PS Modeling

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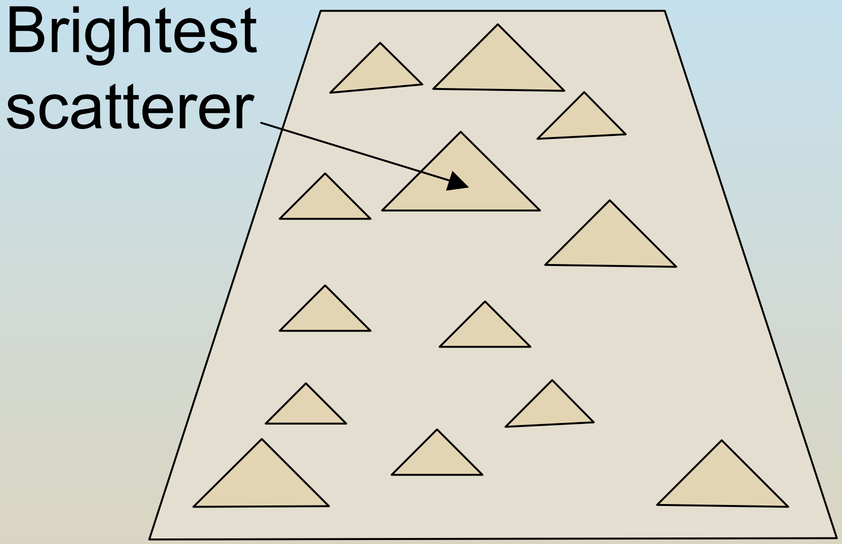
Physical scattering model



A resolution cell

- Each resel comprises many individual scatterers
- All resels have one brightest scatterer
- Brightest scatterers from a collection of resels are exponentially distributed

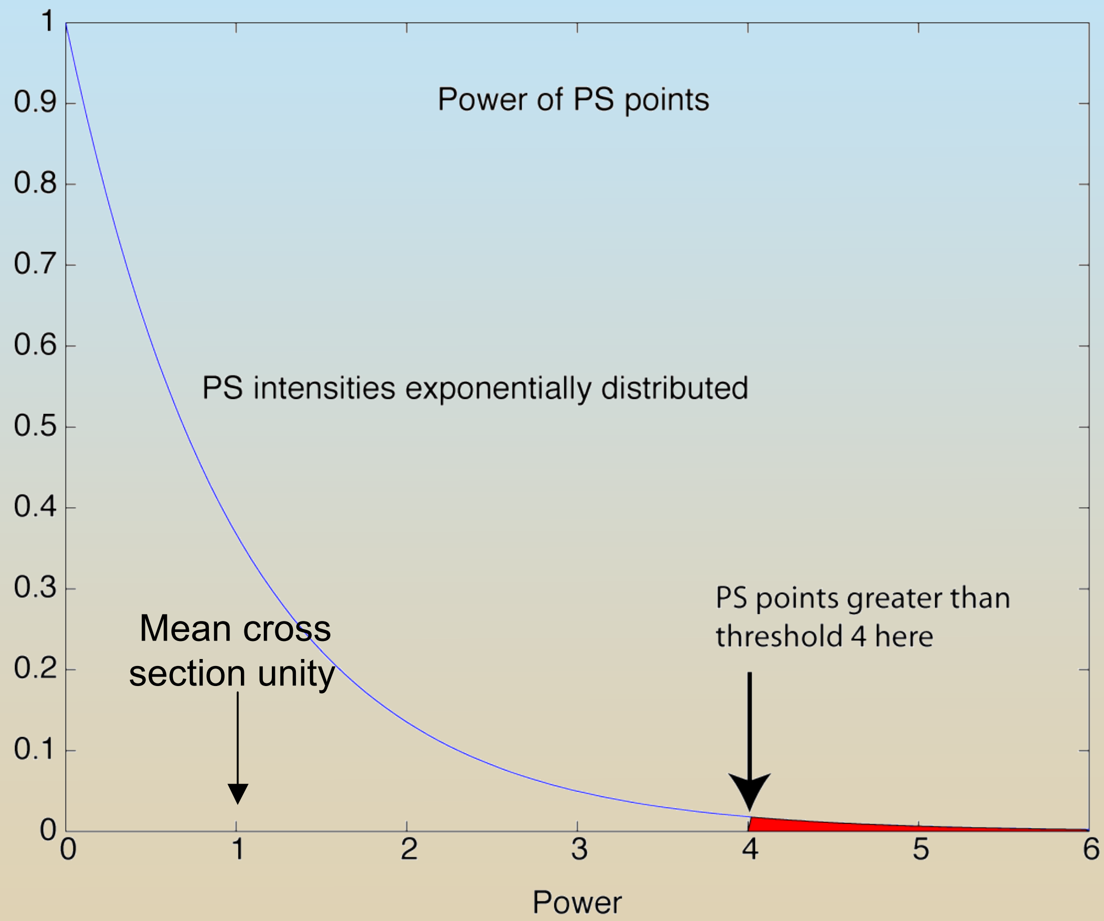
Imaging (or signal) model

- Assume pixels = resels
- Pixels are distributed targets with exponentially distributed power
- All pixels have one brightest scatterer
- Brightest scatterer is also exponentially distributed
- If brightest scatterer is brighter than background, it is a PS

Probability of PS pixels

- Assume PS brightest scatterers exponentially distributed in intensity
- Mean cross section σ_r
- Uniformly everywhere in location
- PS are pixels with cross section greater than some threshold σ_{min}

PS brightest scatterers exponentially distributed



σ_{min} - fluctuating noise model

- Single realization part σ_b (mainly the non-PS clutter in a resel)
- Fluctuating part σ_n across time series (thermal noise, decorrelation)
- Noises add

