Intro to Soil Mechanics: the what, why & how

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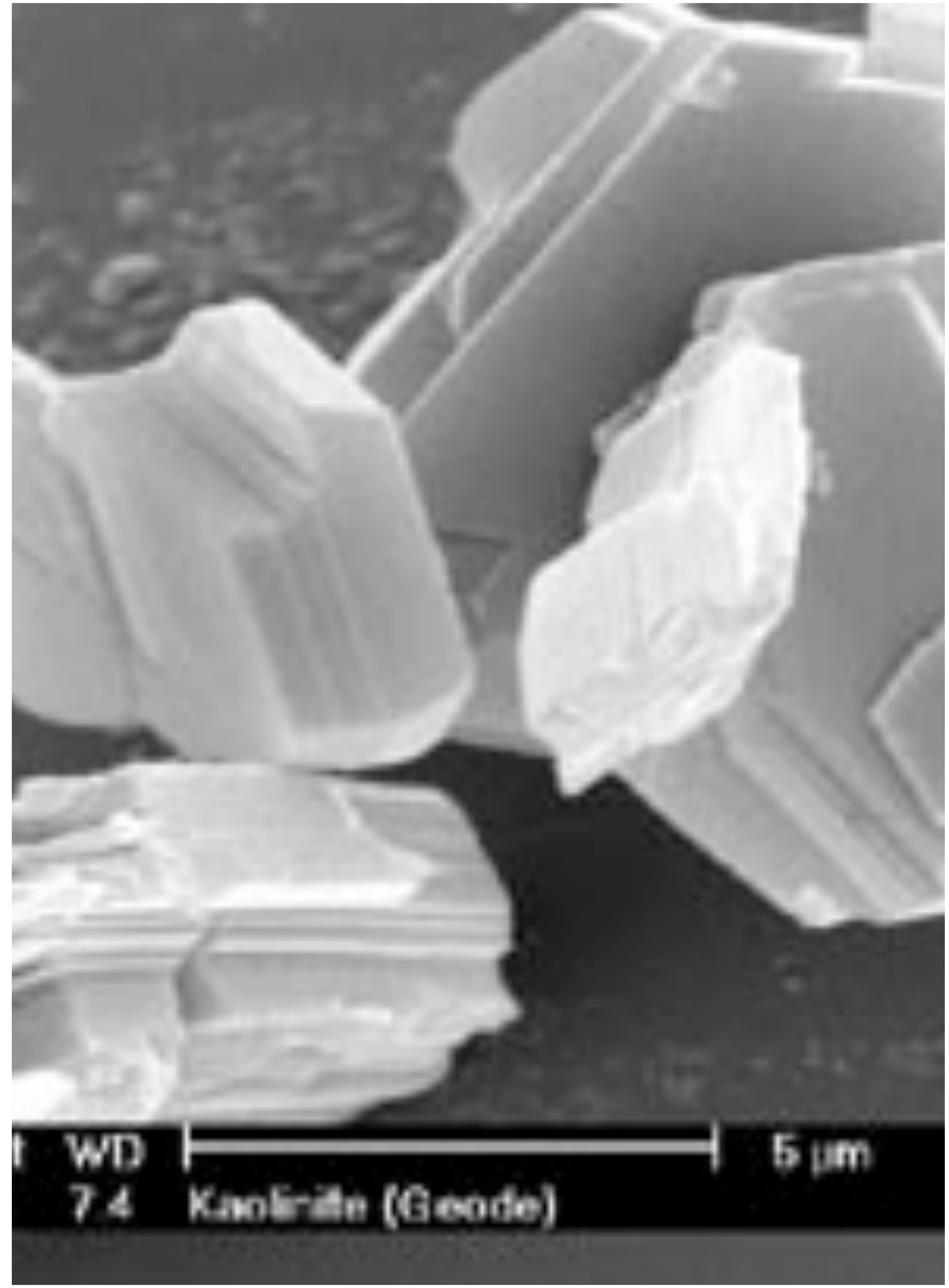
The What?

What is Soil Mechanics? *erdbaumechanik*

The application of the laws of mechanics (physics) to soils as engineering materials

Karl von Terzaghi is credited as the father of erdbaumechanik

sands & gravels



clays & silts

The Why?

Sandcastles | what holds them up?

Palacio de Bellas Artes
Mexico, DF

uniform settlement

The leaning tower of Pisa

differential settlement

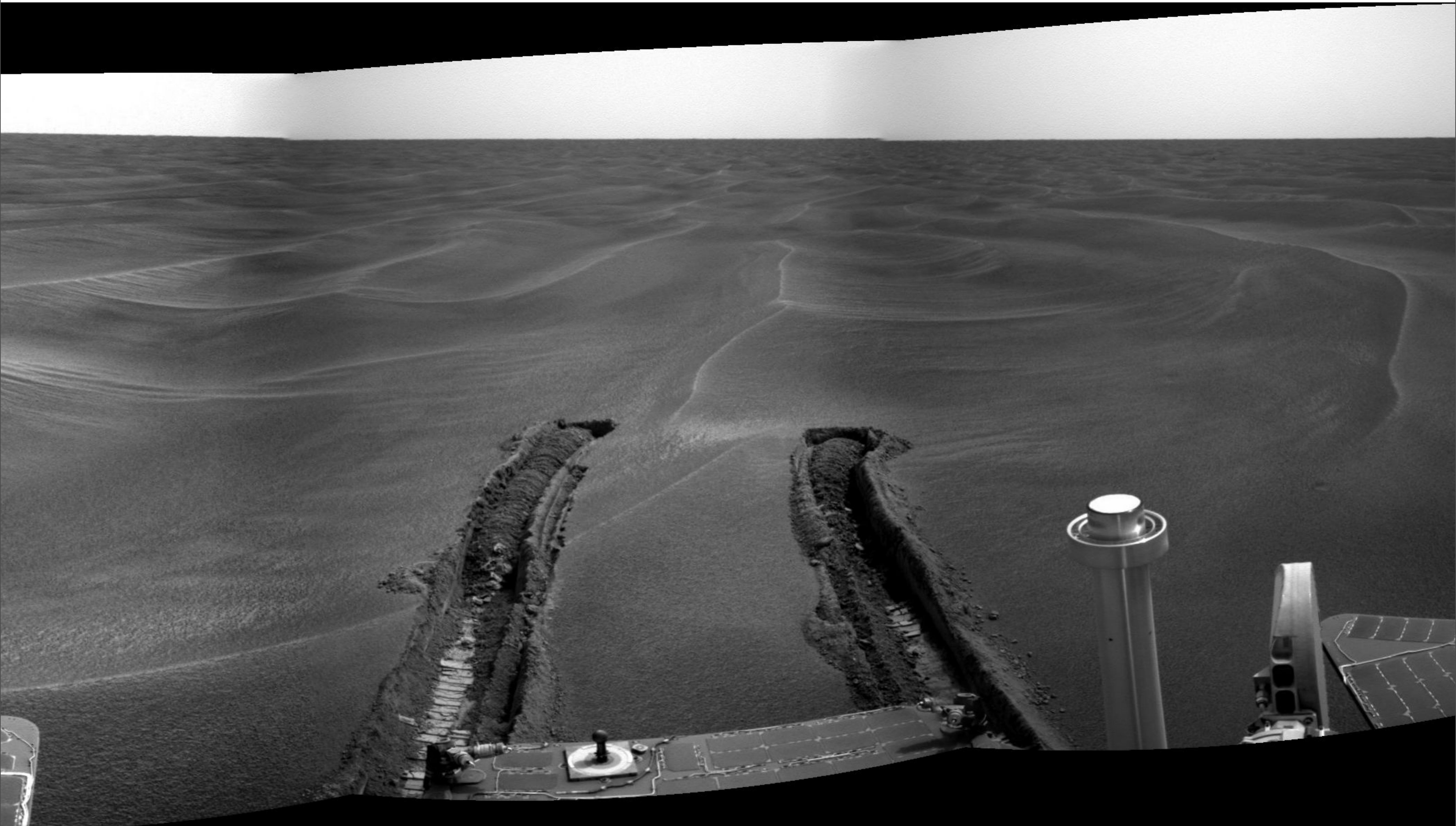
Teton dam | dam failure

Niigata earthquake

liquefaction

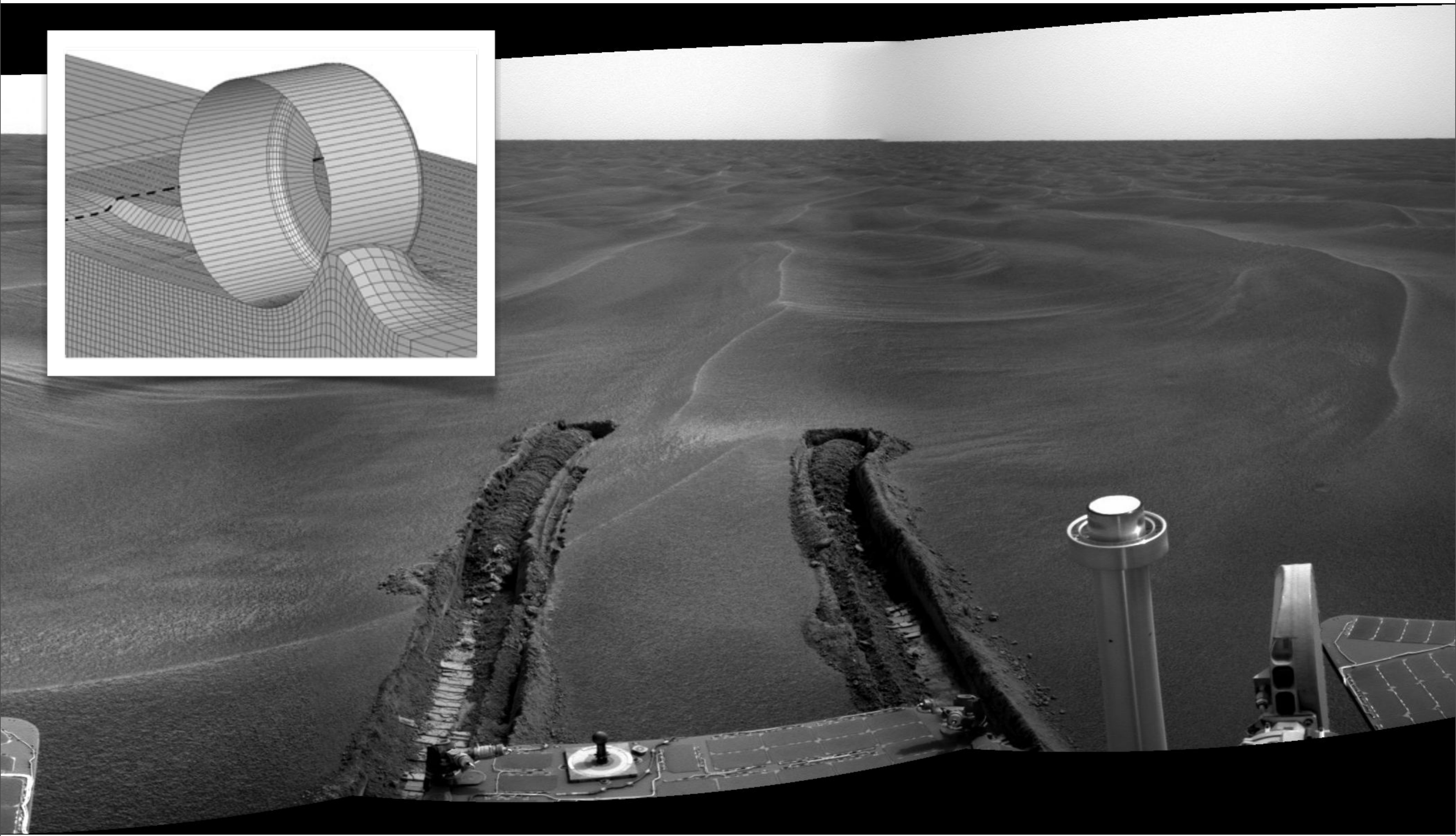
Katrina
New Orleans

levee failure



MER: Big Opportunity

xTerramechanics



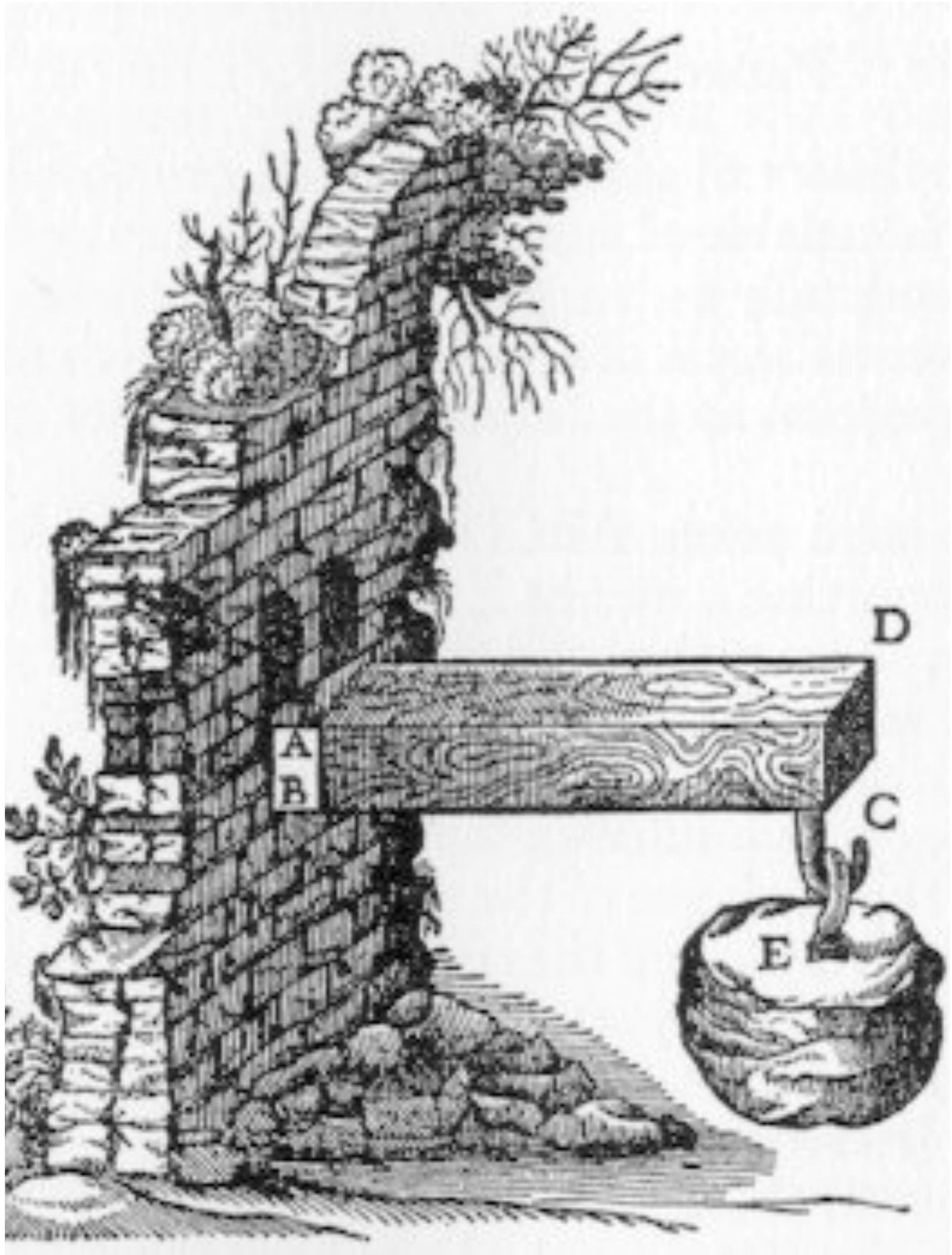
MER: Big Opportunity

xTerramechanics

The How?

