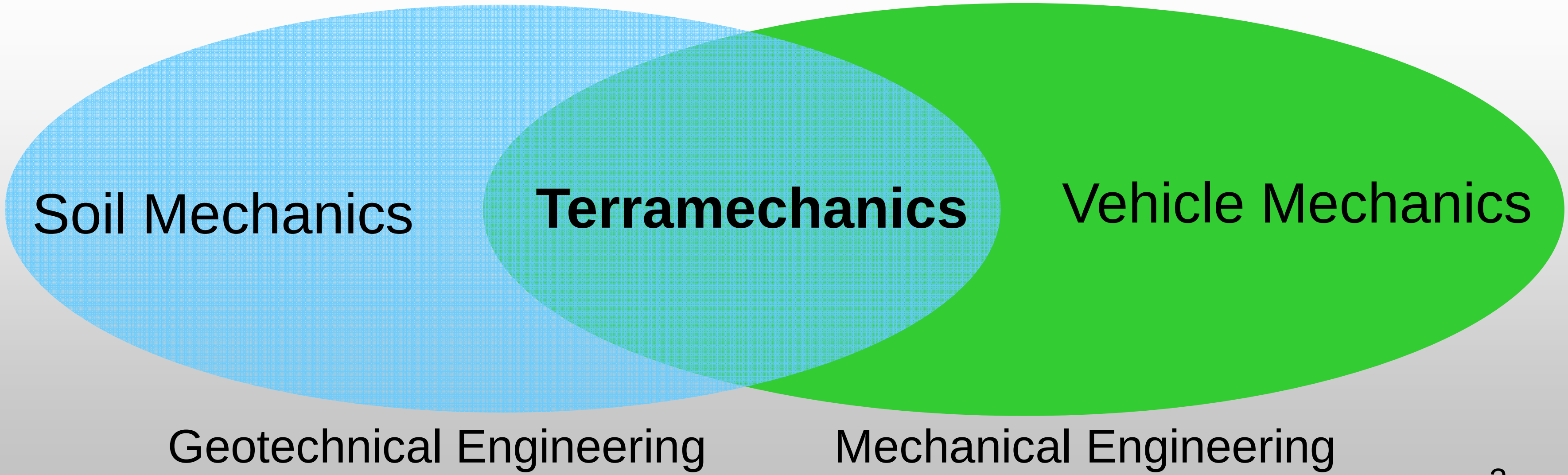
Surface Interaction Modeling

Engineering Methods

Karl Iagnemma, Ph.D.
Massachusetts Institute of Technology

Terramechanics

- Terramechanics
  - Engineering science that studies the interaction between vehicles and (deformable) terrain
- Soil mechanics and vehicle mechanics
- Analysis of wheeled, tracked, legged systems



Soil Mechanics

Terramechanics

Vehicle Mechanics

Geotechnical Engineering

Mechanical Engineering

An Engineer’s Job

- Design vehicle for robust mobility on Mars surface
  - Wheels, tracks, legs?
    - Number, diameter and width?
    - Required nominal torque?
    - Required peak power?
  - Obstacle crossing performance?
    - Suspension configuration?
  - Steering mechanism?
- **How to address in a principled, systematic fashion?**

NASA’s Mars Science Laboratory (MSL)
Design/Test Model (DTM) in the sandy Mars Yard at JPL

An Engineer’s Reality

- How to model this scenario?
  - High sinkage
  - High slip ratio
  - Material transport effects
    - Clogged grousers
  - Variables of interest
    - Soil properties
    - Soil state
    - Wheel load
    - Wheel geometric properties
    - Wheel linear and angular velocity