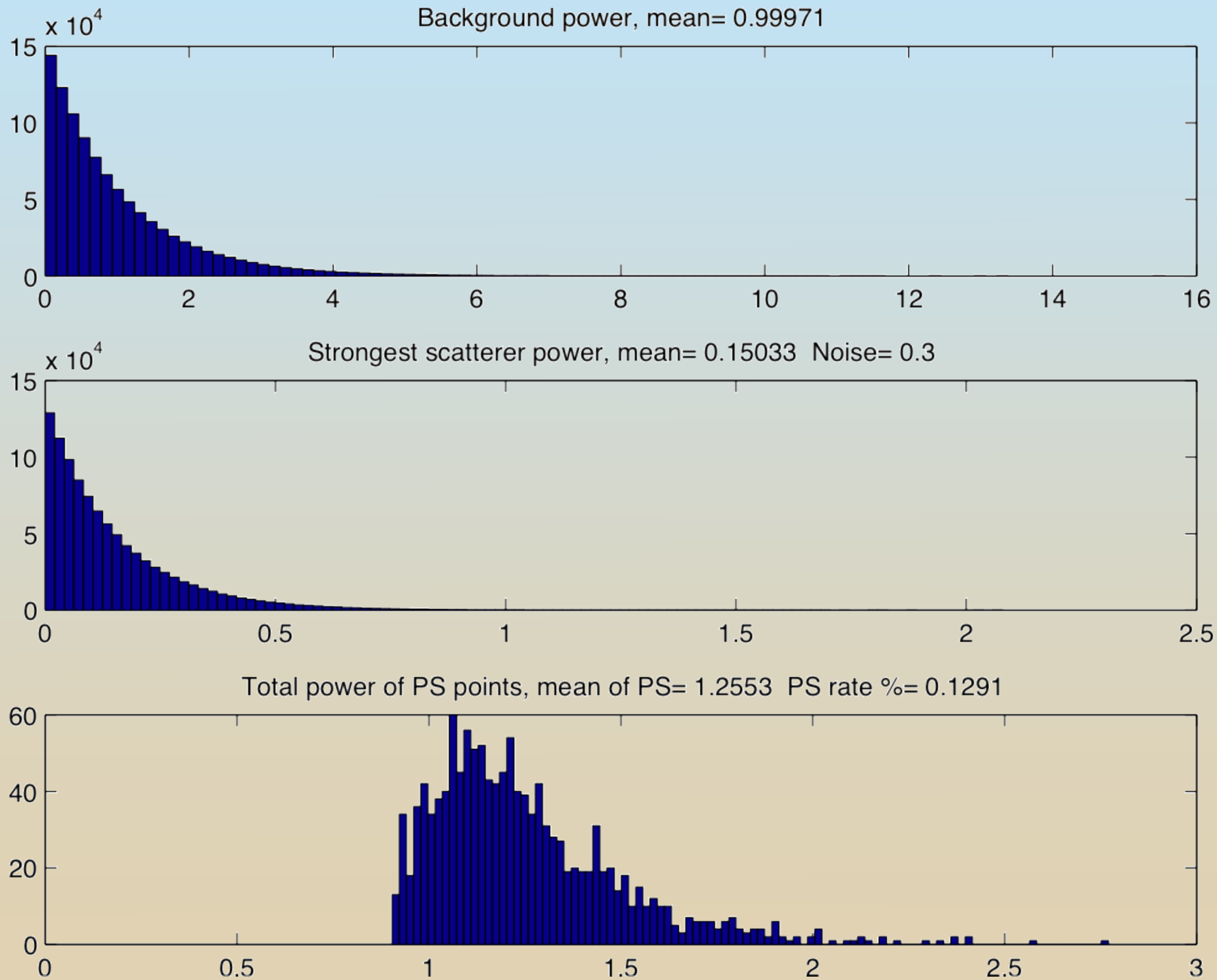
Alternate model for σ_{min}

- Background clutter σ_b exponentially distributed
- “Noise” powers σ_n also exponentially distributed across time series but one realization (single look statistics)