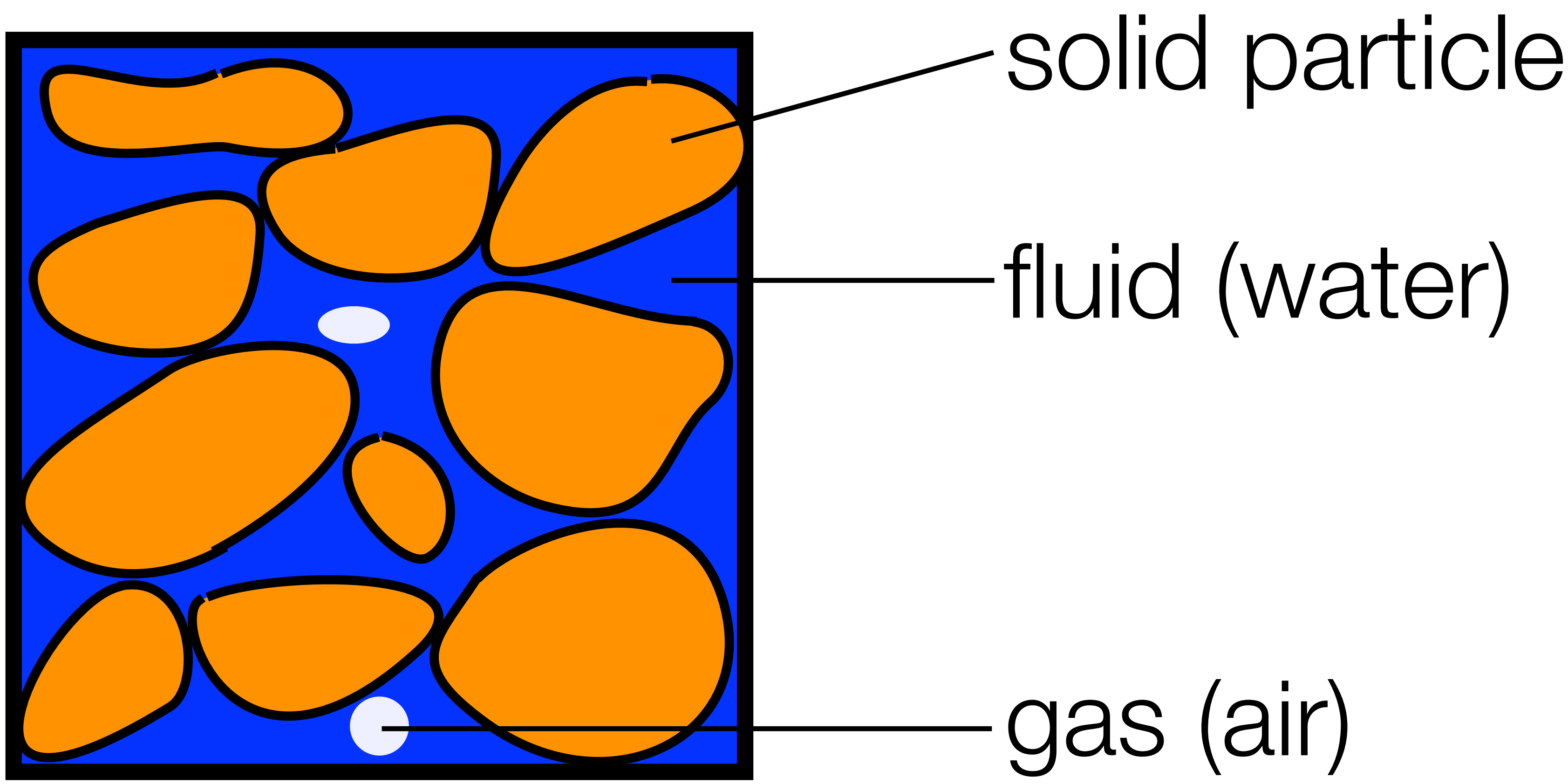
Topics in classic Soil Mechanics

- Index & gradation
- Soil classification
- Compaction
- Permeability, seepage, and effective stresses
- Consolidation and rate of consolidation
- Strength of soils: sands and clays



Index & gradation

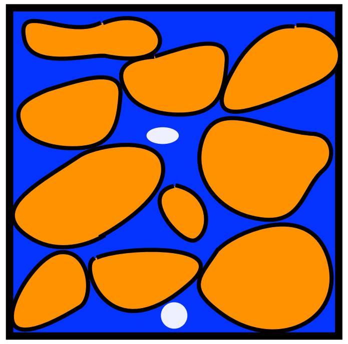
Definition: soil mass is a collection of particles and voids in between (voids can be filled w/ fluids or air)



Each phase has volume and mass

Mechanical behavior governed by phase interaction

Index & gradation



solid

water+air=voids

Key volumetric ratios

$$e = \frac{V_v}{V_s}$$

void ratio

[0.4,1] sand

[0.3,1.5] clays

$$\eta = \frac{V_v}{V_t}$$

porosity

[0,1]

$$S = \frac{V_w}{V_v}$$

saturation

[0,1]

Key mass ratio

$$w = \frac{M_w}{M_s}$$

water content

<1 for most soils

>5 for marine, organic

Key link mass & volume

$$\rho = M/V$$

moist, solid, water, dry, etc.

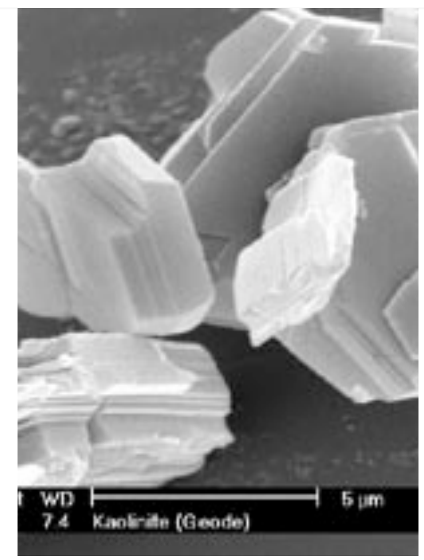
ratios used in practice to characterize soils & properties

Gradation & classification

Grain size is main classification feature

sands & gravels

- can see grains
- mechanics~texture
- d>0.05 mm



clays & silts

- cannot see grains
- mechanics~water
- d<0.05 mm

Soils are currently classified using USCS (Casagrande)

Fabric in coarsely-grained soils

“loose packing”, high e
“dense packing”, low e
} relative

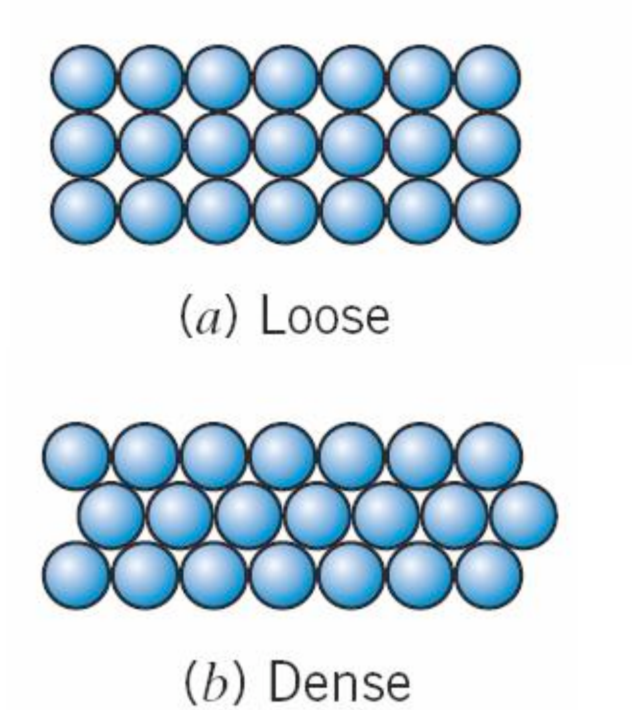
$$e = \frac{V_v}{V_s}$$

e_{max} greatest possible, loosest packing

e_{min} lowest possible, densest packing

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$ relative density

strongly affects engineering behavior of soils

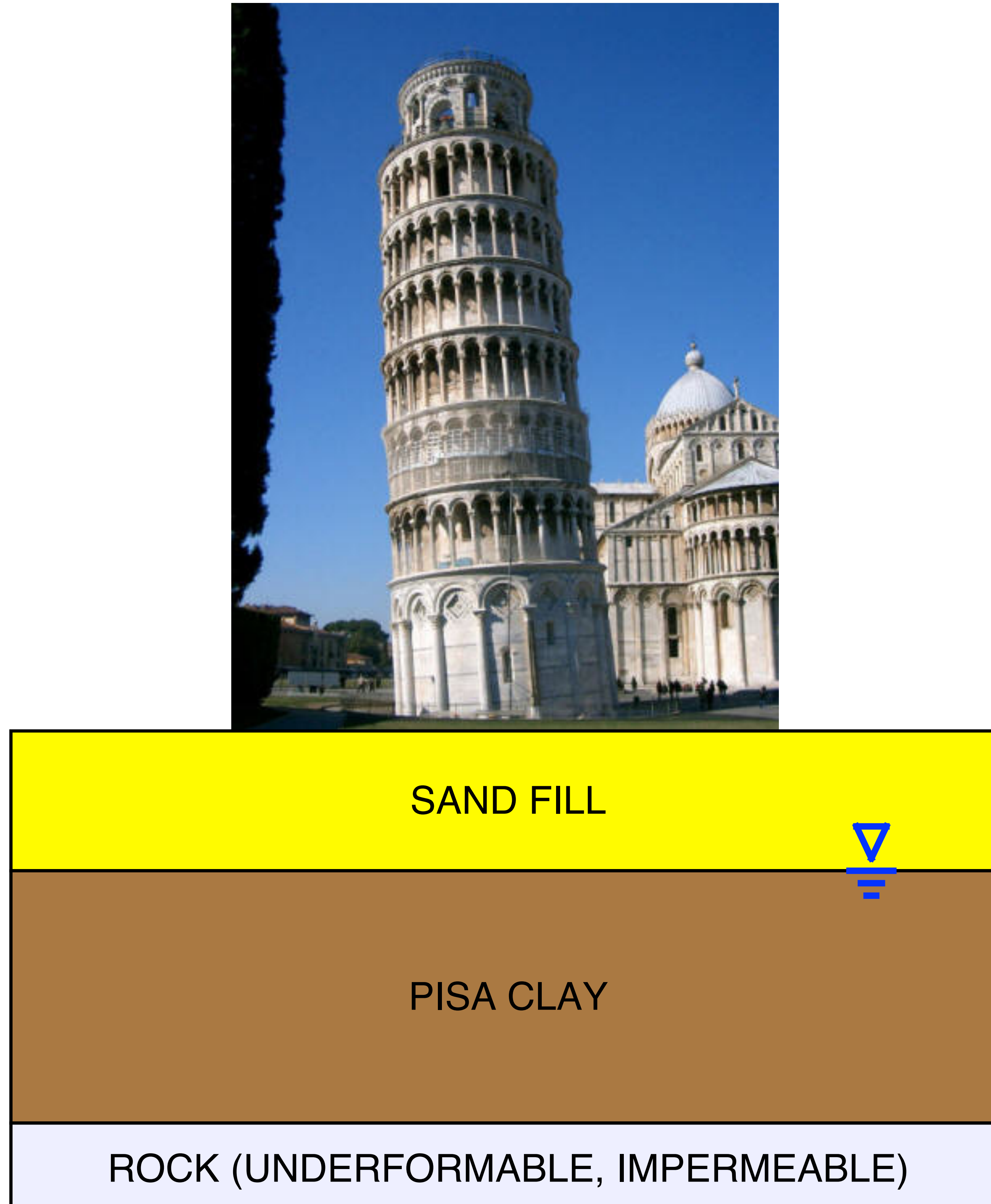


(*a*) Loose

(*b*) Dense

Typical problem(s)xb in Soil Mechanics

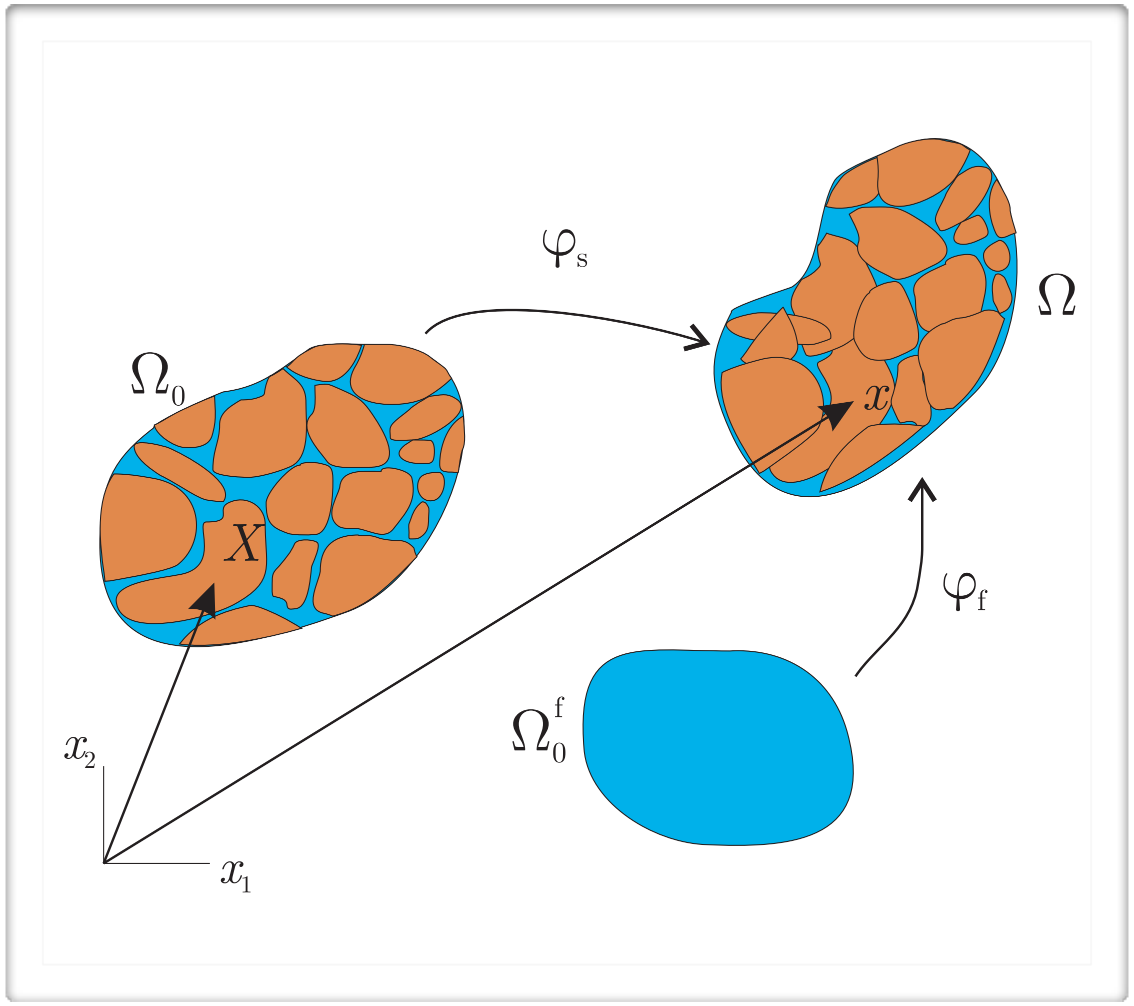
- Compact sand fill
- Calculate consolidation of clay
- Calculate rate of consolidation
- Determine strength of sand
- Calculate F.S. on sand (failure?)
- Need: stresses & matl behavior



Modeling tools

Theoretical framework

- continuum mechanics
- constitutive theory
- computational inelasticity
- nonlinear finite elements



Theoretical framework

- continuum mechanics
- constitutive theory
- computational inelasticity
- nonlinear finite elements

$$\overbrace{\phi \frac{\dot{p}}{K_f} + \nabla \cdot \boldsymbol{v} \quad = \quad -\nabla \cdot \boldsymbol{q}}^{\text{balance of mass}}$$

$$\underbrace{\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{\gamma} \quad = \quad \mathbf{0}}_{\text{balance of momentum}}$$

Theoretical framework

- continuum mechanics
- constitutive theory
- computational inelasticity
- nonlinear finite elements

$$\boldsymbol{q} \quad = \quad \boldsymbol{k} \cdot \nabla h \longleftarrow \text{darcy}$$

$$\dot{\boldsymbol{\sigma}}' \quad = \quad \boldsymbol{c}^{\mathrm{ep}} : \dot{\boldsymbol{\epsilon}} \longleftarrow \text{hooke}$$