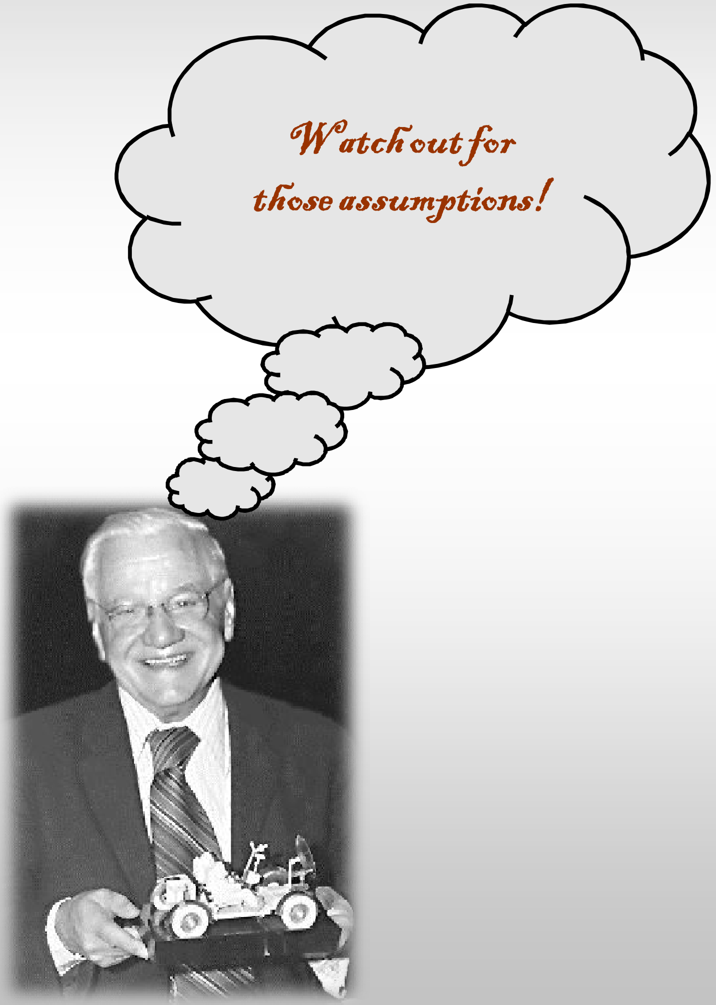
Opportunity
Maneuvers
out of
Sand Trap

Terramechanics

- Limitations of terramechanics modeling
  - Attempt to model all soil types with single set of relations
    - Frictional soils, crusty materials, clay
  - Assumption of homogeneity
  - Attempt to apply (semi)-empirical models in predictive manner
  - Little consideration of off-nominal operation
  - Difficulty in employing quasi-static models for dynamic simulation
- Assertion: General approach remains valid
  - Not all limitations are fundamental
- Goals
  - Understand limits of applicability of terramechanics
  - Identify areas requiring new research

Terramechanics Principles

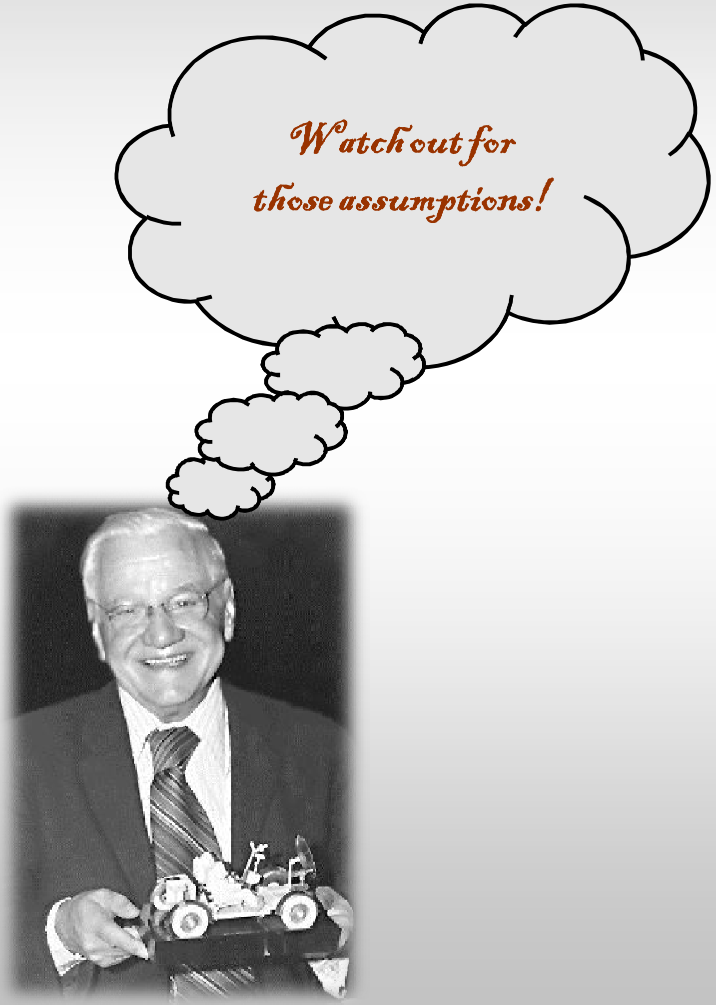
- Fundamental relations
  - – Pressure-sinkage
  - – Shear stress-shear displacement
    - Wheel slip
- Other effects
  - – Grousers/lugs
  - – Lateral forces
  - – Repetitive loading
- Limitations
  - – Inhomogeneity
  - – Scale effects
  - – Slipping and sinking



M.G. Bekker

Terramechanics Principles

- Fundamental relations
  - Pressure-sinkage
  - Shear stress-shear displacement
    - Wheel slip
- Other effects
  - Grousers/lugs
  - Lateral forces
  - Repetitive loading
- Limitations
  - Inhomogeneity
  - Scale effects
  - Slipping and sinking