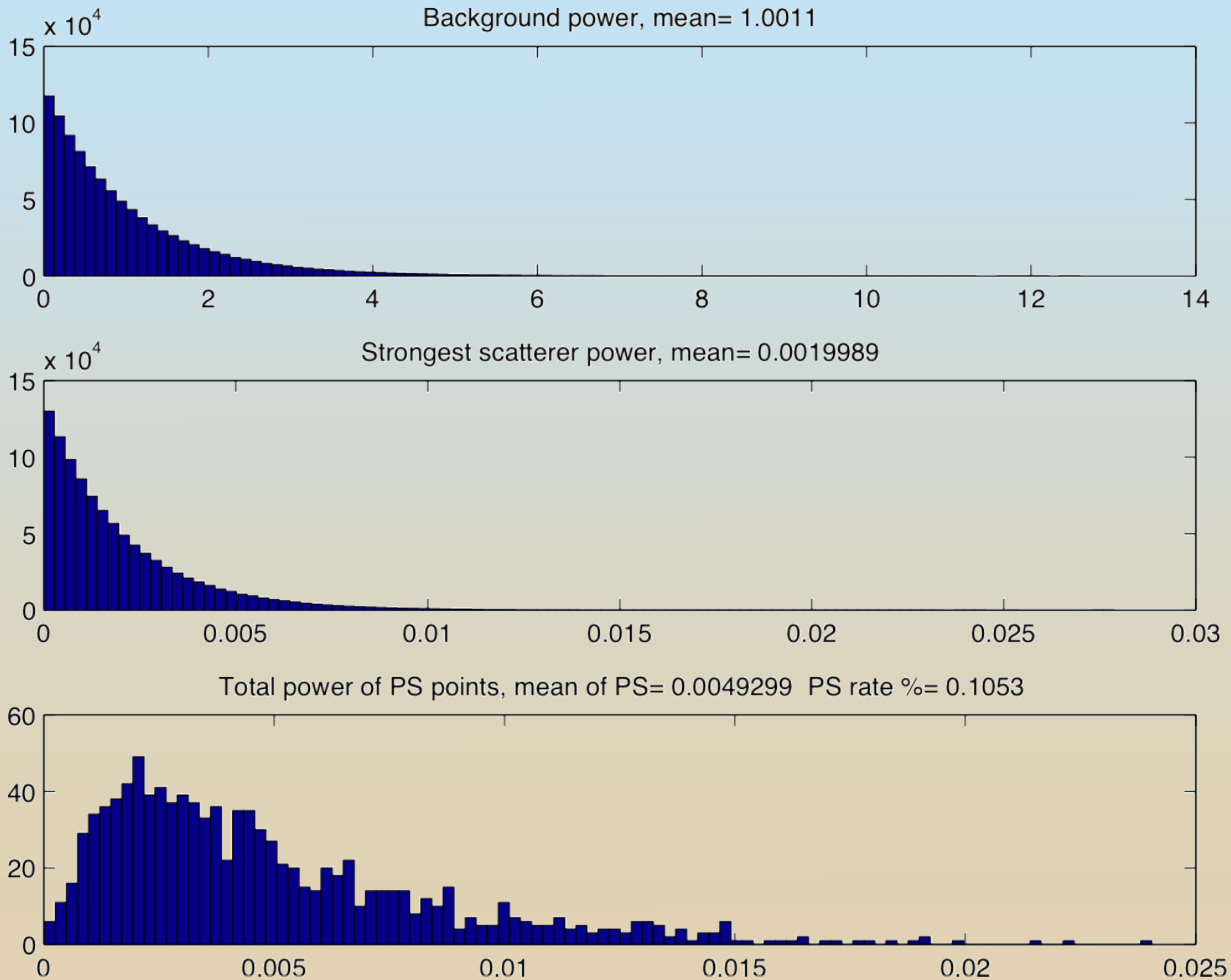
Alternate model interpretation

- In alternate model, PS points are any in which PS is brighter than background
- PS points tend to be those in which background is dark rather than where PS are bright – many more dark pixels than bright in exponential distribution