$\boldsymbol{k}$ permeability tensor

controls fluid flow

$\boldsymbol{c}^{\mathrm{ep}}$ mechanical stiffness

controls deformation

Theoretical framework

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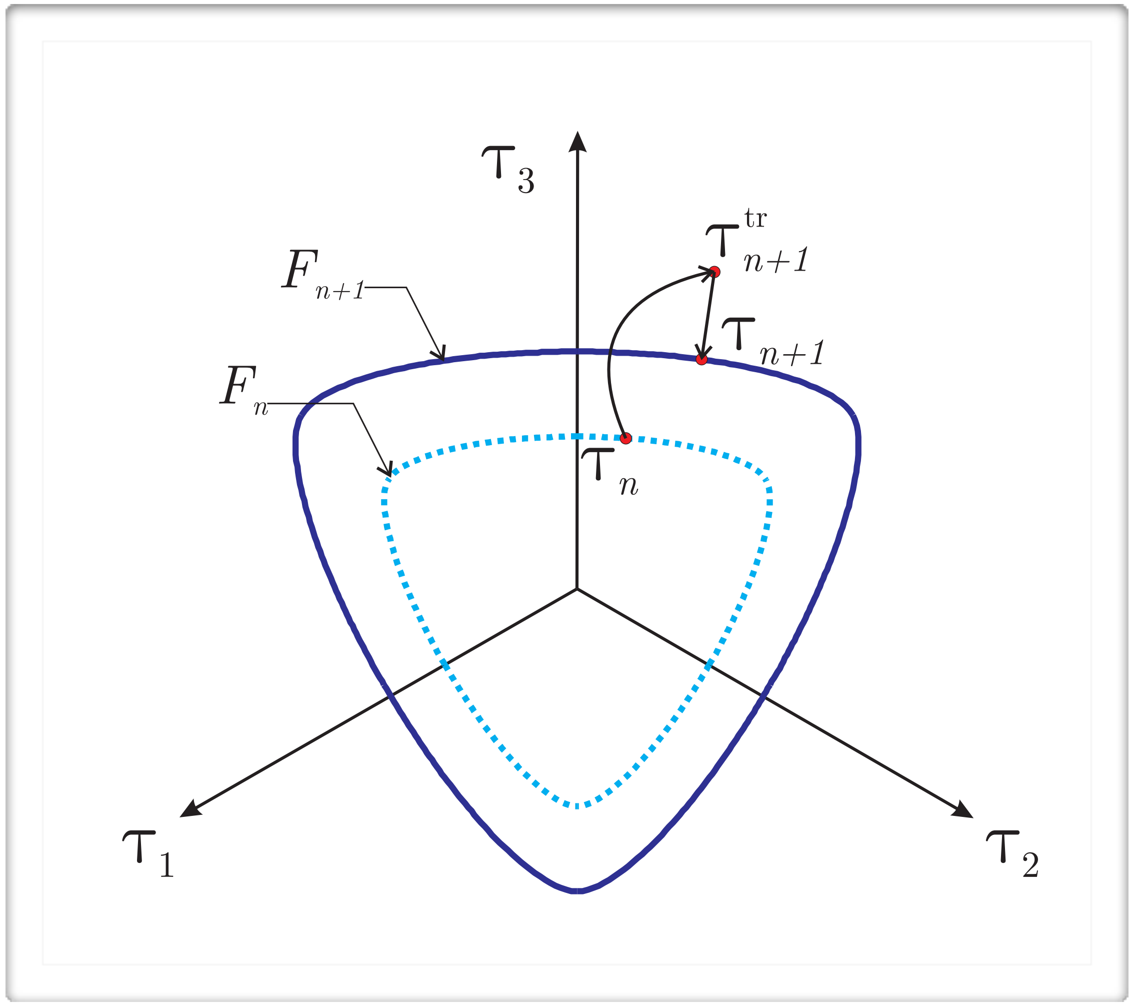

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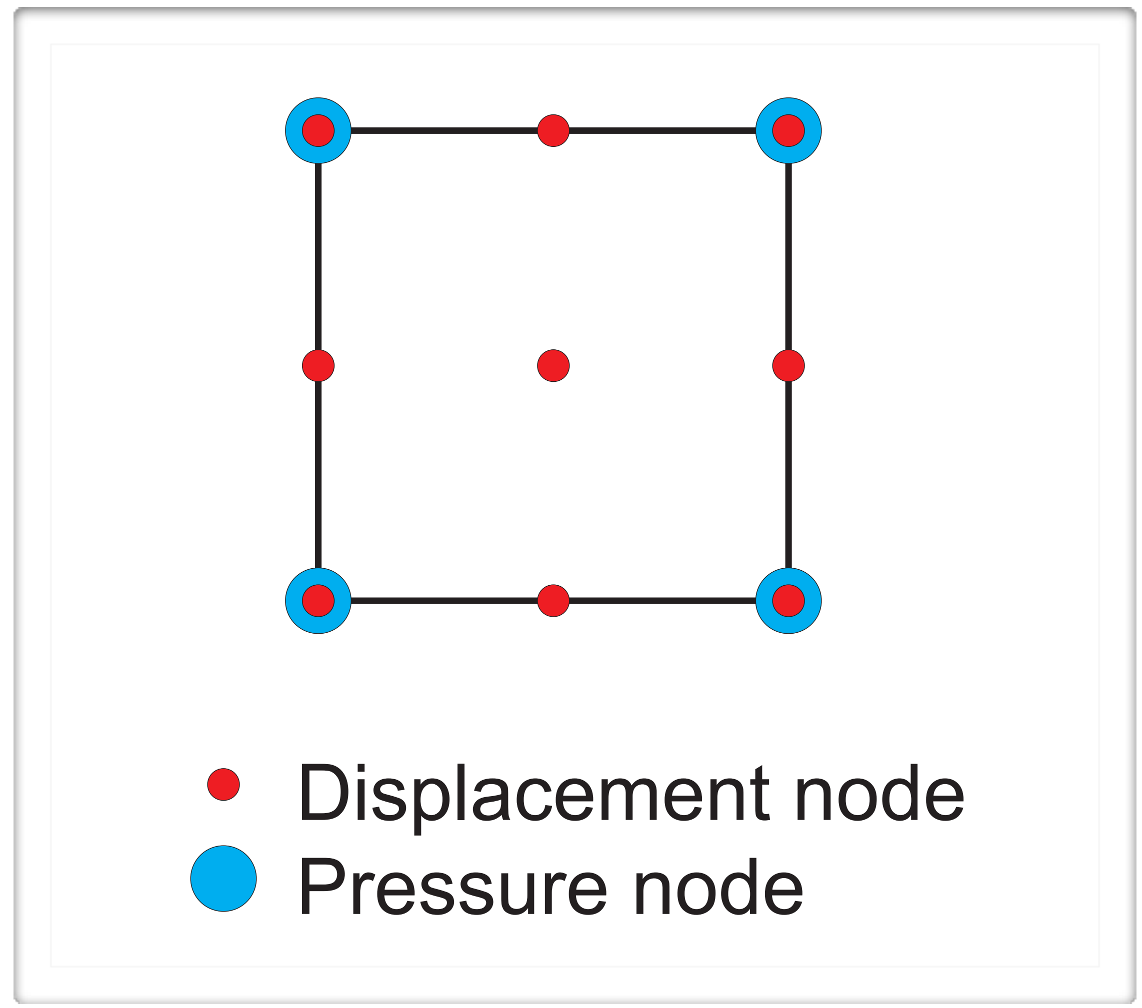
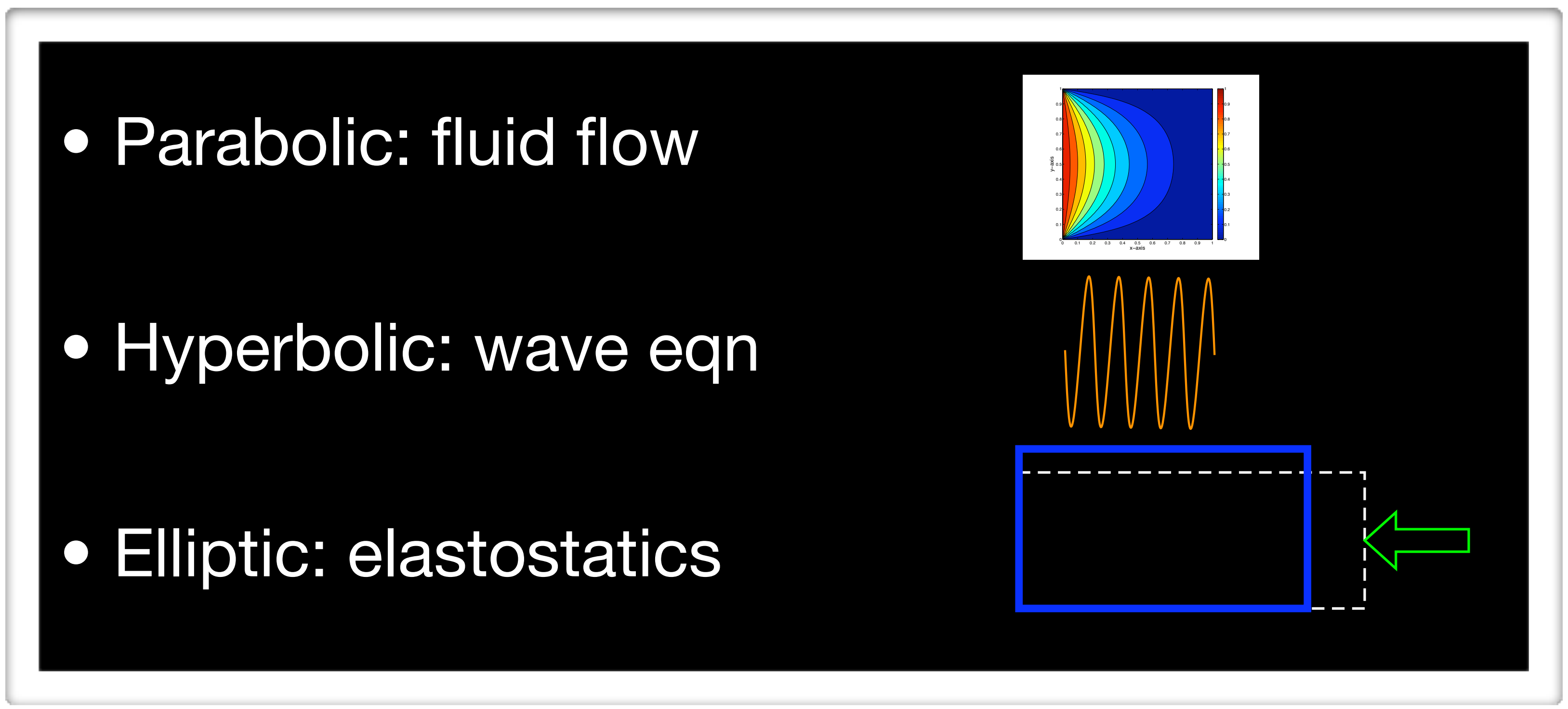


Finite Element Method (FEM)

- Designed to *approximately* solve PDE's
- PDE's model physical phenomena
- Three types of PDE's:



FEM recipe

Strong from

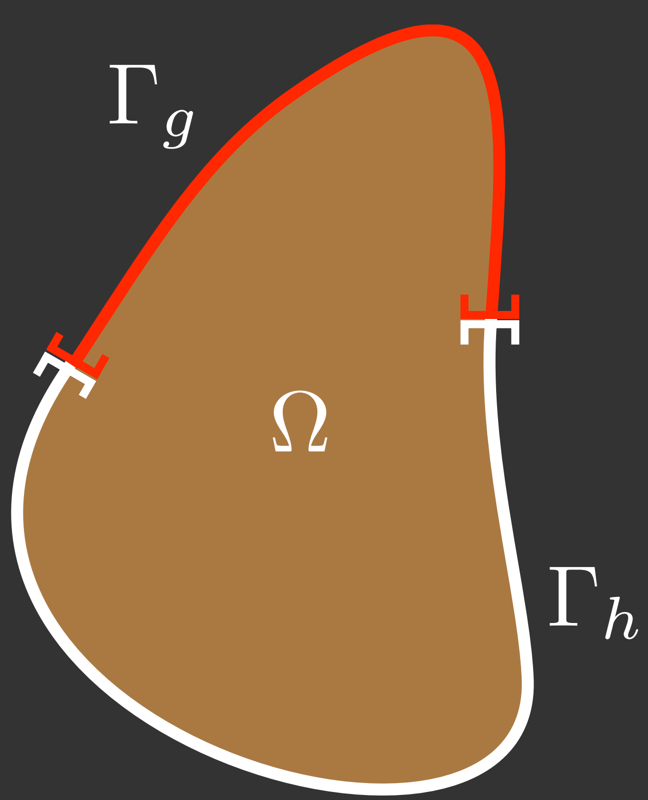
Weak form

Galerkin form

Matrix form

Multi-D deformation with FEM

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f} &= \boldsymbol{0} \quad \text{in } \Omega && \longleftarrow \text{equilibrium} \\ \boldsymbol{u} &= \boldsymbol{g} \quad \text{on } \Gamma_g && \longleftarrow \text{e.g., clamp} \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} &= \boldsymbol{h} \quad \text{on } \Gamma_h && \longleftarrow \text{e.g., confinement} \end{aligned}$$



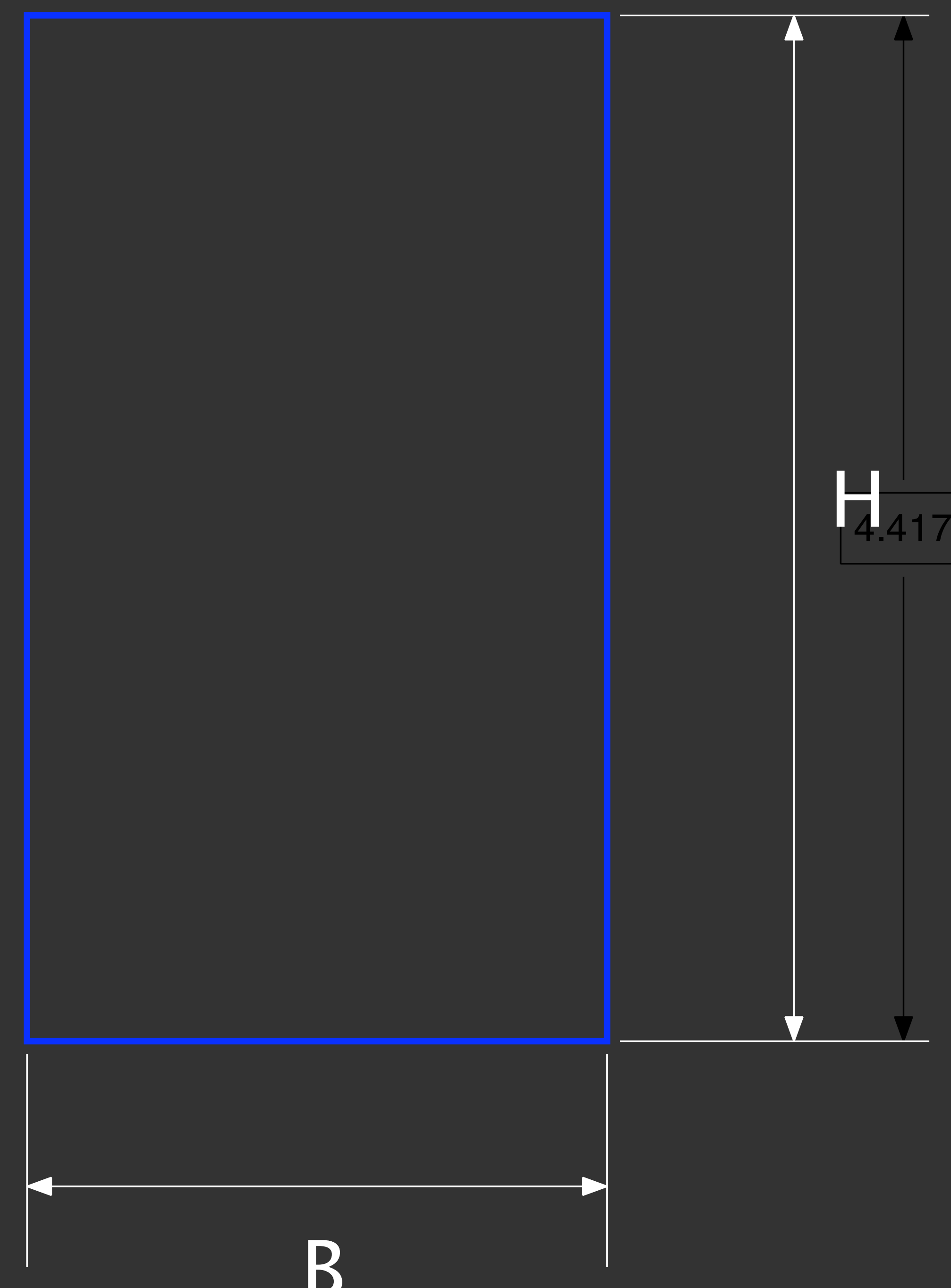
Constitutive relation:

given $\boldsymbol{u}$ ➡ get $\boldsymbol{\sigma}$

e.g., elasticity, plasticity

Modeling Ingredients

1. Set geometry
2. Discretize domaiin
3. Set matl parameters
4. Set B.C.'s
5. Solve



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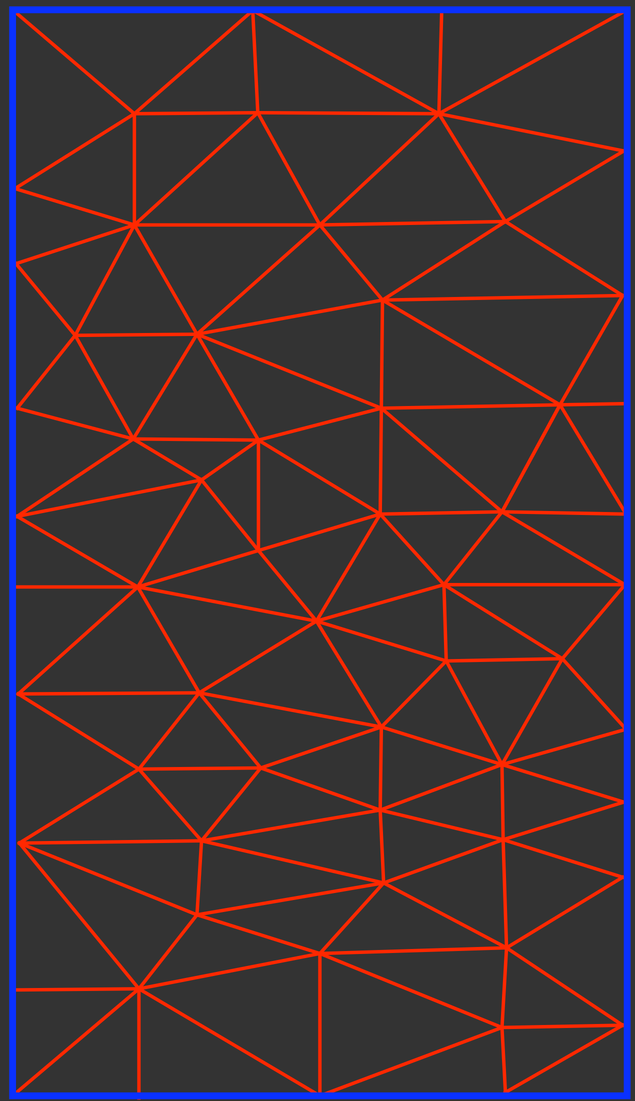

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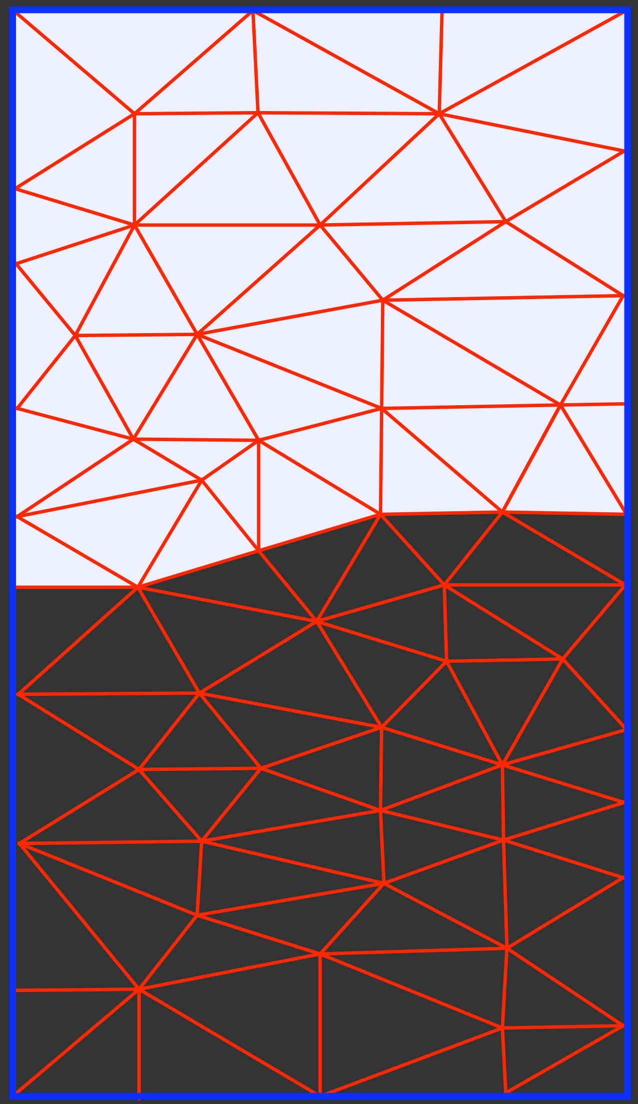

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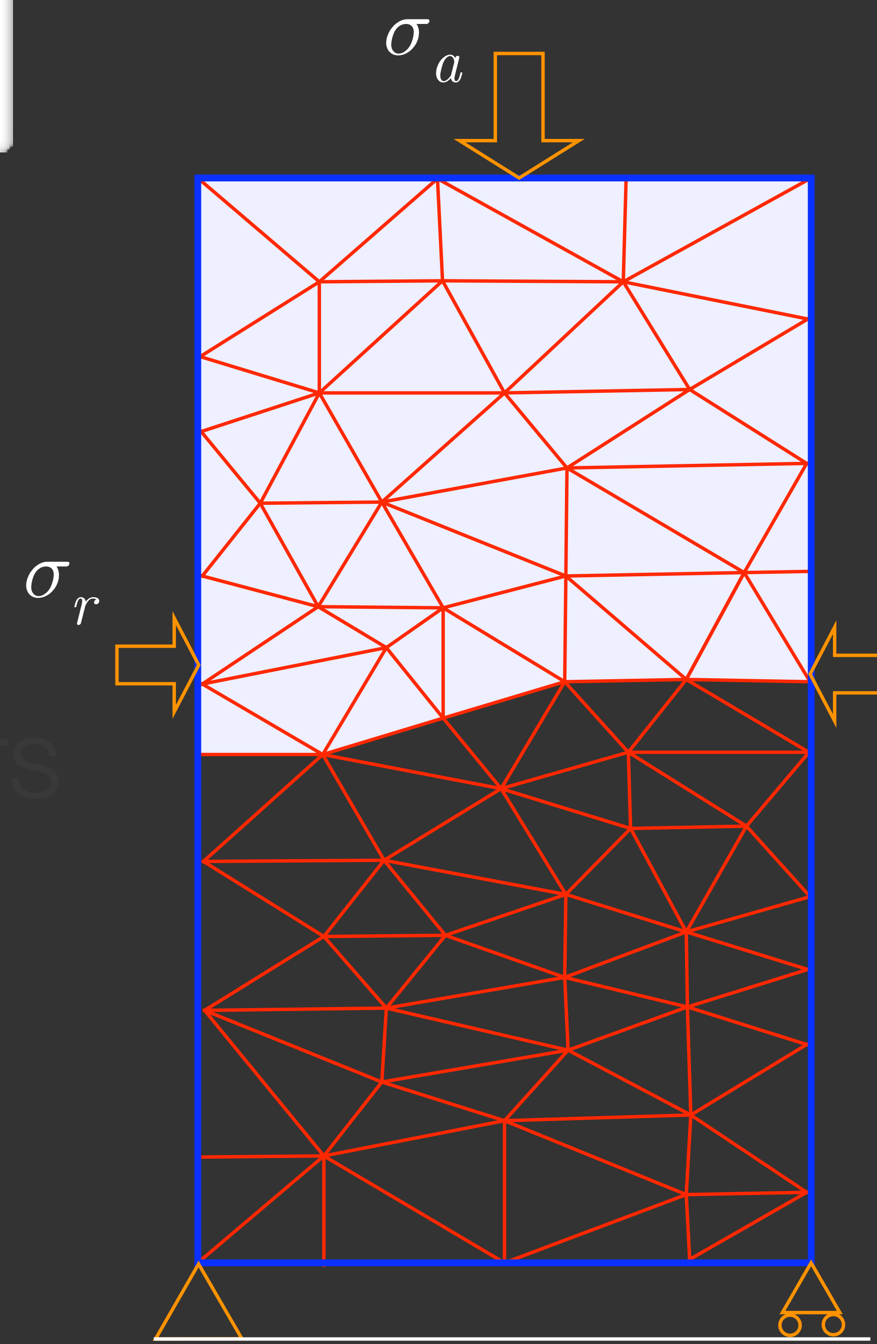

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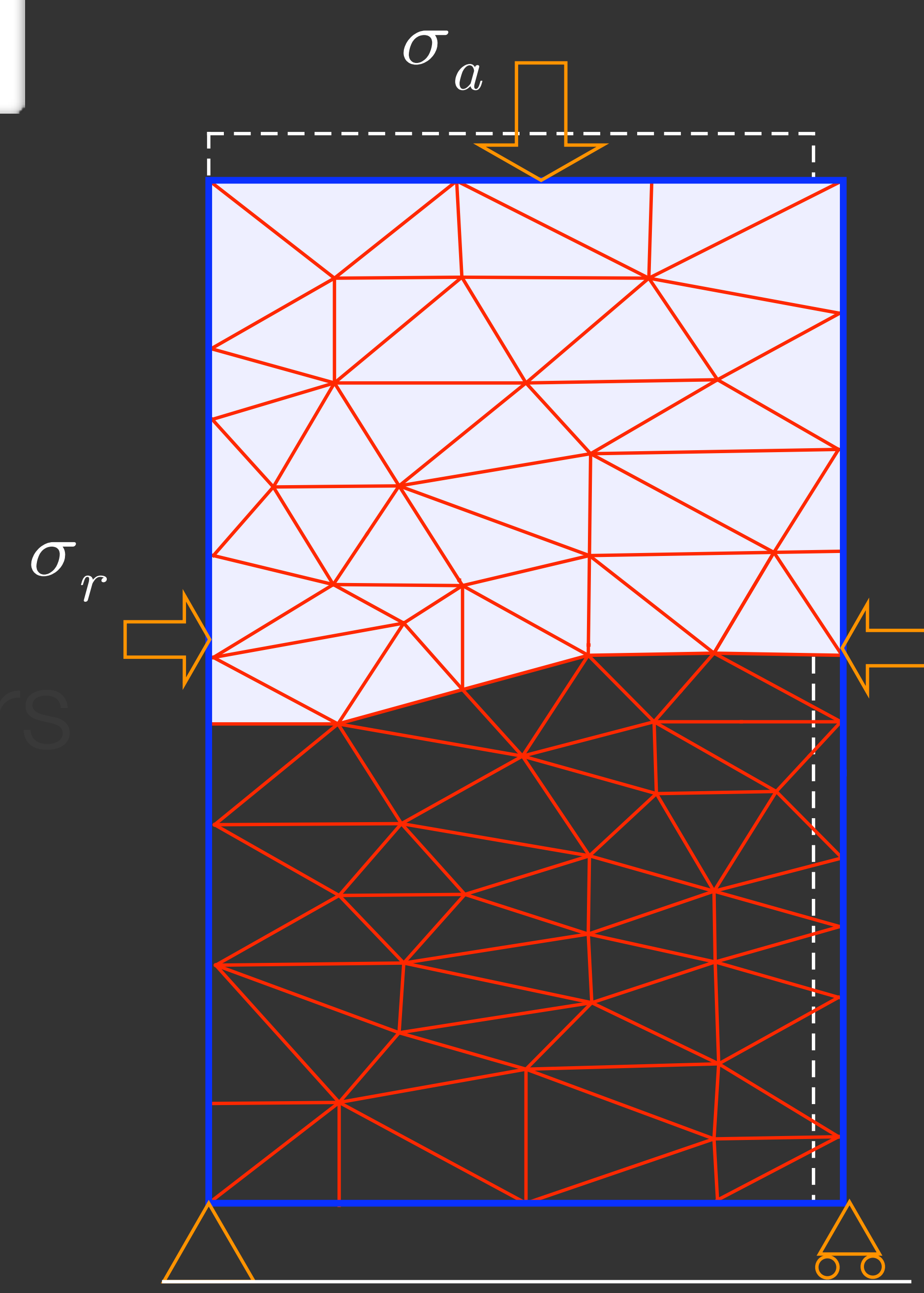
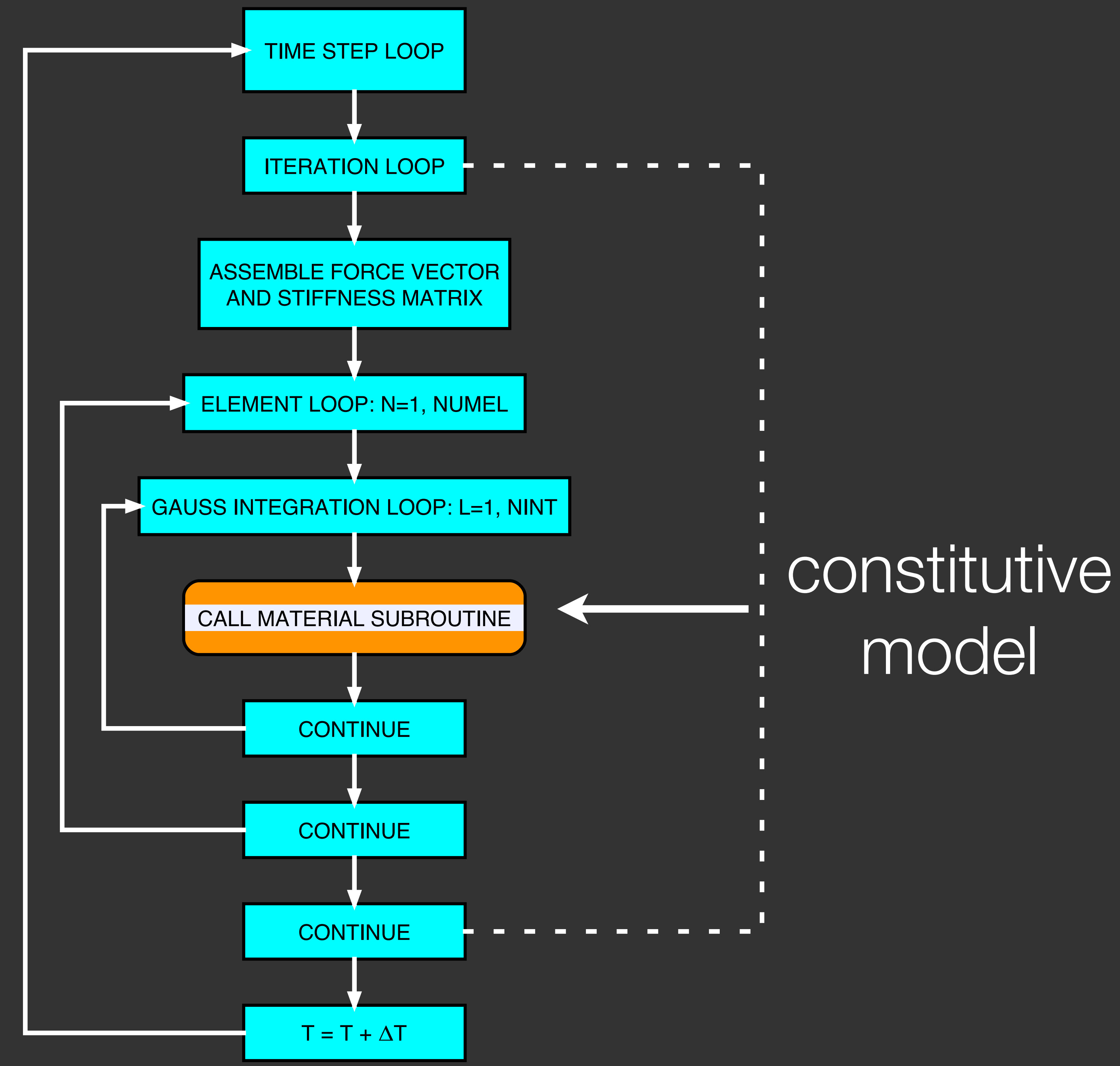