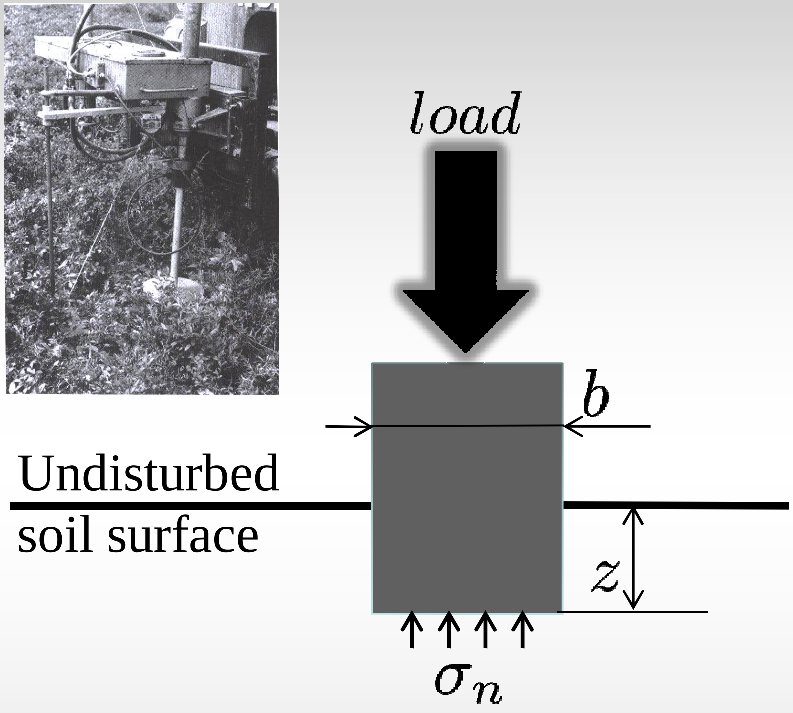
M.G. Bekker

Pressure-Sinkage

- Pressure-sinkage relationship for geomaterials

$$\sigma = kz^n$$

  - σ is normal pressure
  - *k* is empirical constant
  - *z* is sinkage from free surface
- Bekker proposed semi-empirical formulation



Cohesion-dependent soil coefficient

Friction-dependent soil coefficient

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

Sinkage exponent

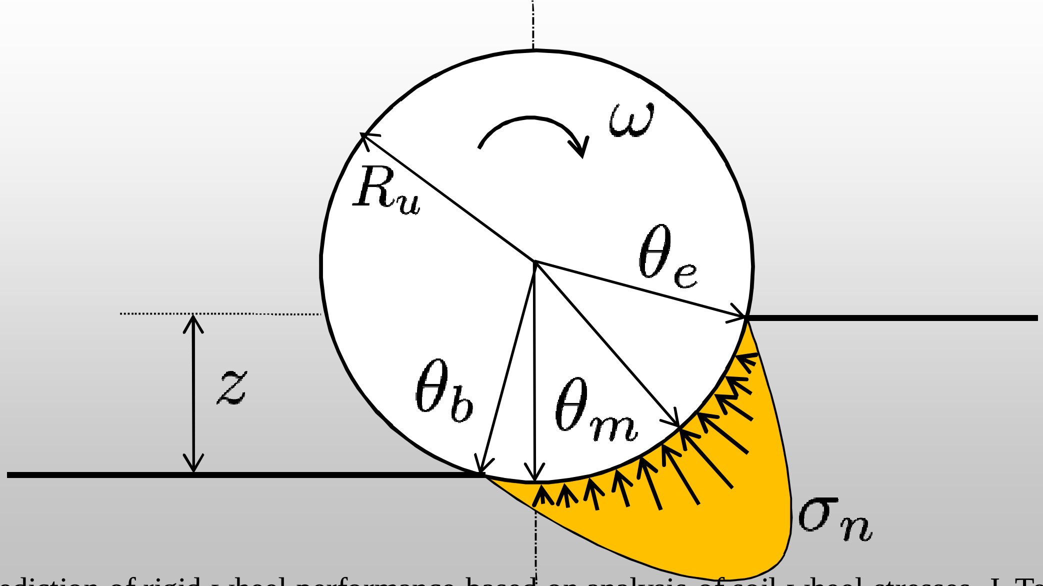
M. G. Bekker. *Theory of Land Locomotion*. Ann Arbor, University of Michigan Press, 1950.

Pressure-Sinkage for Wheels

- Can compute normal stress for wheels along terrain interface

$$\sigma_{nf} = \left(\frac{k_c}{b} + k_\phi\right)\left[R\left(\cos(\theta) - \cos(\theta_e)\right)\right]^n \qquad \theta_m < \theta \leq \theta_e$$

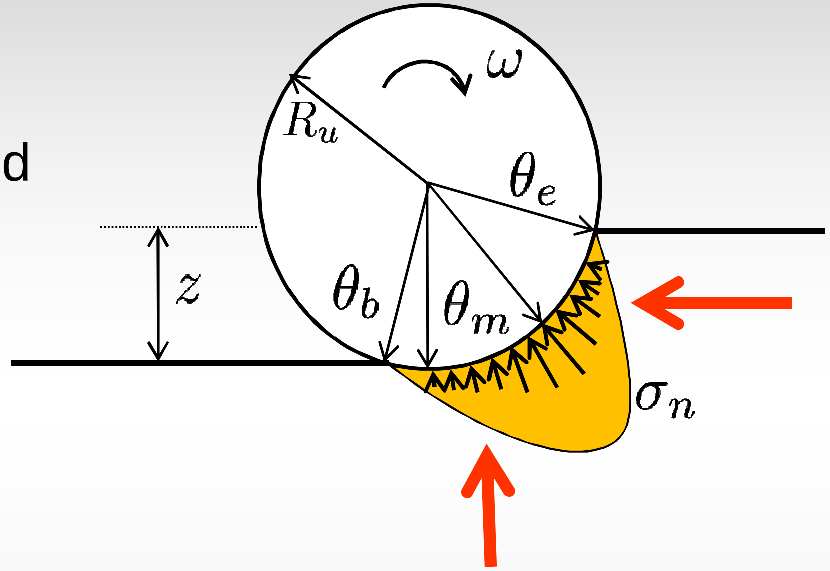
$$\sigma_{nr} = \left(\frac{k_c}{b} + k_\phi\right)\left[R\left(\cos\left(\theta_e - \left(\frac{\theta - \theta_r}{\theta_N - \theta_r}\right)(\theta_e - \theta_N)\right) - \cos(\theta_e)\right)\right]^n \qquad \theta_b \leq \theta \leq \theta_m$$