PS distribution – constant noise model

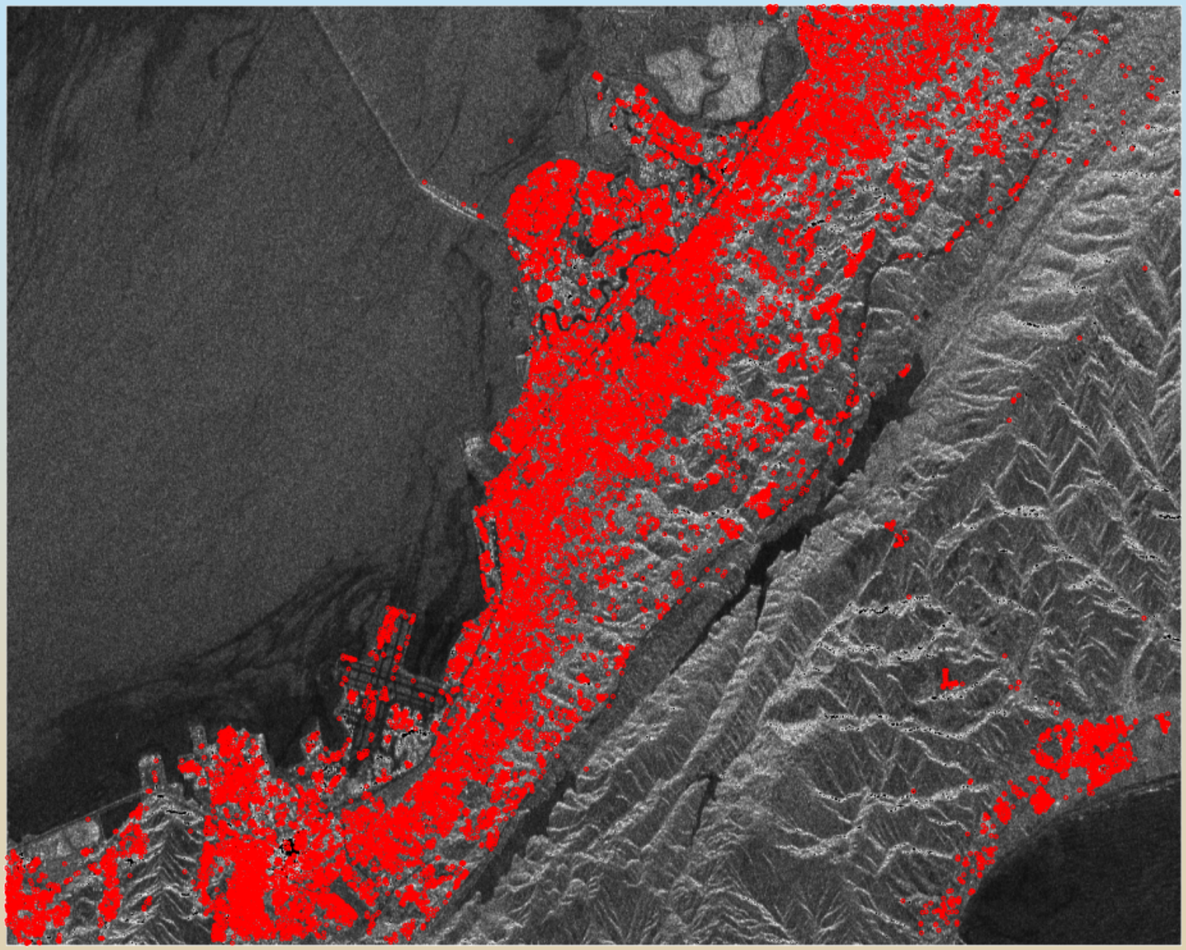


PS distribution alternate model

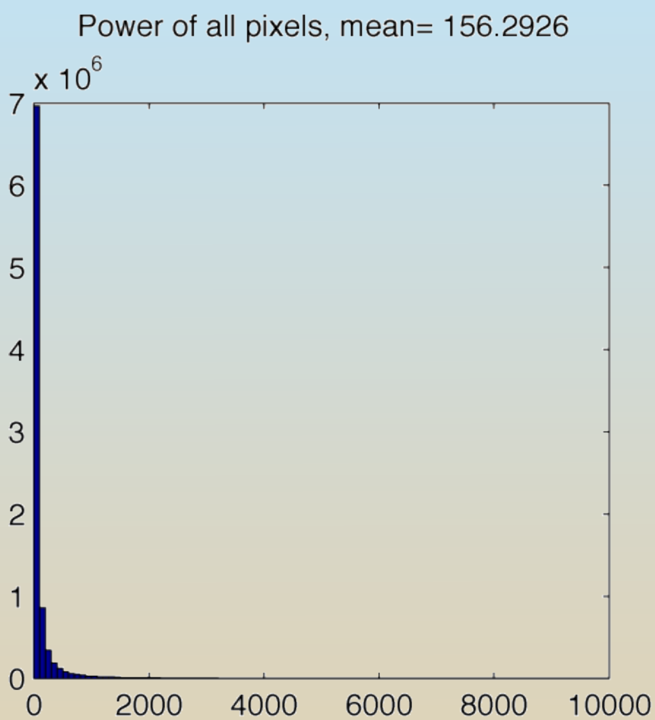


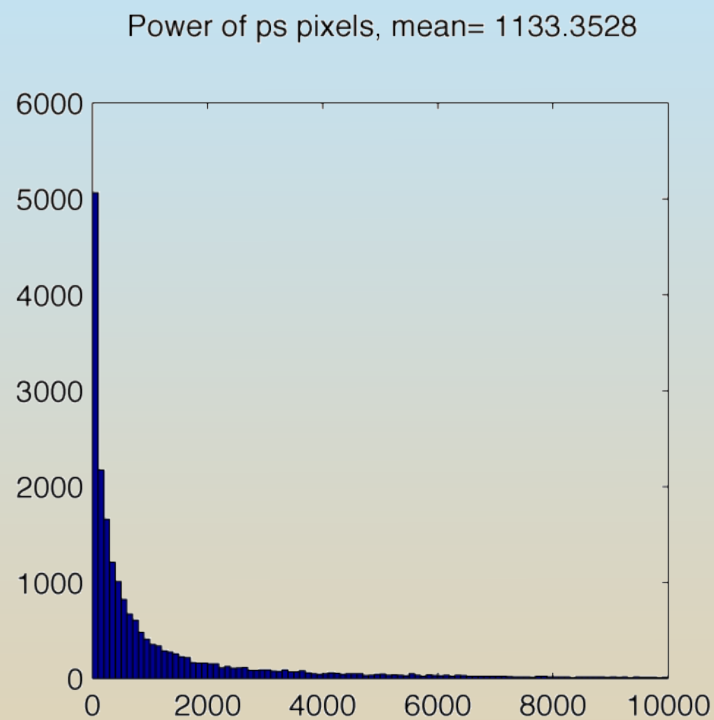
PS points found in SFO image

PS points in SFO area



Measured histograms of PS fit neither model well





But closer to initial model, esp. if we note PS from brighter region in image

Computing prob(PS) for radar systems

- Cross sections derive from equivalent sigma-zeros multiplied by pixel area

$$\sigma_n = \sigma_{0,n} \cdot A$$

$$\sigma_b = \sigma_{0,b} \cdot A$$

- $\sigma_{0,x}$ is the noise or clutter equivalent value of the normalized cross section

- Therefore, system performance depends on radar resolution as we will see below

PS algorithm

- Following Hooper, use low or bandpass filter to remove spatially uncorrelated signal
- Filter size approx. one atmosphere correlation length
- Residual PS points have standard deviation less than atmosphere over filter

Velocity error - what we often want to measure

- Vandermonde matrix of measurement times:

$$x = \begin{bmatrix} 1 & 0 \\ 1 & x_1 \\ 1 & \ldots \\ 1 & x_{N-1} \end{bmatrix}$$

- For equal weights

$$y = \left(x^T x\right)^{-1}$$

- Time series error

$$\sigma_{PSvelocity} = \sqrt{y(2,2)} \cdot \sigma_{PS}$$

Adding noises

- Thermal SNR: compute signal to noise ratio from the radar equation:

$$BW = c/(2\sin(\theta)\ \delta_{range})$$

$$A_{ground} = [c \bullet P_{len}/(2\sin(\theta))]\ \ [R\lambda/L_{ant}]$$

$$\gamma_{thermal} = P_t G_t/(4\pi R^2) * A_{ground} * A_r/(4\pi R^2)\ /(k * T_{sys} * BW)$$

- Equivalent noise $P_{thermal}$ is:

$$P_{thermal} = 1/\gamma_{thermal}$$

Spatial and temporal decorrelation

- Similarly, for spatial decorrelation we have

$$B_{crit} = \lambda R/(2\delta_{range}\cos^2(\theta))$$

$$\rho_{spatial} = 1\text{-}B_{nominal}/B_{crit}$$

$$P_{spatial} = 1/\rho_{spatial}\text{-}1$$

- Temporal decorrelation, assuming exponential decay

$$\rho_{temporal} = \exp(-T_{repeat}/T_{decorr})$$

$$P_{temporal} = 1/\rho_{temporal}\text{-}1$$

Total effective noise power

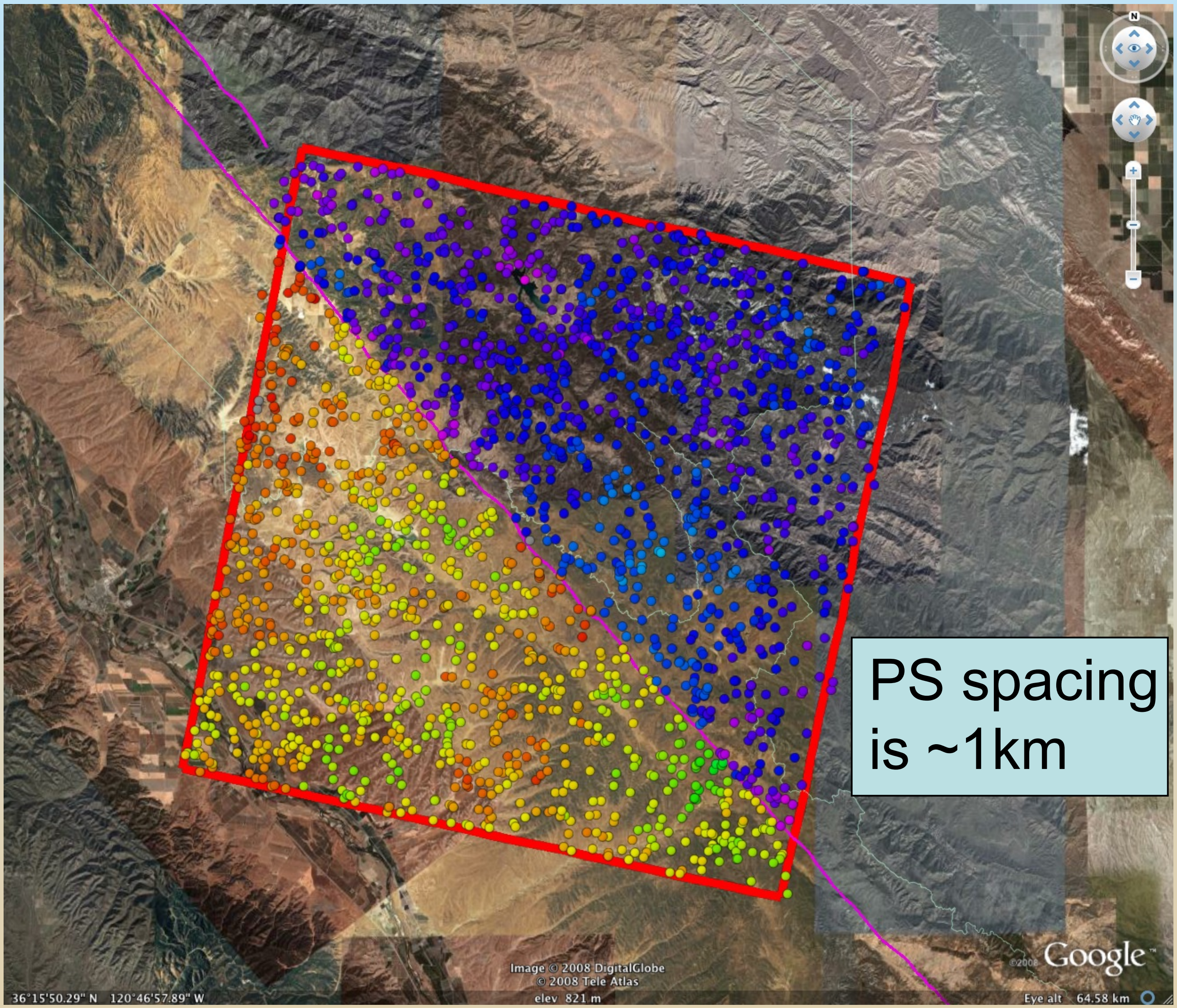
- Total effective noise power is

$$P_{total} = P_{\gamma} + P_{thermal} + P_{spatial} + P_{temporal}$$

- Effective total signal to noise for the probability calculation above is

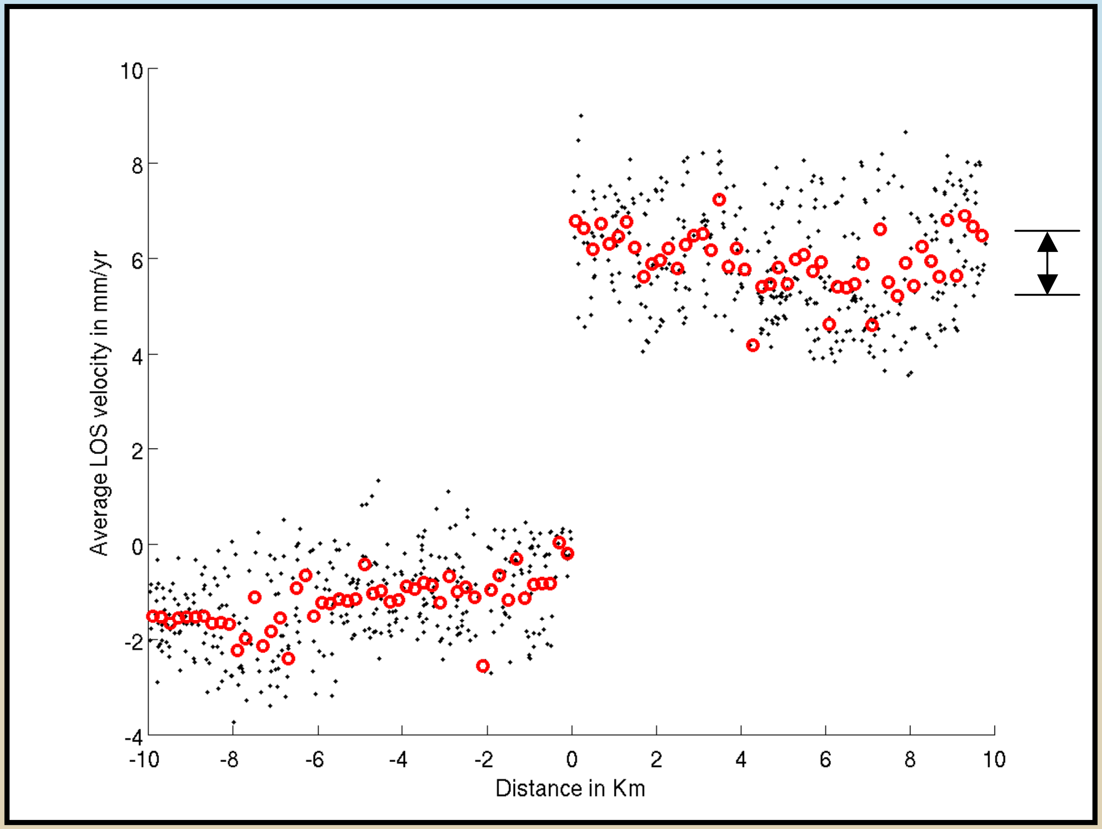
$$\gamma_{total} = 1/P_{total}$$

PS image of San Andreas Fault - ERS satellite



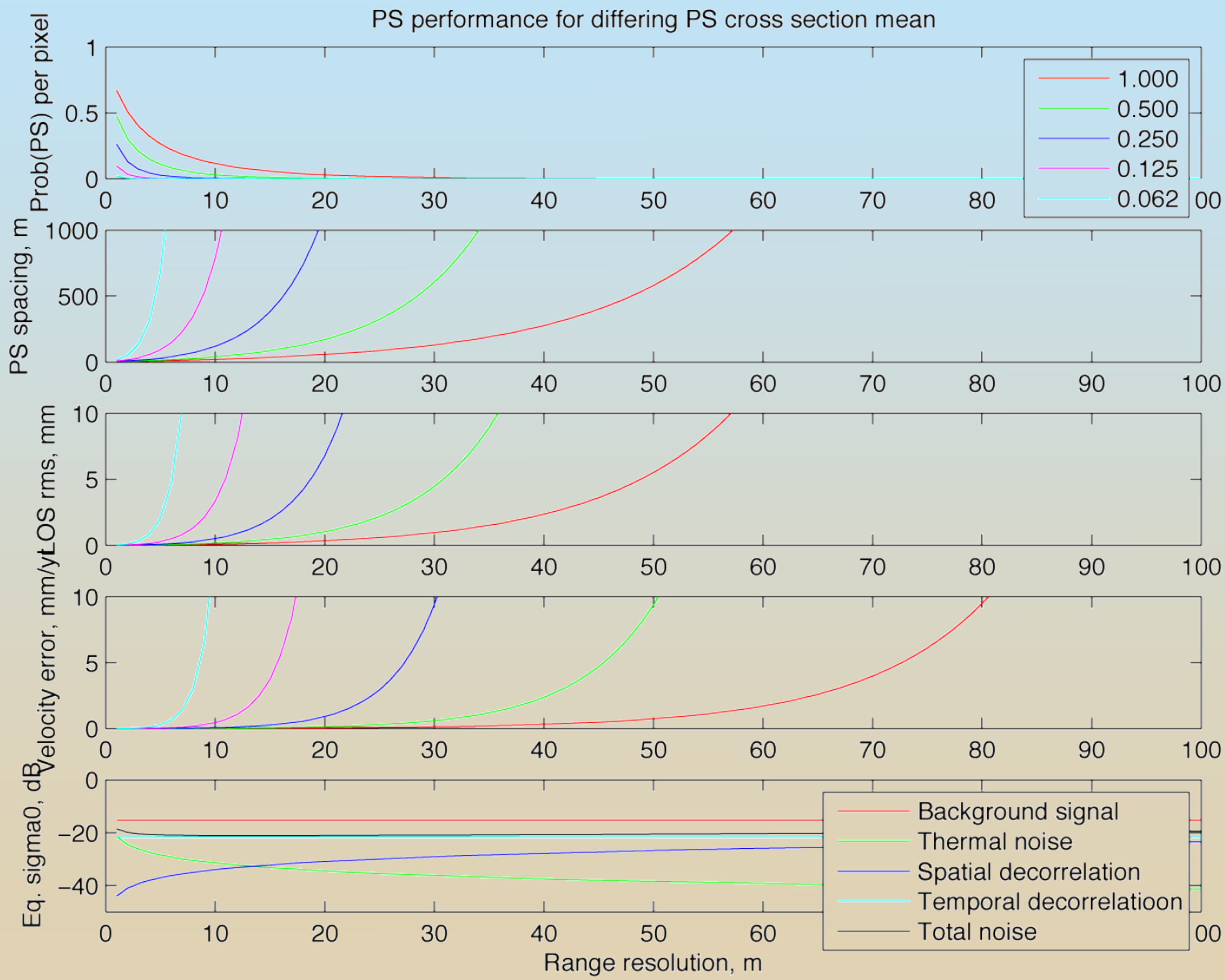