FEM Program



TIME STEP LOOP
ITERATION LOOP
ASSEMBLE FORCE VECTOR AND STIFFNESS MATRIX
ELEMENT LOOP: N=1, NUMEL
GAUSS INTEGRATION LOOP: L=1, NINT
CALL MATERIAL SUBROUTINE
CONTINUE
CONTINUE
CONTINUE
T = T + ΔT
constitutive model

Material behavior: shear strength

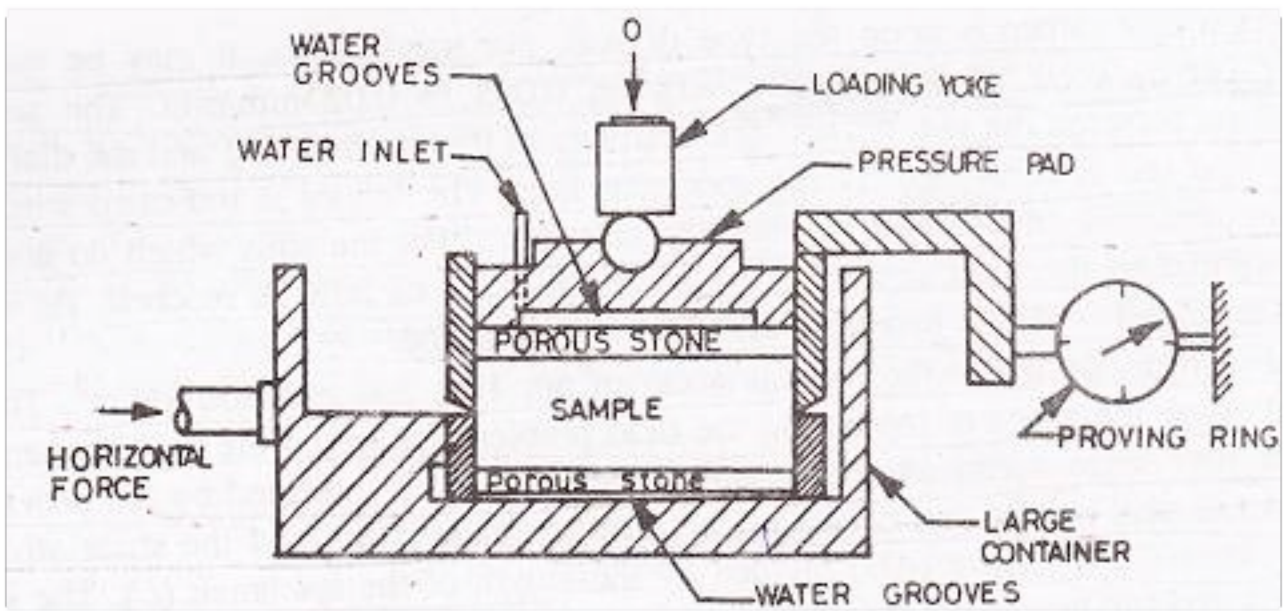
- Void ratio or relative density
- Particle shape & size
- Grain size distribution
- Particle surface roughness
- Water
- Intermediate principal stress
- Overconsolidation or pre-stress

Engineers have developed models to account for most of these variables

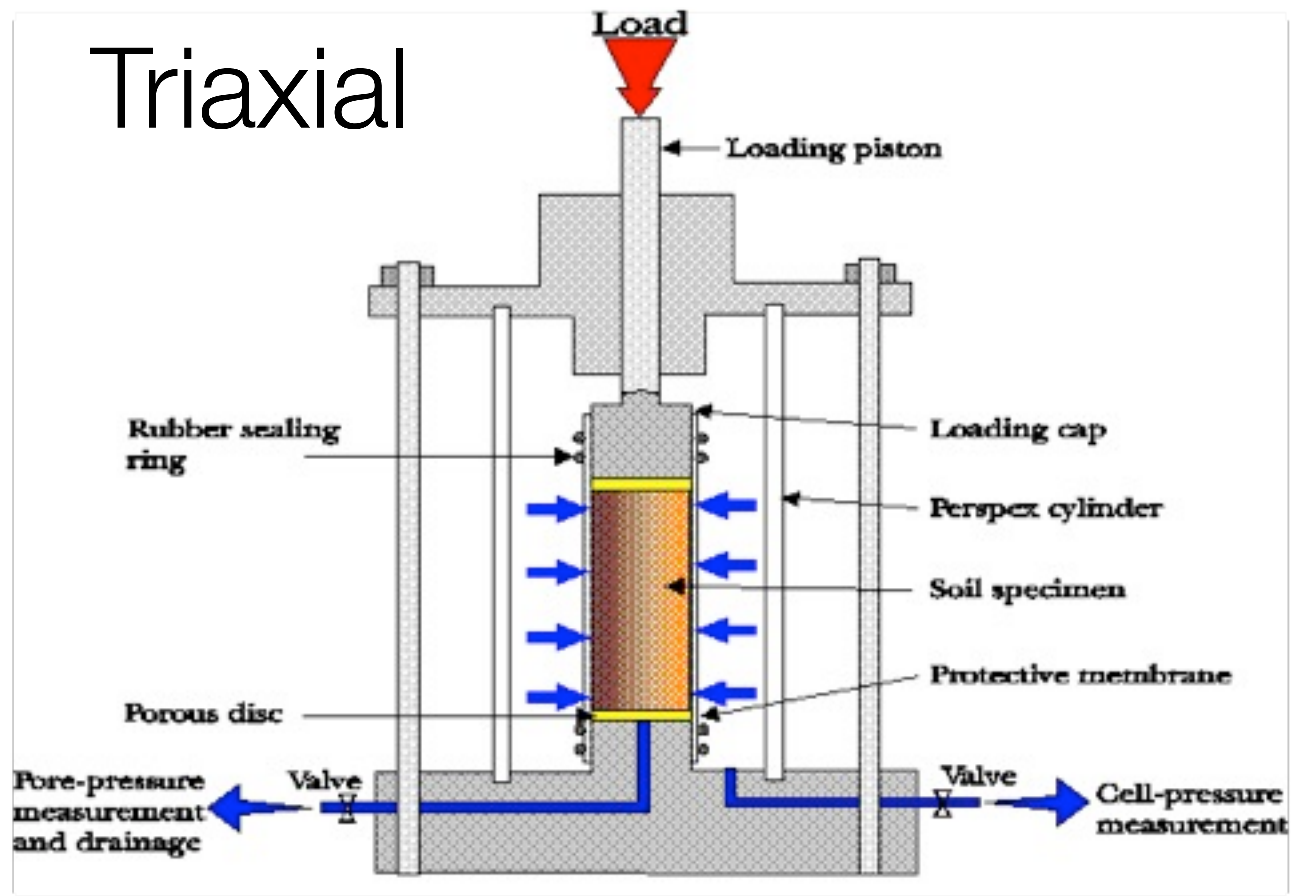
Elasto-plasticity framework of choice

A word on current characterization methods

Direct Shear



Triaxial

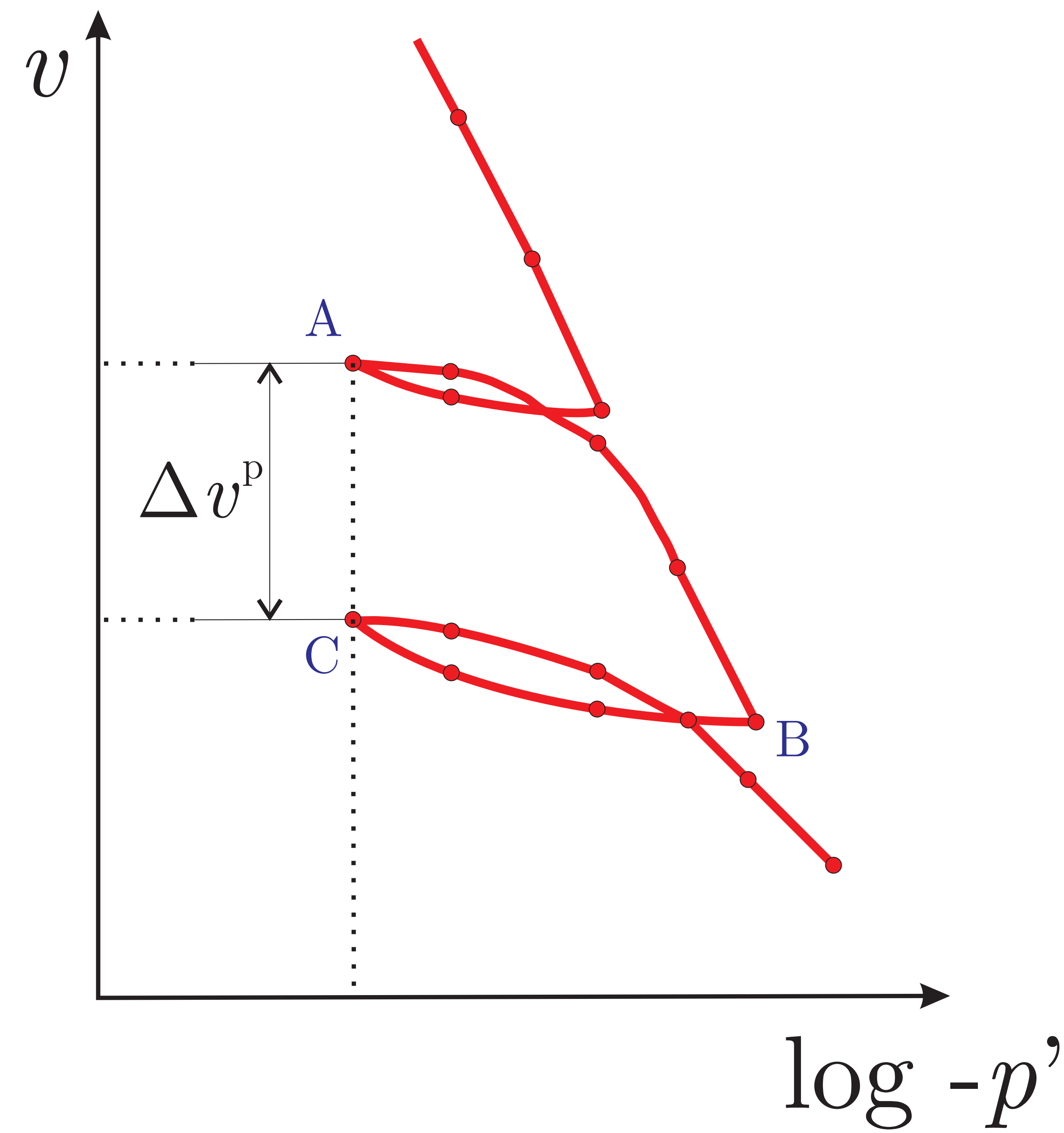


Pros: cheap, simple, fast, good for sands
Cons: drained, forced failure, non-homogeneous

Pros: control drainage & stress path, principal dir. cnst., more homogeneous
Cons: complex

Material models for sands should capture

- Nonlinearity and irrecoverable deformations
- Pressure dependence
- Difference tensile and compressive strength
- Relative density dependence
- Nonassociative plastic flow



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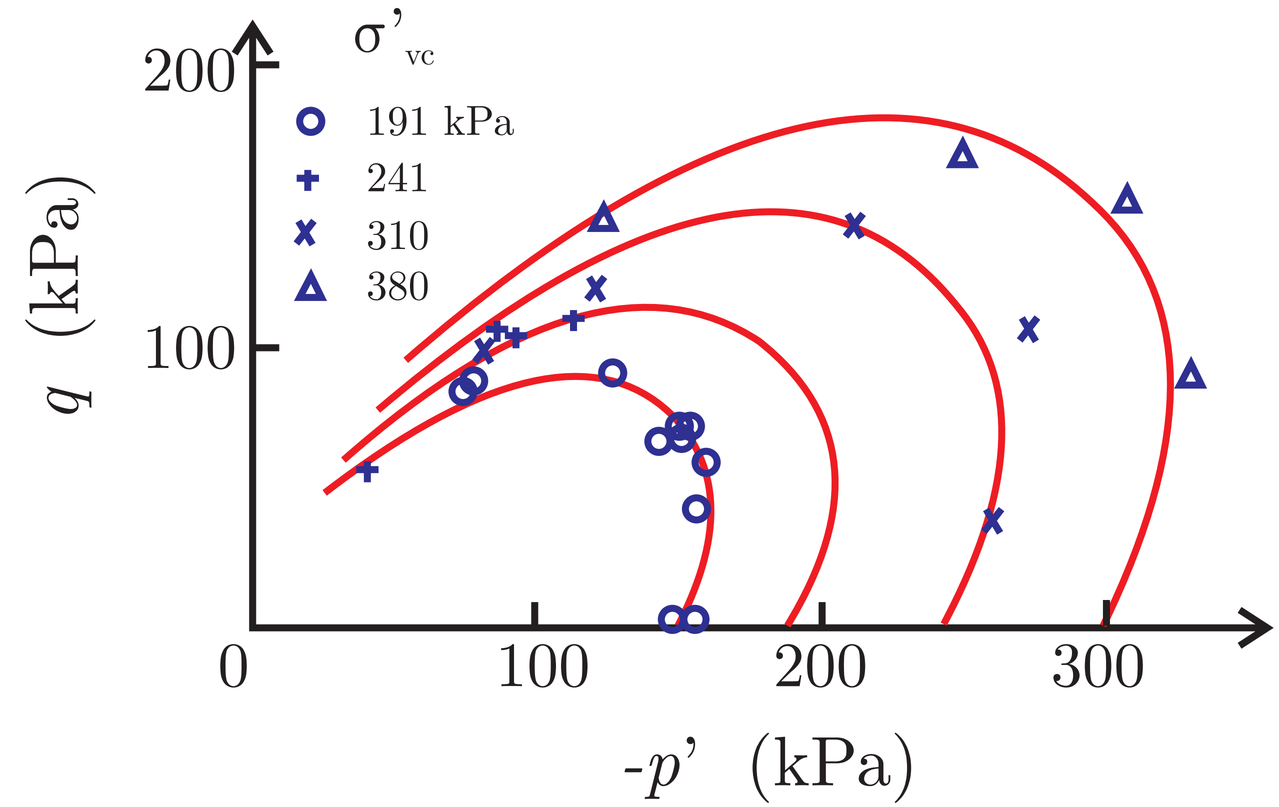