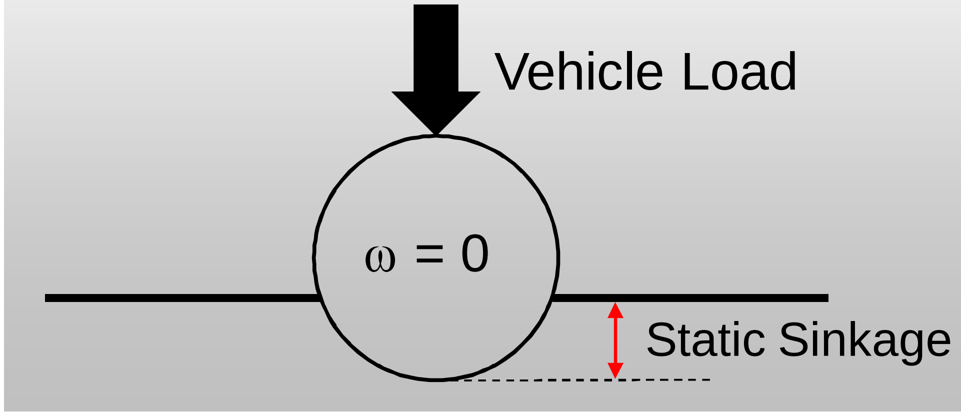


J. Y. Wong and A. R. Reece. Prediction of rigid wheel performance based on analysis of soil-wheel stresses. J. Terramechanics,1967

Pressure-Sinkage for Wheels

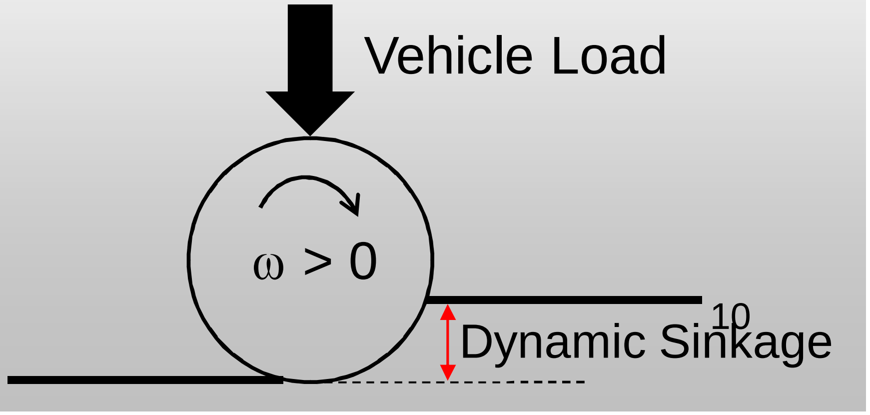
- Sinkage plays critical role in mobility
  - Increased sinkage causes increased motion resistance
  - Energy lost in terrain compaction
- Sinkage can be divided in two components
  - Static sinkage
  - Dynamic sinkage (or slip-sinkage)