PS spacing is ~1km

PS performance over San Andreas Fault - ERS satellite

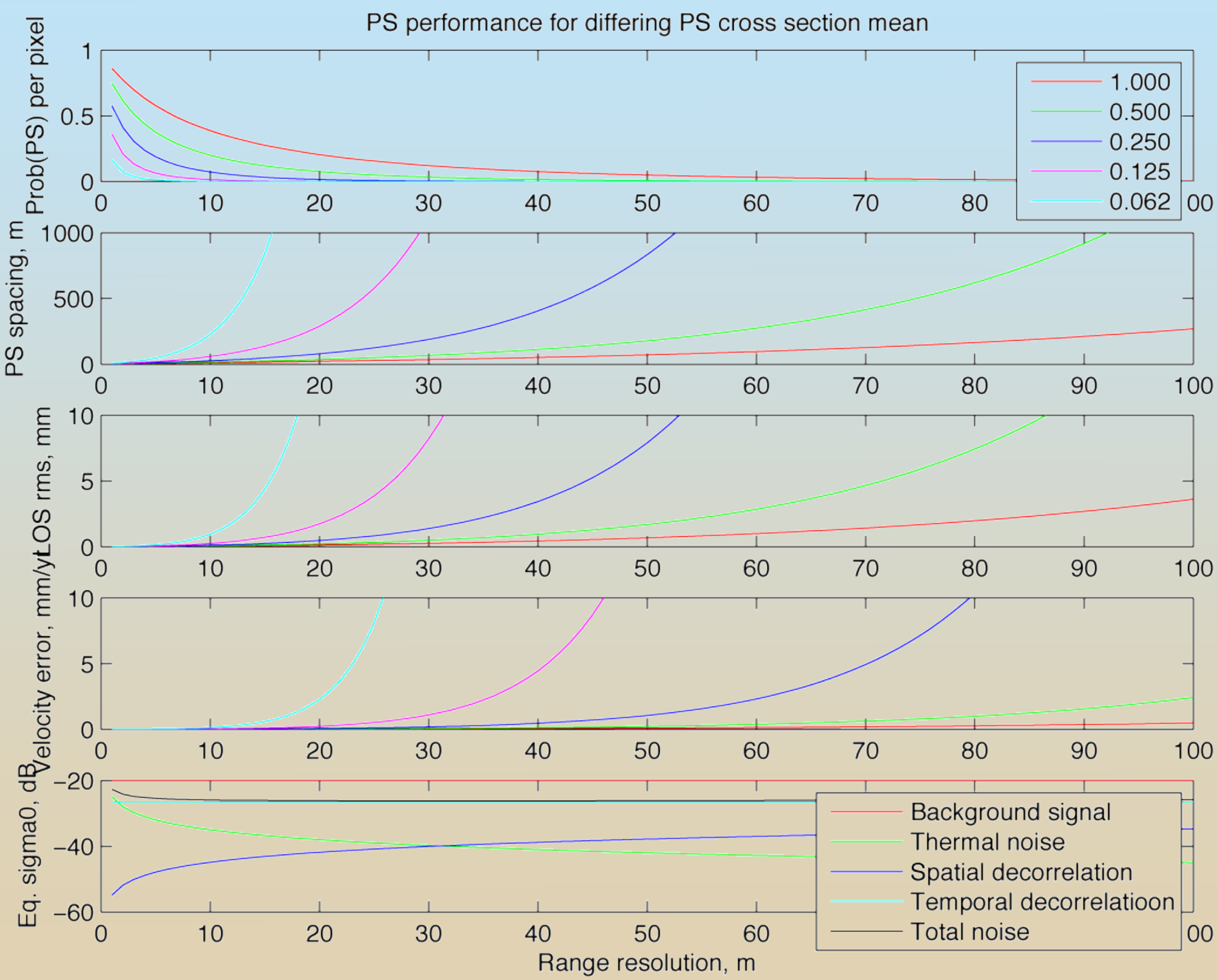


RMS error ~1 mm/yr

PS performance for ERS - a graph only an engineer could love



PS peformance for DESDynI

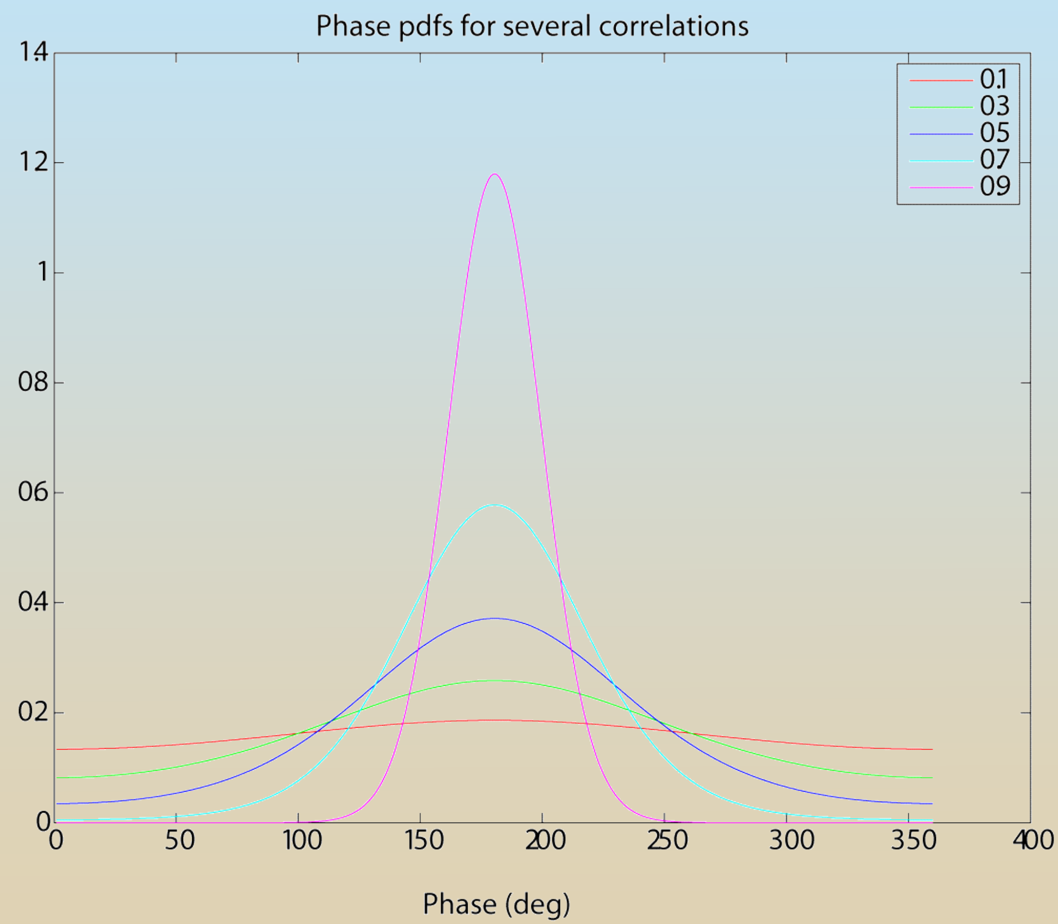


Ongoing work – Detection theory

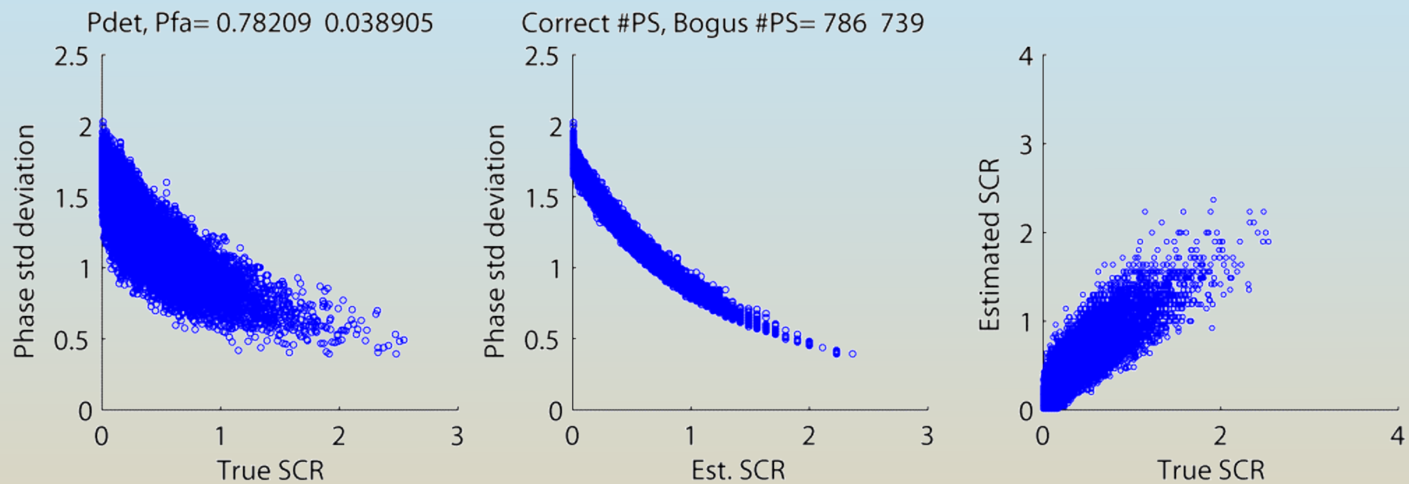
- We will not find all PS points in an image, as there is noise