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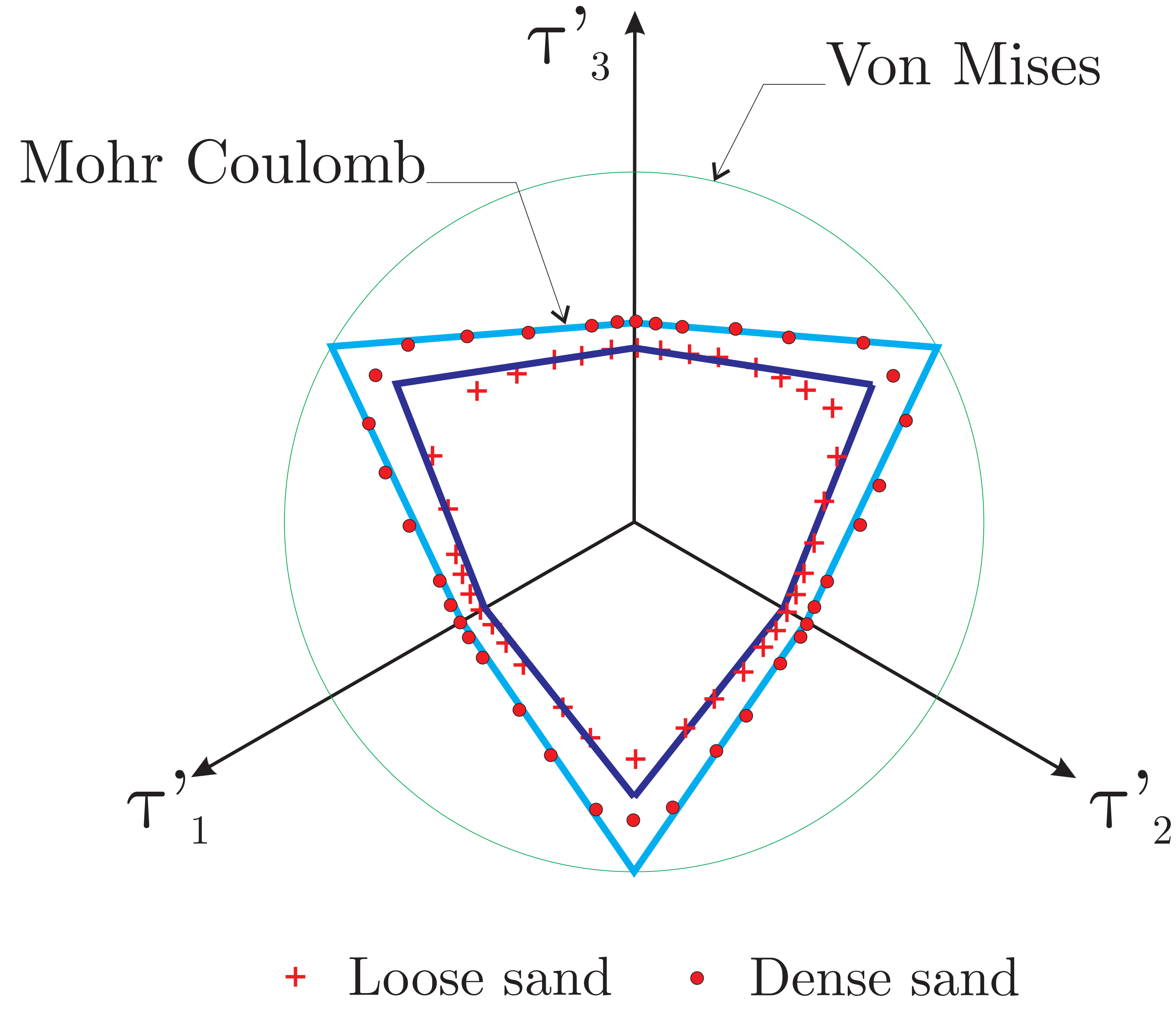

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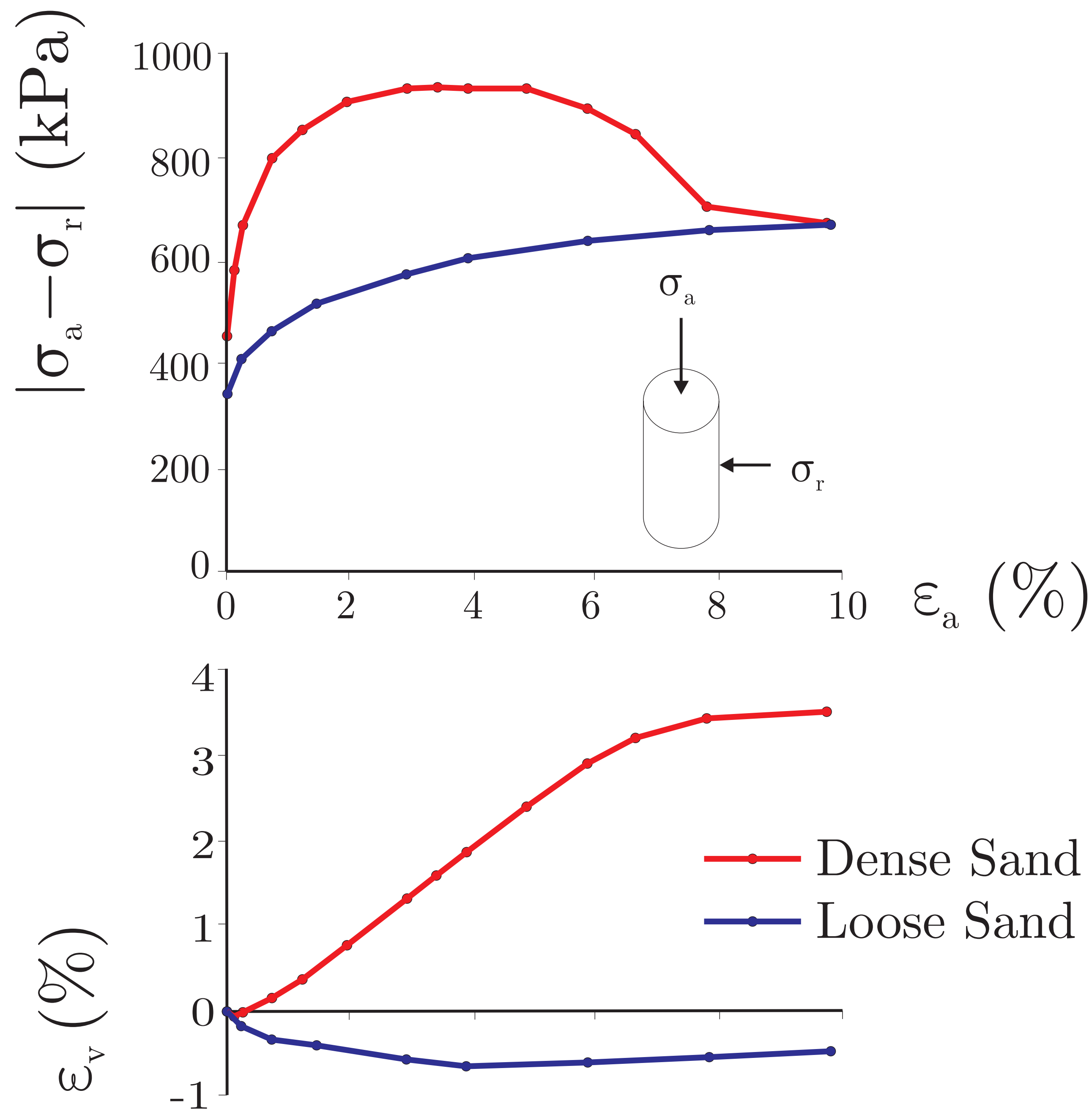

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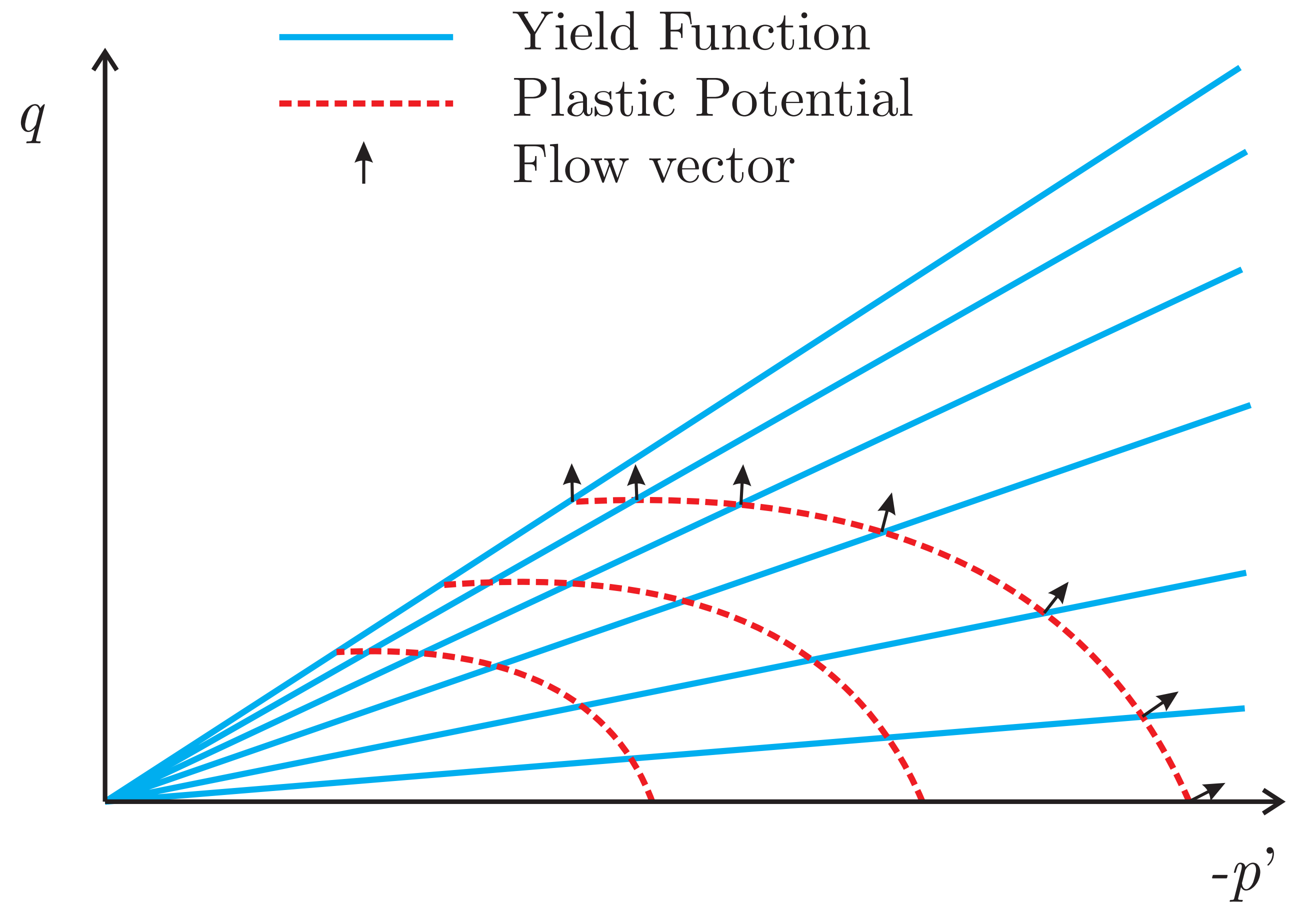


Elasto-plasticity in one slide

Hooke's law $\dot{\boldsymbol{\sigma}} = \boldsymbol{c}^{\mathrm{ep}} : \dot{\boldsymbol{\epsilon}}$

Additive decomposition of strain $\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^{\mathrm{e}} + \dot{\boldsymbol{\epsilon}}^{\mathrm{p}}$

Convex elastic region $F(\boldsymbol{\sigma}, \boldsymbol{\alpha}) = 0$

Non-associative flow $\dot{\boldsymbol{\epsilon}}^{\mathrm{p}} = \dot{\lambda} \boldsymbol{g}, \qquad \boldsymbol{g} := \partial G / \partial \boldsymbol{\sigma}$

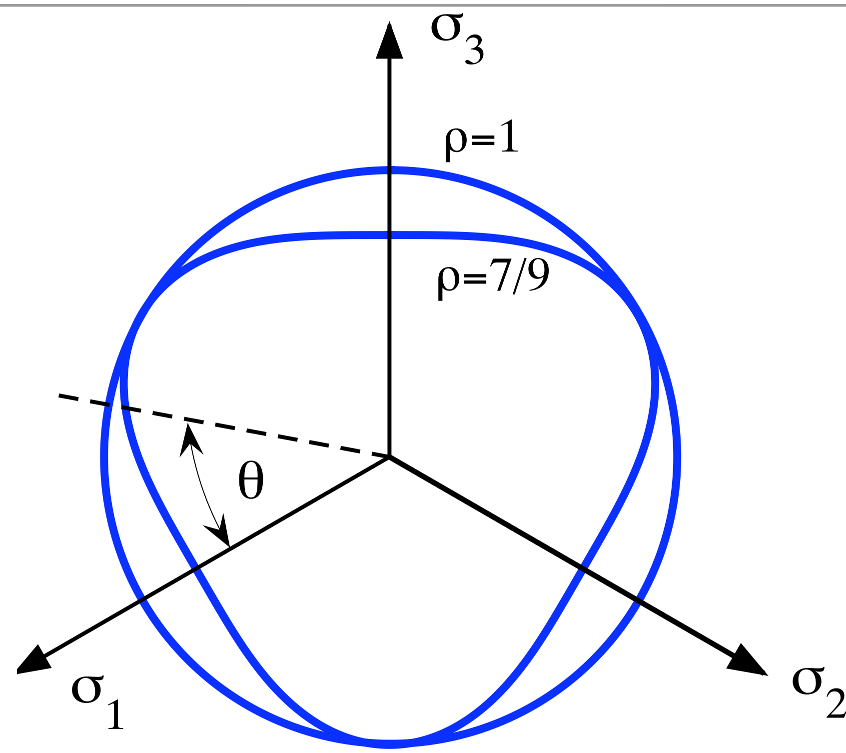
K-T optimality $\dot{\lambda} F = 0 \qquad \dot{\lambda} H = -\partial F / \partial \boldsymbol{\alpha} \cdot \dot{\boldsymbol{\alpha}}$