Vehicle Load

ω = 0

Static Sinkage

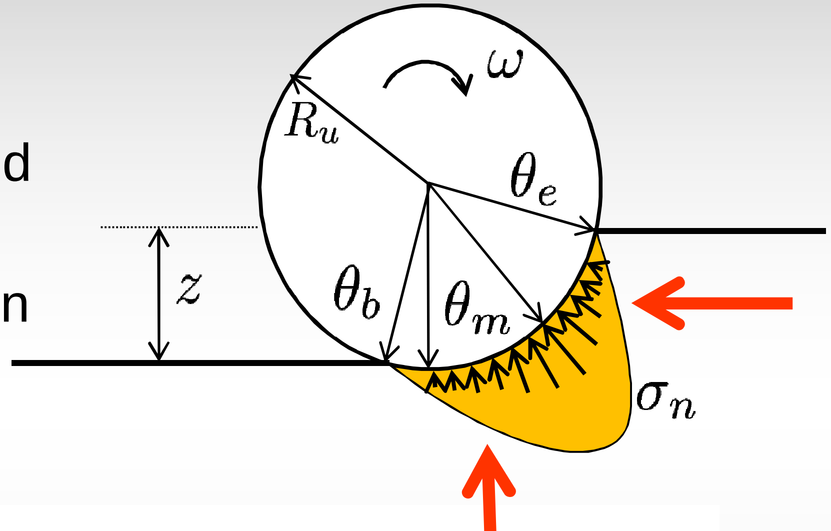

Vehicle Load

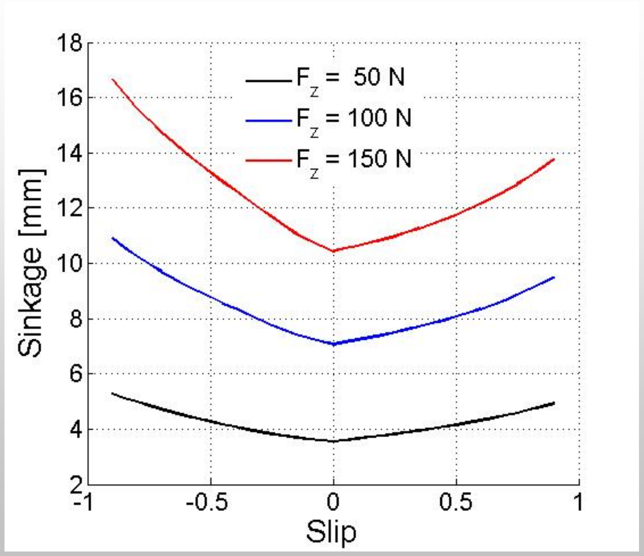
ω > 0

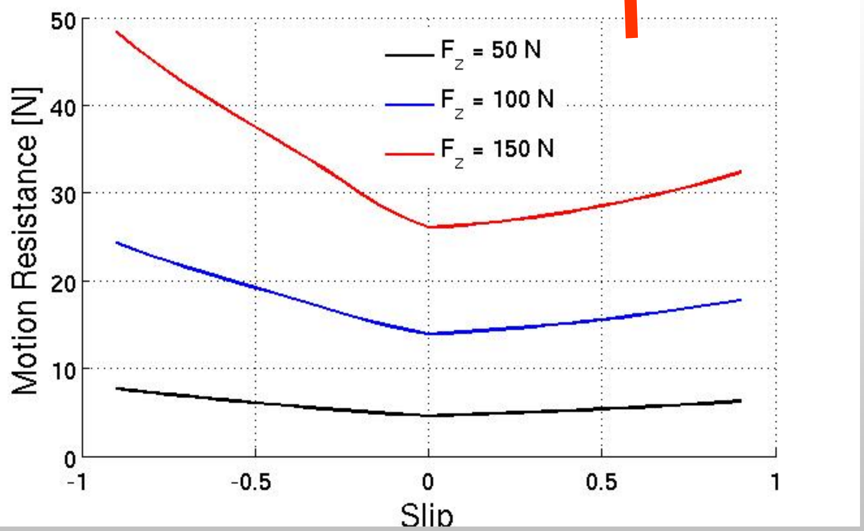
Dynamic Sinkage

Pressure-Sinkage for Wheels

- Sinkage plays critical role in mobility
  - Increased sinkage causes increased motion resistance
    - Energy lost in terrain compaction

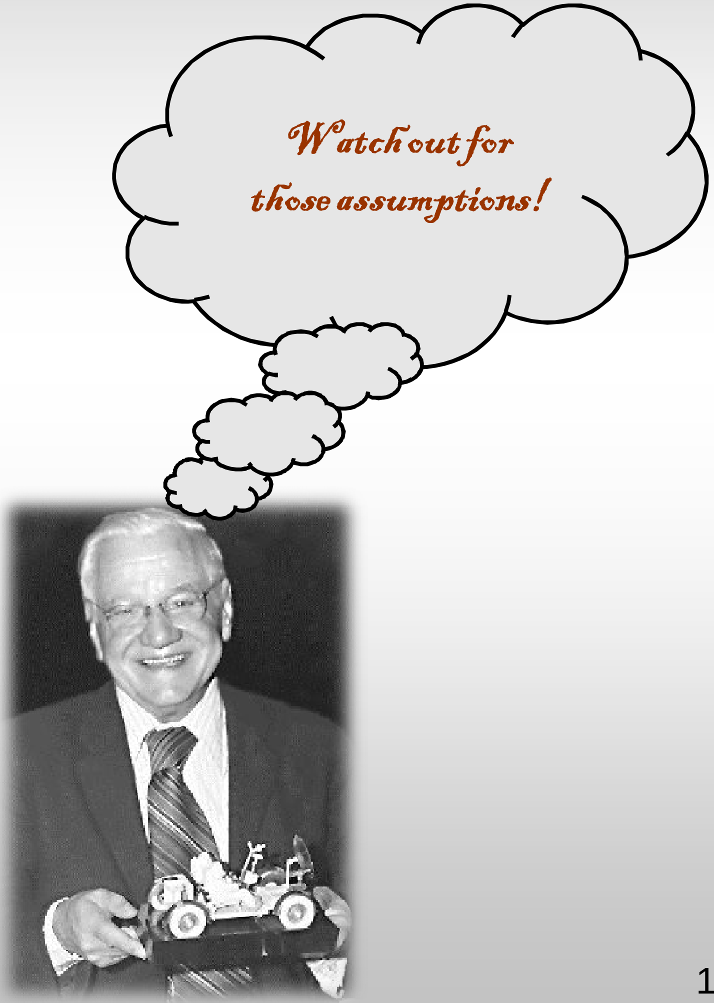






Terramechanics Principles

- Fundamental relations
  - Pressure-sinkage
  - Shear stress-shear displacement
    - Wheel slip
- Other effects
  - Grousers/lugs
  - Repetitive loading
  - Lateral forces
- Limitations
  - Inhomogeneity
  - Scale effects
  - Slipping and sinking



M.G. Bekker

Shearing Properties of Soil

- Motion of a wheel or track causes shearing at the soil interface
  - Resistance forces generated by soil mass
  - Depends on slip, loading conditions

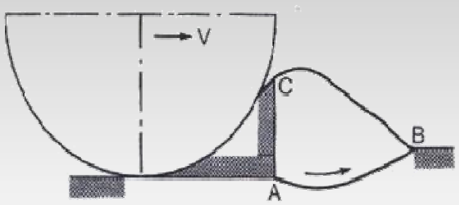


Figure 2.20: Flow patterns and soil wedge formed in front of a locked rigid wheel at 100% skid in sand