- Again, model each pixel as containing a random background+noise plus a single coherent point
- So each pixel is possible PS
- How many of these will we find?

PDF distributions for interferograms

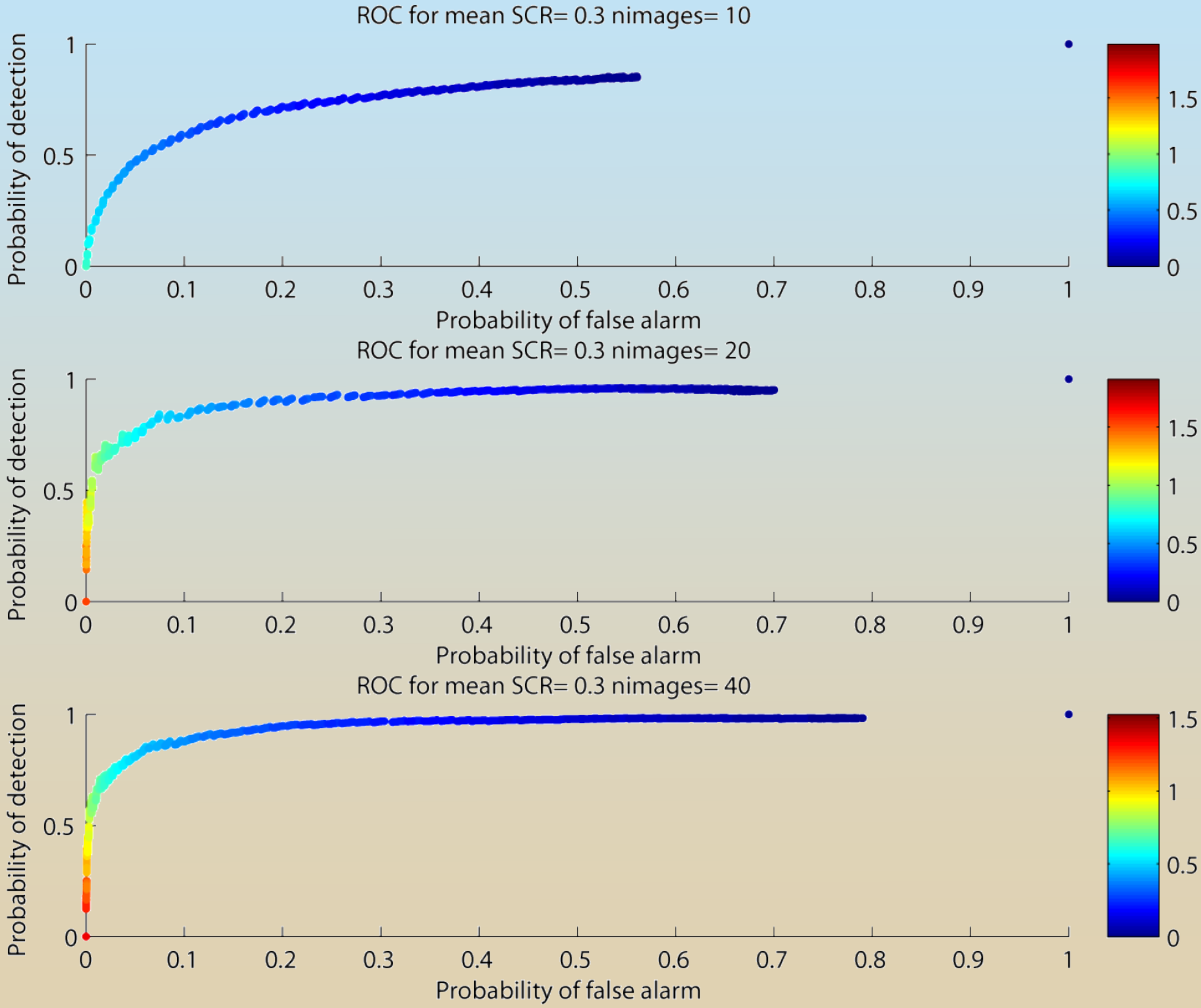
- Compute interferogram pdf from two SLC pdfs and assume independence of phases



Phase noise in PS pixels



Compute probability of false alarm and detection



Compare ROC curves