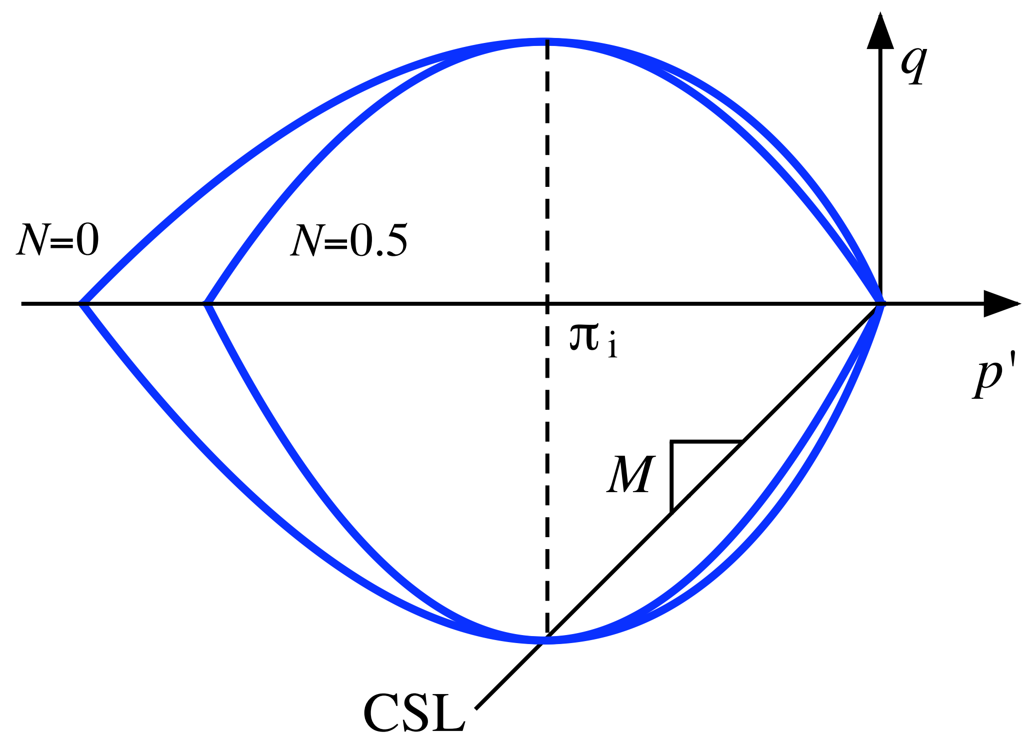
Elastoplastic constitutive tangent

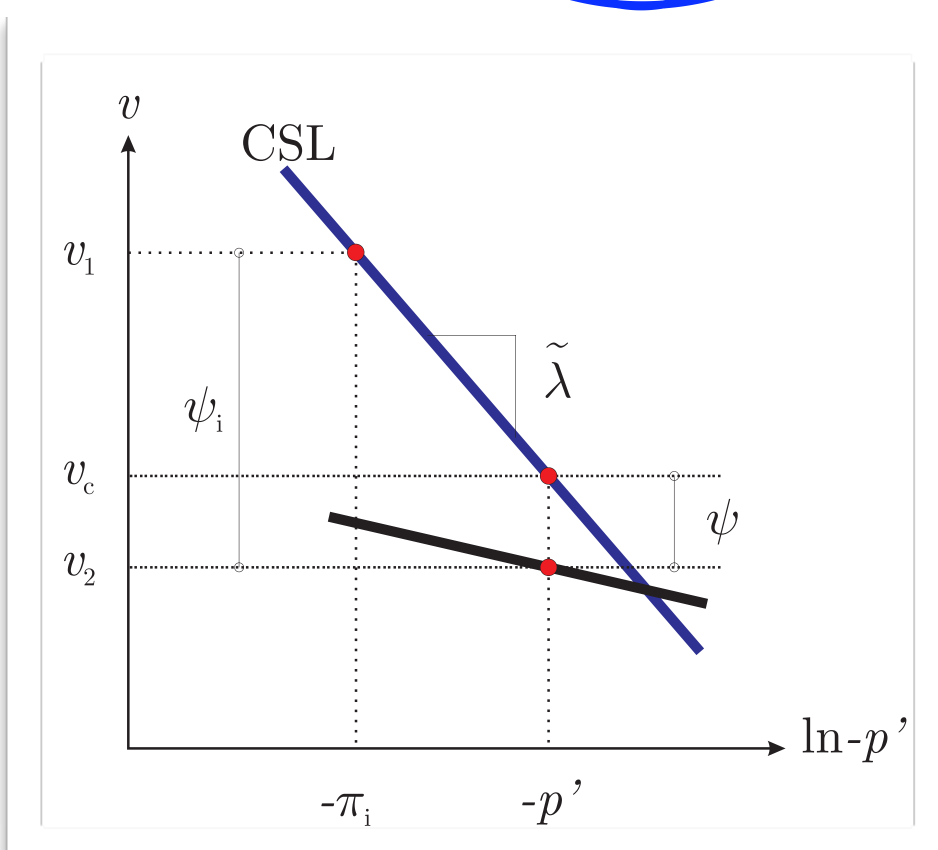
$$\boxed{\boldsymbol{c}^{\mathrm{ep}} = \boldsymbol{c}^{\mathrm{e}} - \frac{1}{\chi} \boldsymbol{c}^{\mathrm{e}} : \boldsymbol{g} \otimes \boldsymbol{f} : \boldsymbol{c}^{\mathrm{e}}, \qquad \chi = H - \boldsymbol{g} : \boldsymbol{c}^{\mathrm{e}} : \boldsymbol{f}}$$

Examples

Example of elasto-plastic model



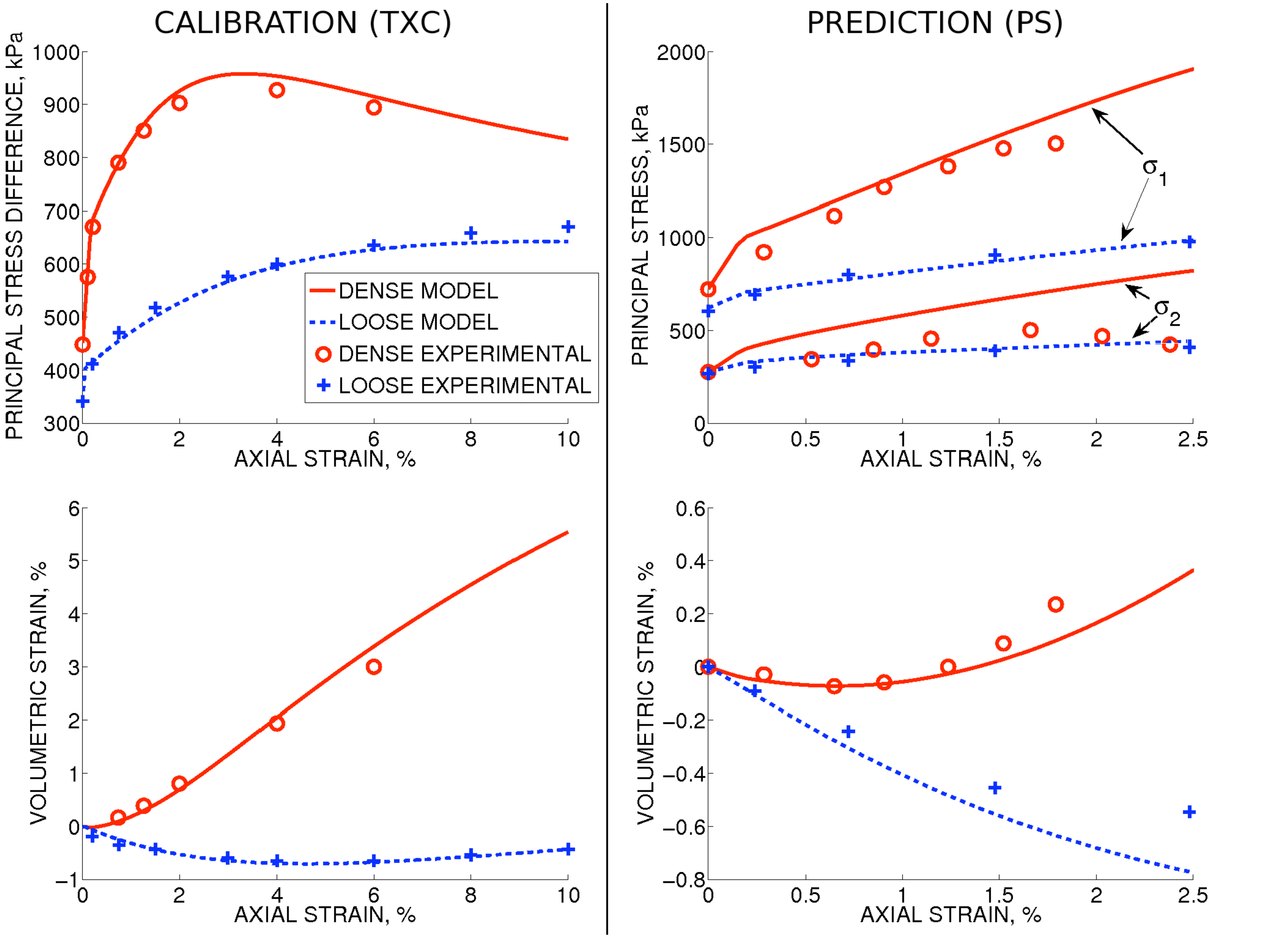




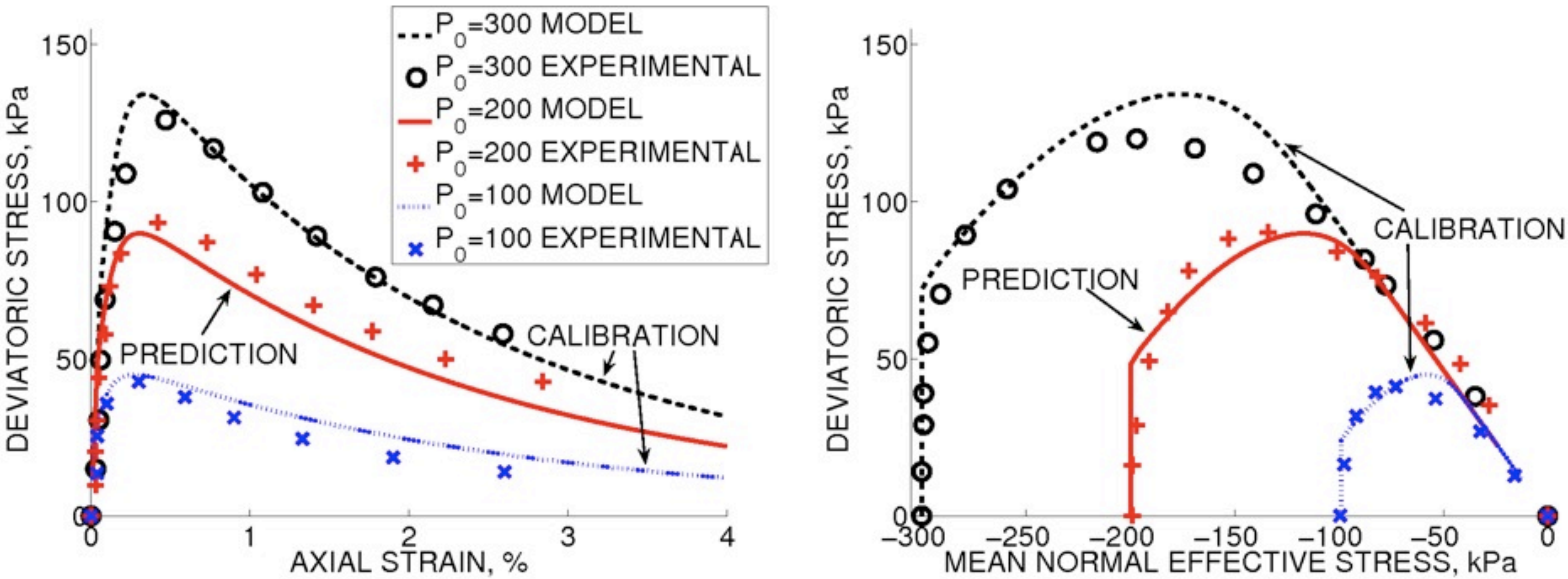
$$F = F(\boldsymbol{\sigma}', \pi_{\mathrm{i}})$$