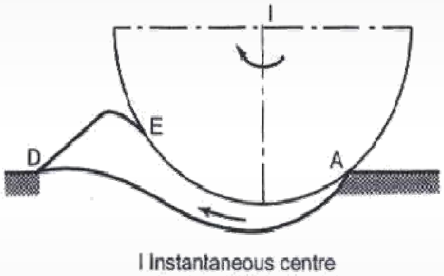


Figure 2.19: Flow patterns beneath a driven rigid wheel at 100% slip in sand

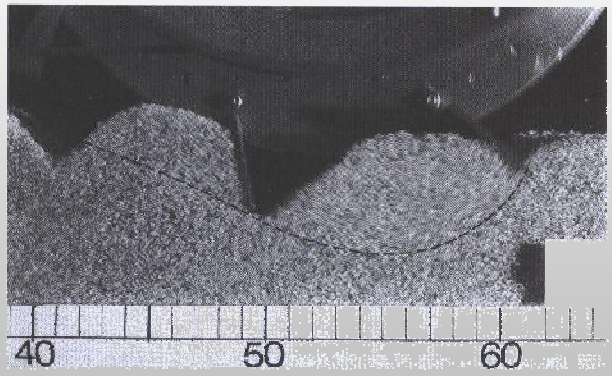
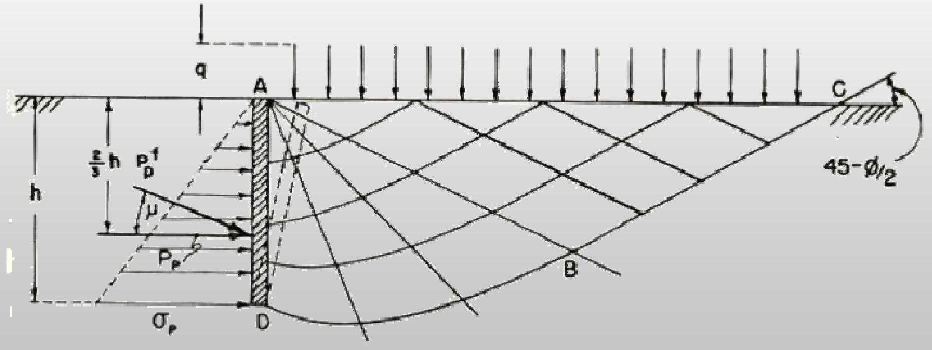


Figure 1.12: Soil flow under the action of grousers of a wheel in sand (Reprinted by permission of *ISTVS* from Wu et al., 1984)

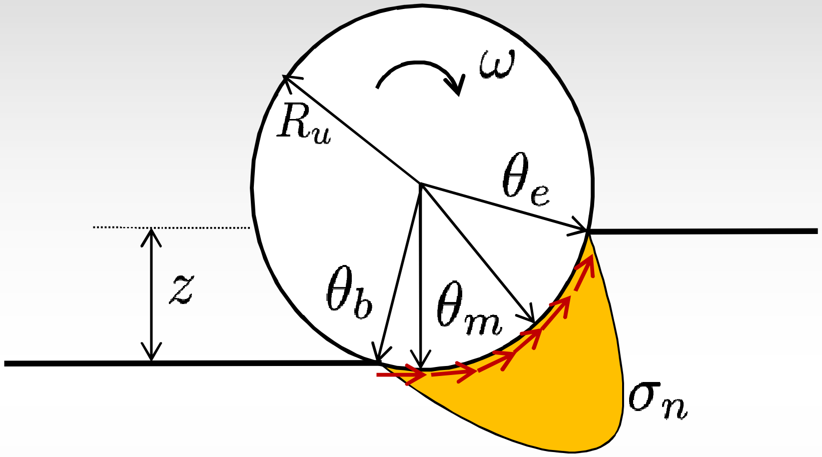


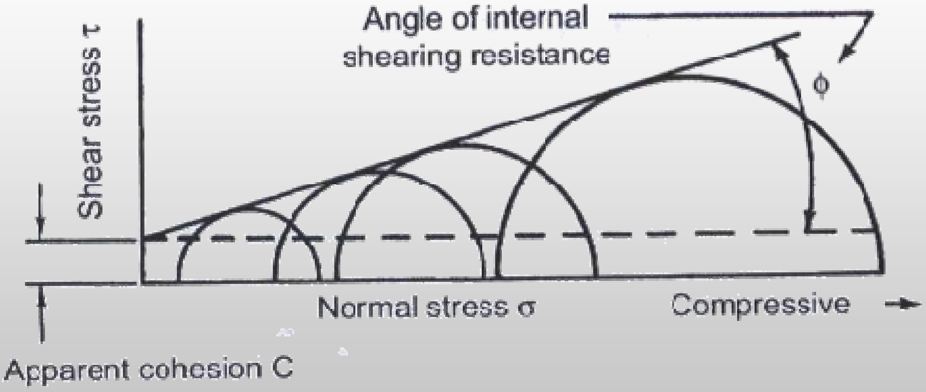
Shearing Properties of Soil

- Shear stress at wheel-soil interface produces traction
- Shear stress is a function of shear displacement
  - Relative motion required to generate traction
    - Non-zero slip ratio
- Soil failure estimated through Mohr-Coulomb failure criterion

$$\tau = c + \sigma \tan \phi$$

  - τ is failure stress
  - c is soil cohesion
  - ϕ is soil internal friction angle





