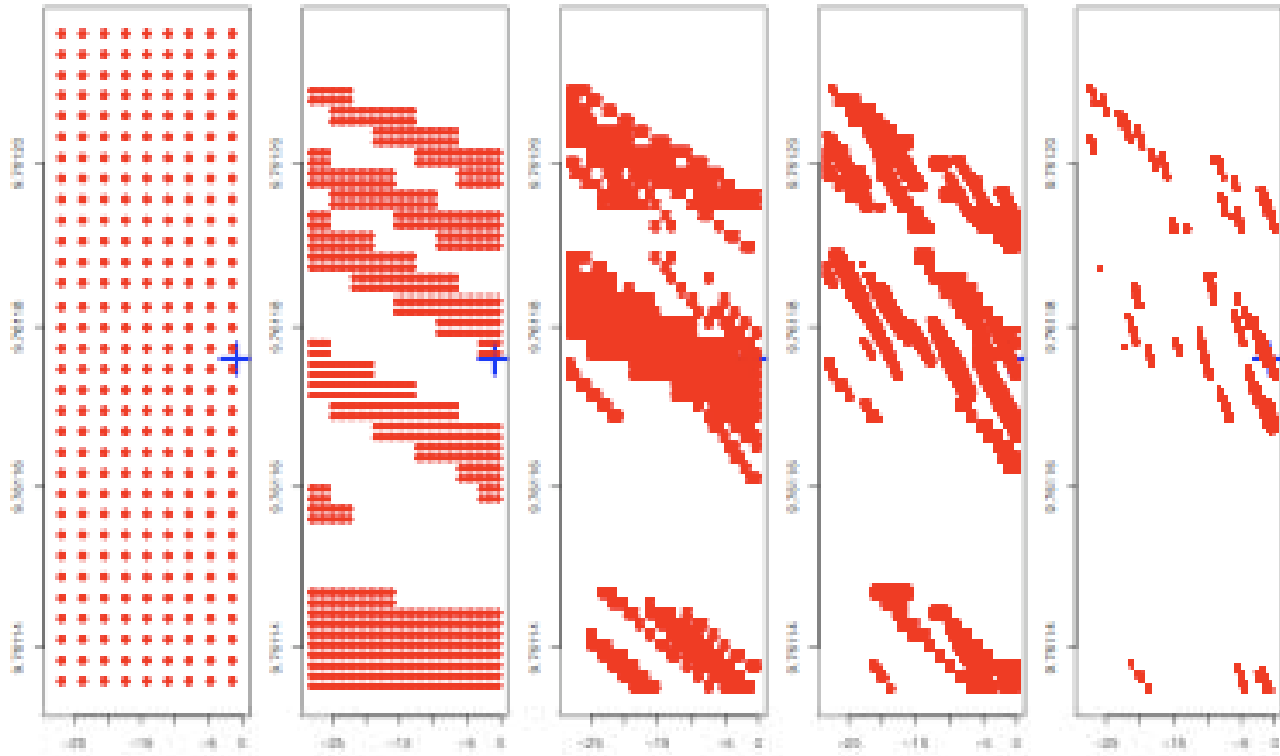
Crab: 25 blocks

Hierarchical Search

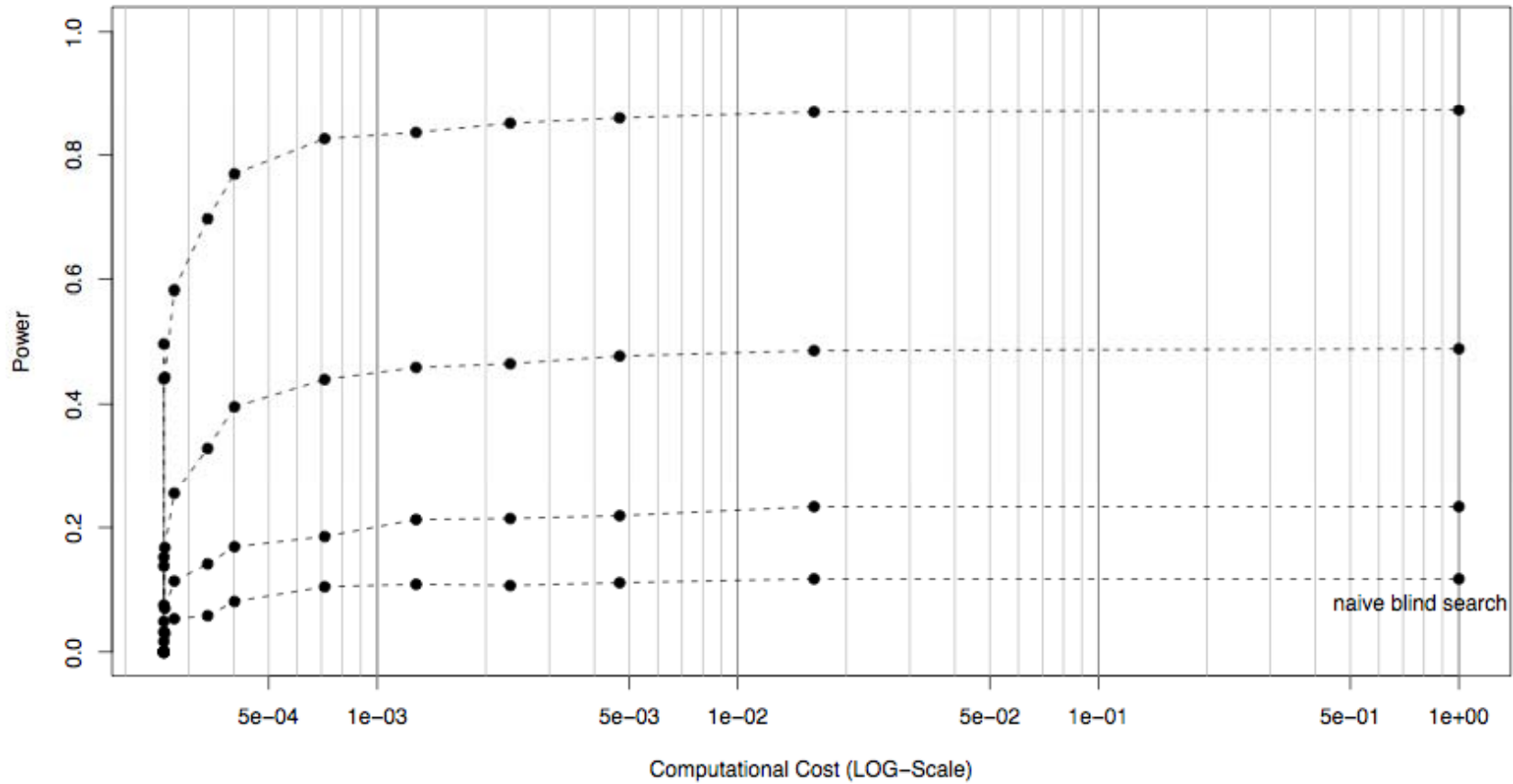
Maximize power subject to computational constraint. Optimal paths through resolution levels found by dynamic programming.



Refinement of search space (local)



Power versus cost



Simulation with 4 different signal strengths