$$G = G(\boldsymbol{\sigma}', \bar{\pi}_{\mathrm{i}})$$

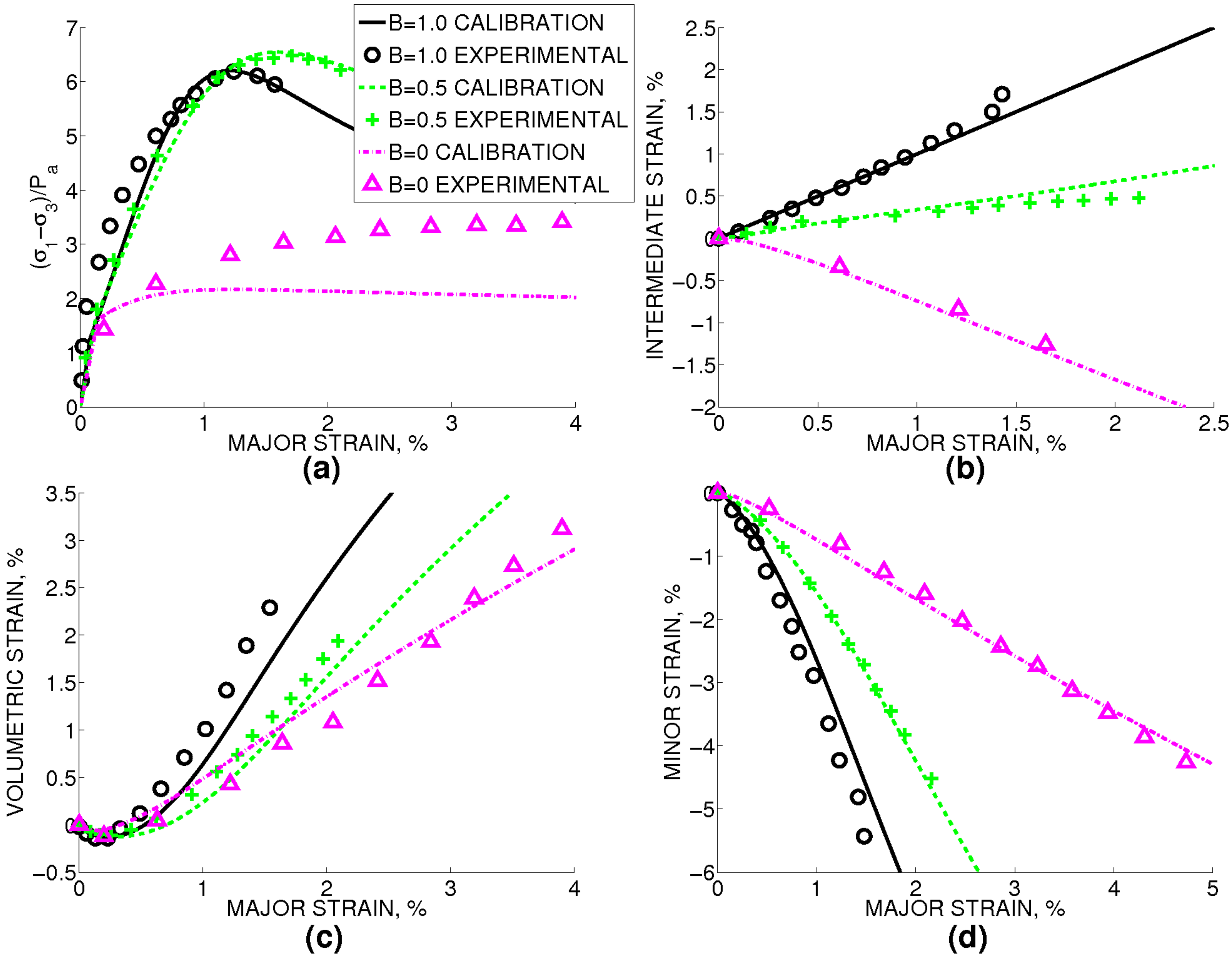
$$H = H(p', \pi_{\mathrm{i}}, \psi)$$



model validation: drained txc and ps

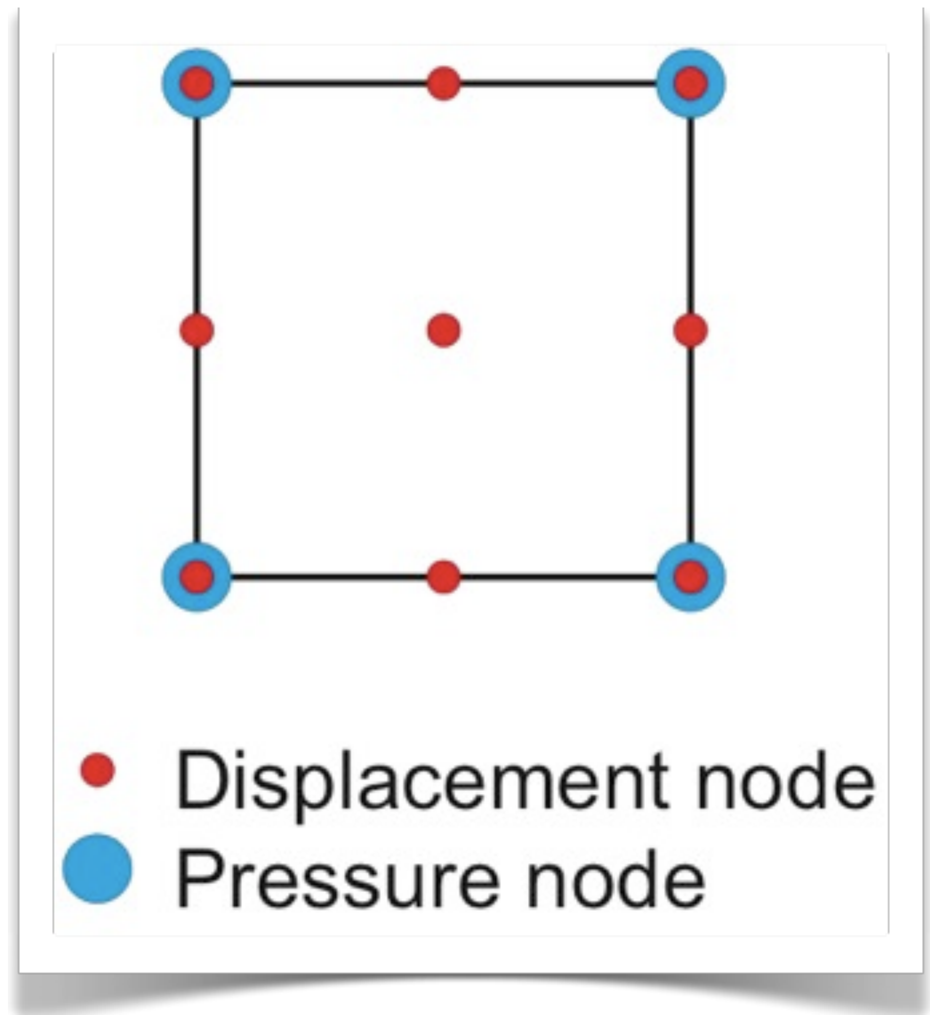


undrained txc loose sands

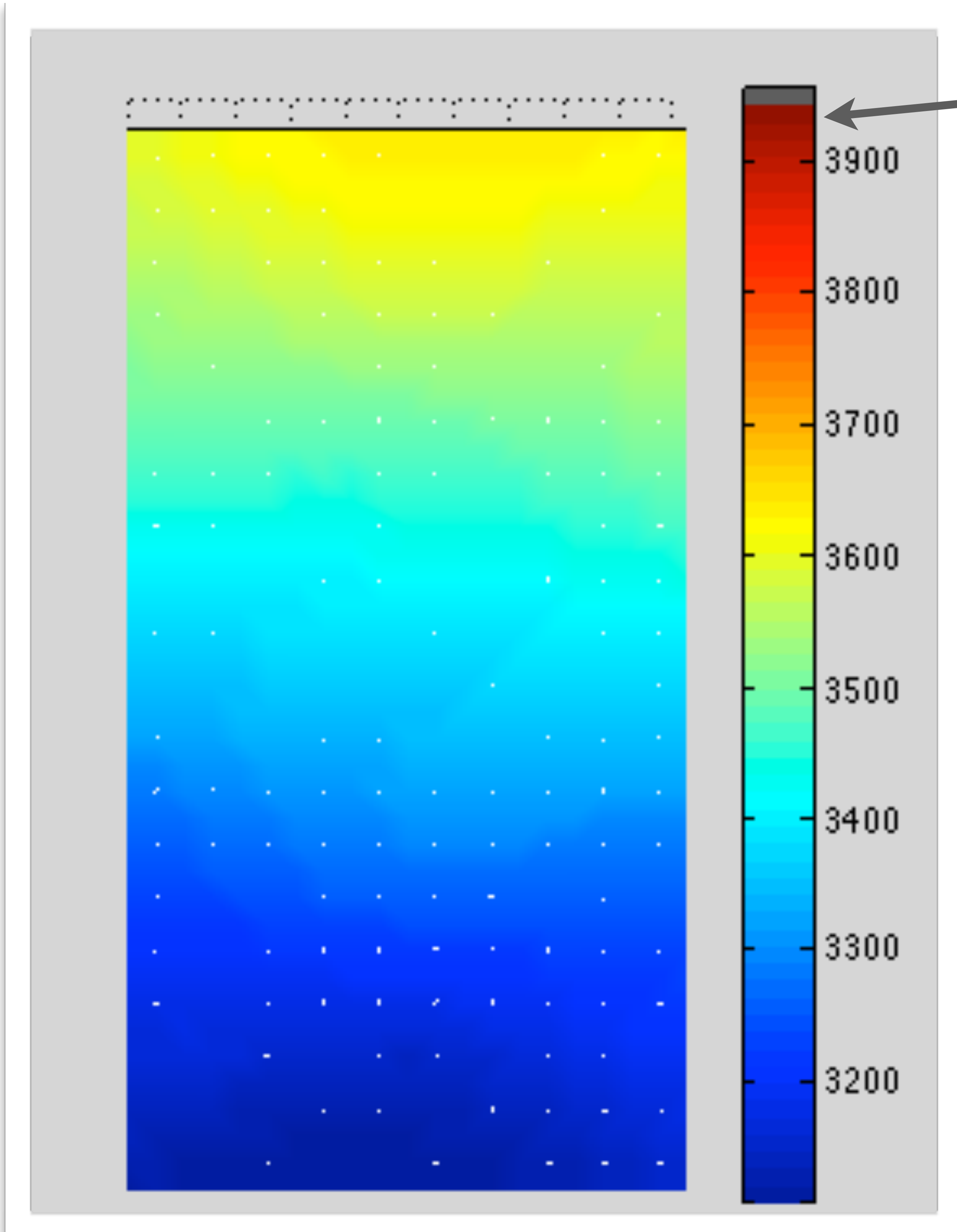


true triaxial b=constant

$H - \bar{H}_L$



Plane-strain liquefaction numerical simulation



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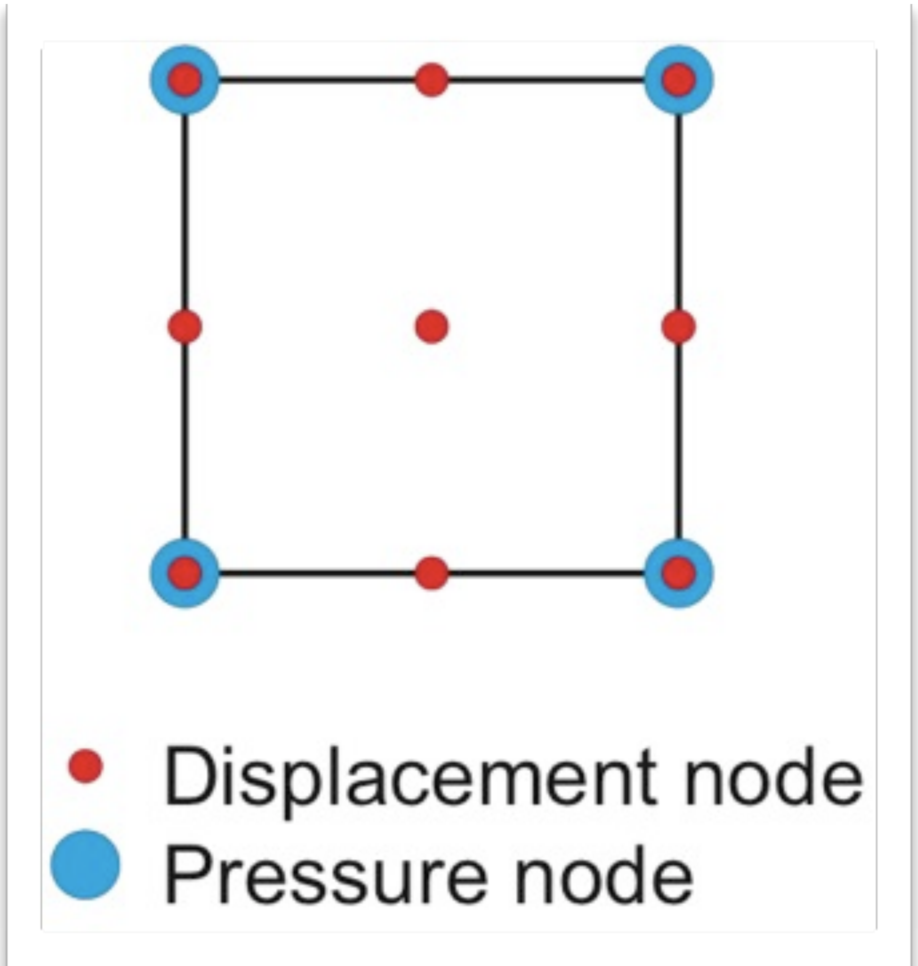

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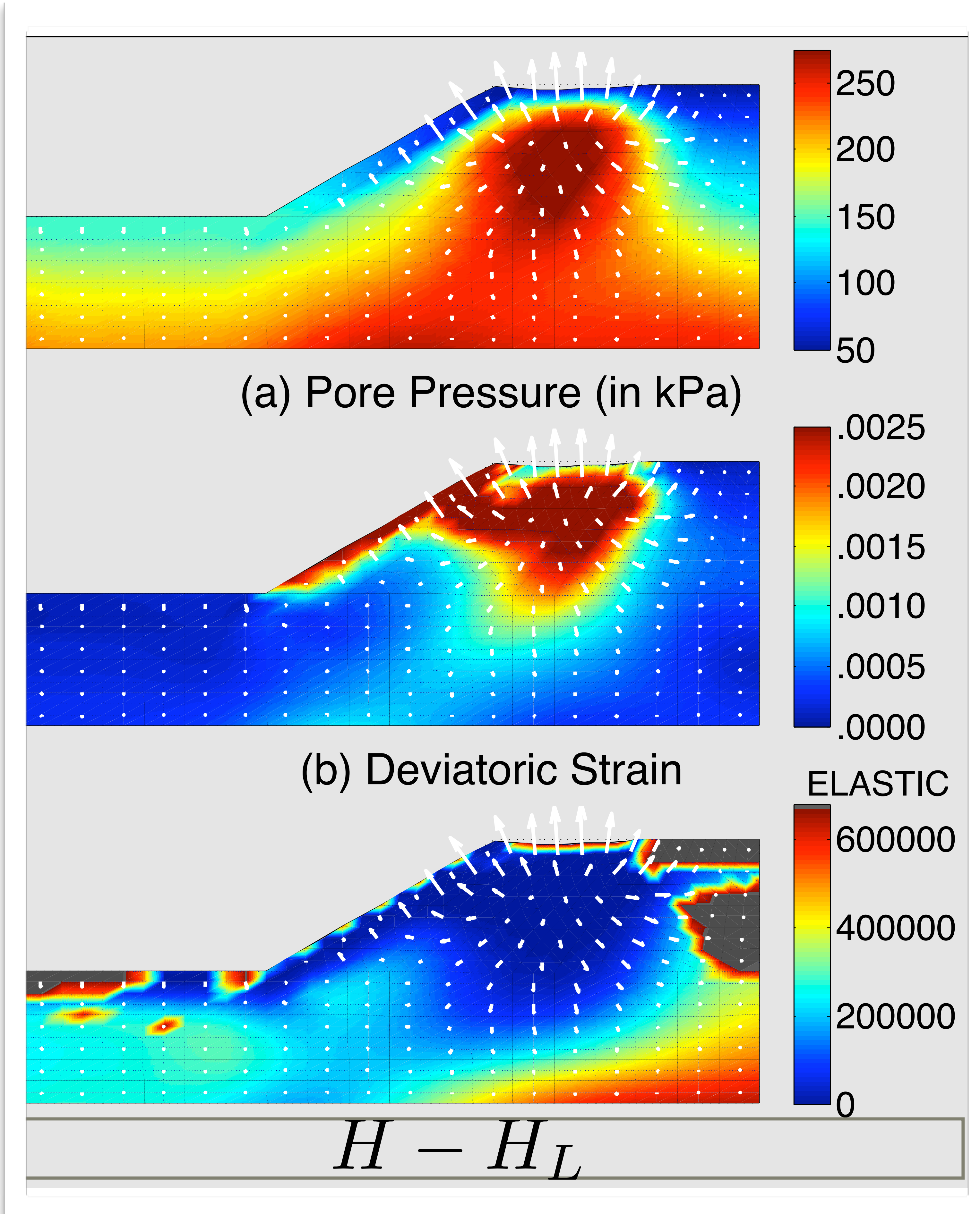
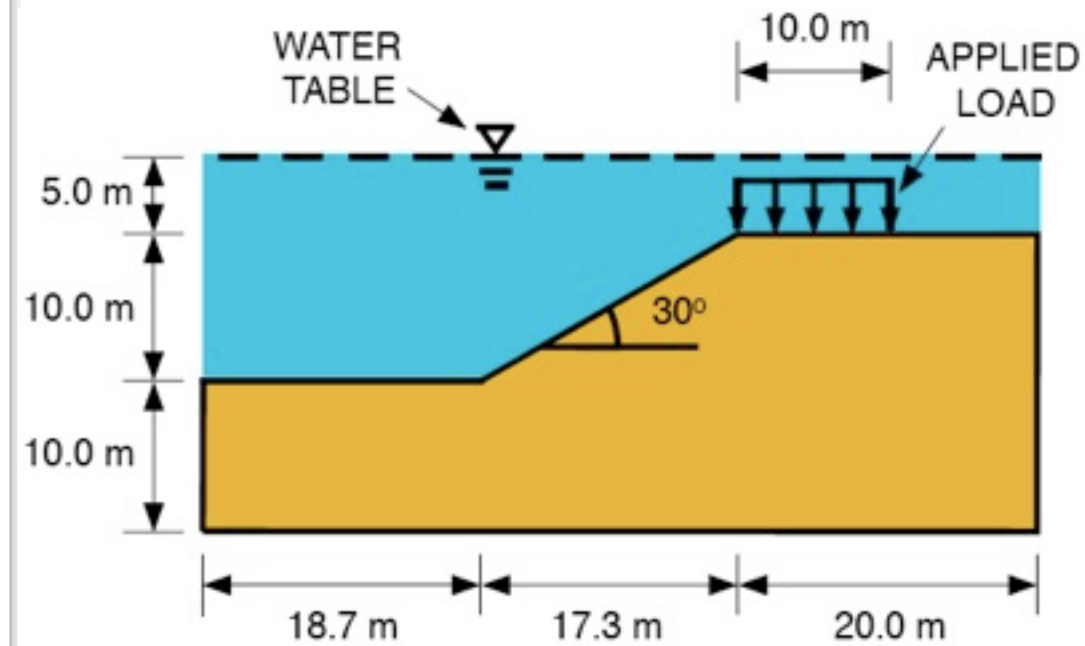
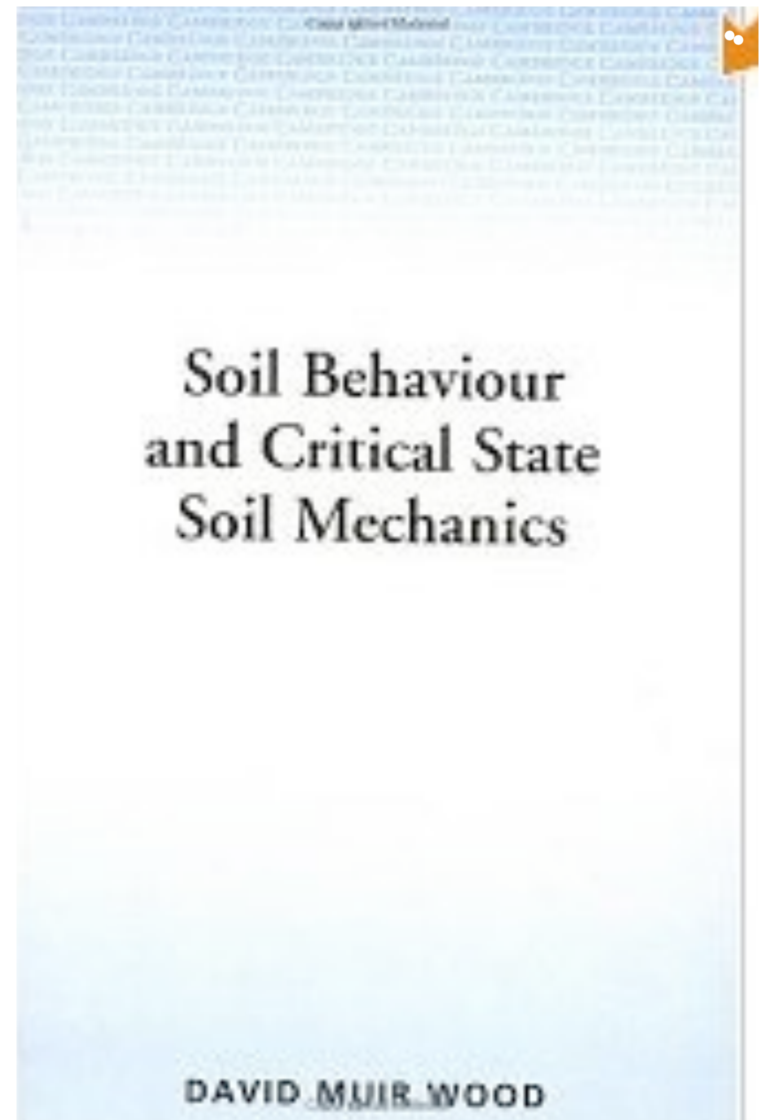




Field scale prediction
Levee failure
(recall Katrina)

References


AN INTRODUCTION TO
Geotechnical Engineering
Robert D. Holtz
William D. Kovacs


Soil Behaviour
and Critical State
Soil Mechanics
DAVID MUIR WOOD