Shearing Properties of Soil

- Can compute shear stress at wheel-terrain interface
  - Janosi-Hanamoto formulation

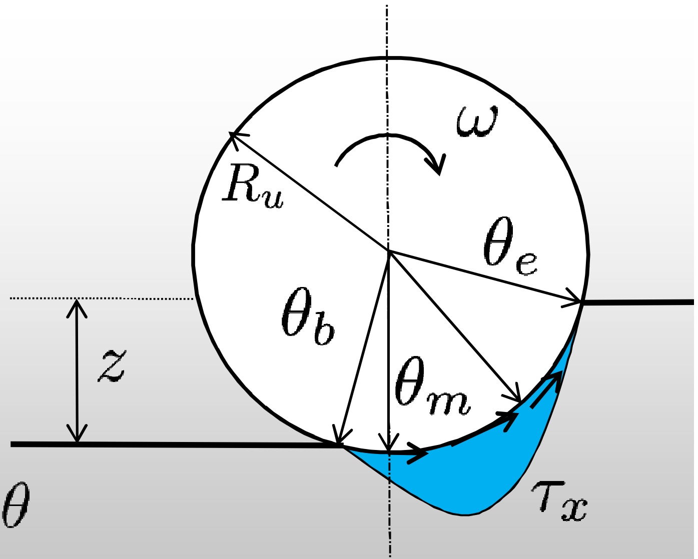
Soil shear displacement

Limit tangential stress

$$\tau_x(\theta) = \tau_{max}\left(1 - e^{\frac{-j_x}{k_x}}\right)$$

Soil shear deformation modulus

$$\tau_{max} = c + \sigma_n(\theta)\tan\phi$$



- Soil shear displacement

$$j_x(\theta) = \int_{\theta_b}^{\theta_e} R_u[1 - (1 - s_d)\cos(\theta)]\,d\theta$$

Z. Janosi and B. Hanamoto. Analytical determination of drawbar pull as a function of slip for tracked vehicles in deformable soils, Proc. ISTVS

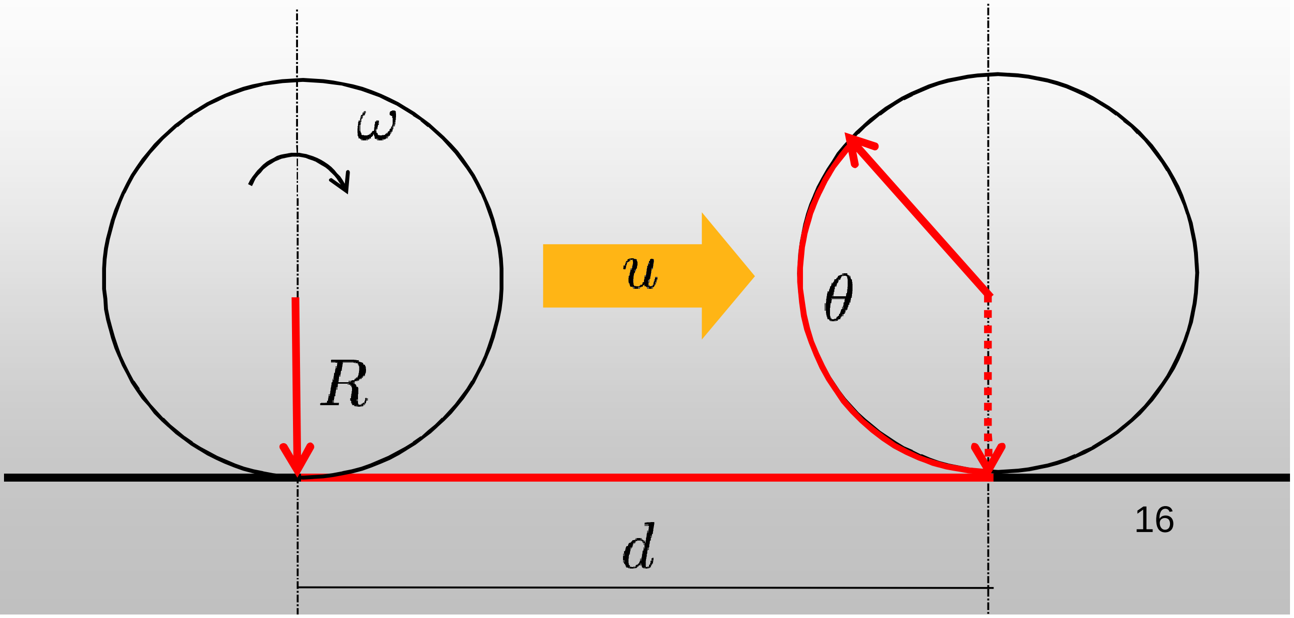
Slip Ratio

- Slip ratio is measure of relative motion between wheel and terrain surface
  - For driven wheel, distance traveled is less than that in free rolling
  - When slip ratio = 1, spinning in place
  - When slip ratio = 0, pure rolling
  - When slip ratio = -1, skidding

$$u \leq \omega R$$

$$d \leq \theta R$$

$$s_d = 1 - \frac{u}{\omega R}$$



Terrain Interaction Forces

- Forces between wheel and terrain can be computed from stress distribution along contact path
- Vertical load

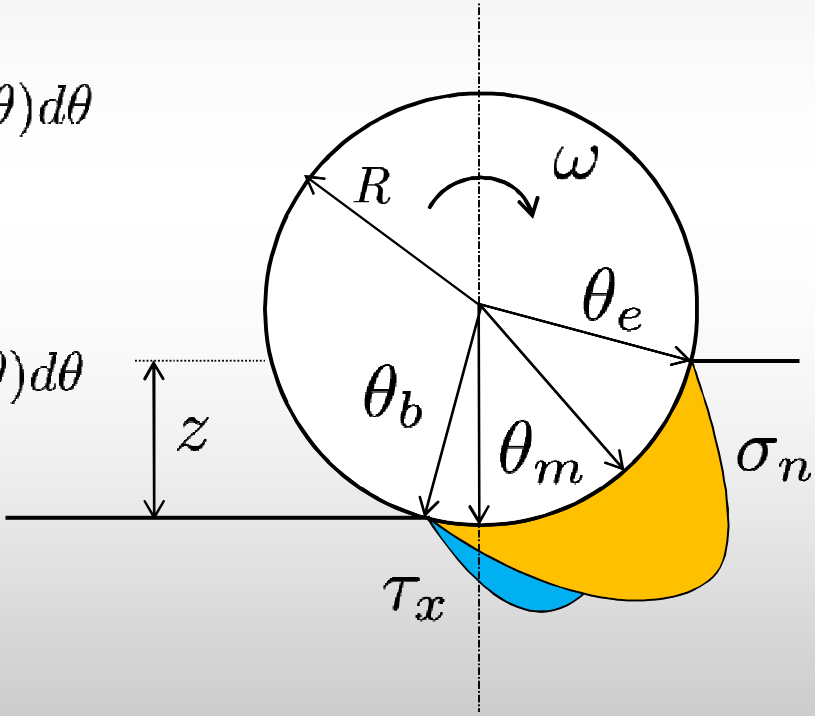
$$W = bR\int_{\theta_b}^{\theta_e} \tau_x(\theta)\sin(\theta) + \sigma_n(\theta)\cos(\theta)d\theta$$

- Longitudinal force

$$F_x = bR\int_{\theta_b}^{\theta_e} \tau_x(\theta)\cos(\theta) - \sigma_n(\theta)\sin(\theta)d\theta$$

- Torque on wheel axle

$$T = bR^2\int_{\theta_b}^{\theta_e} \tau_x(\theta)d\theta$$



Summary

- Stresses at wheel-terrain interface
  - Decompose into normal and shear stresses
  - Modeled with semi-empirical formulations
  - Integration yields forces acting on vehicle
- Given
  - Terrain properties
  - Slip
  - Loading conditions
- Can compute
  - Sinkage
  - Thrust
  - Required torque

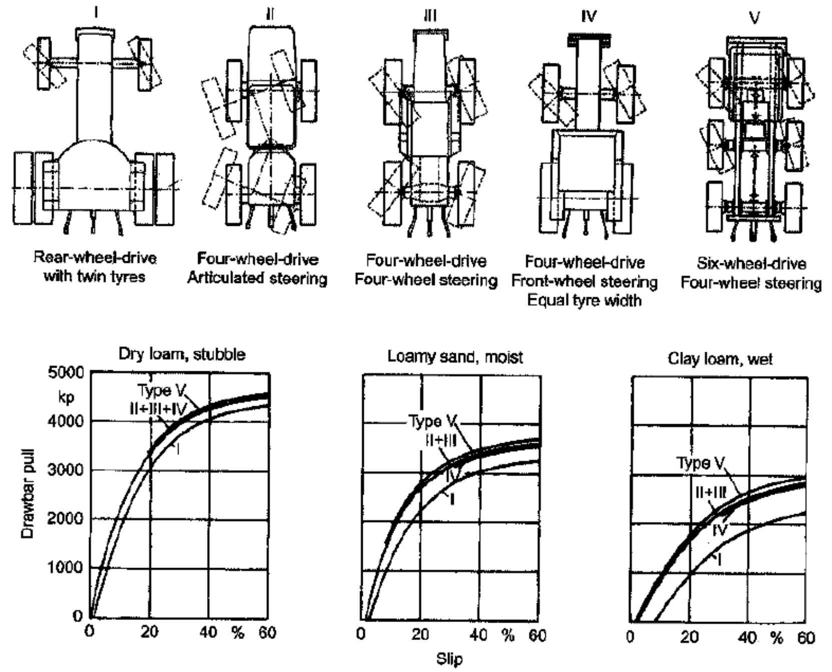


Figure 1.3: Comparison of various configurations for agricultural tractors (Reprinted by permission of *ISTVS* from Sohne, 1976)

Terramechanics Principles

- Fundamental relations
  - Pressure-sinkage
  - Shear stress-shear displacement
    - Wheel slip
- Other effects
  - Grousers/lugs
  - Lateral forces
  - Repetitive loading
- Limitations
  - Inhomogeneity
  - Scale effects
  - Slipping and sinking

M.G. Bekker

Effect of Grousers

- Grousers are small features on wheel surface
  - Designed to improve traction and climbing performance
- Have been modeled through Terzaghi's bearing capacity theory

Pathfinder, MER, and MSL wheels

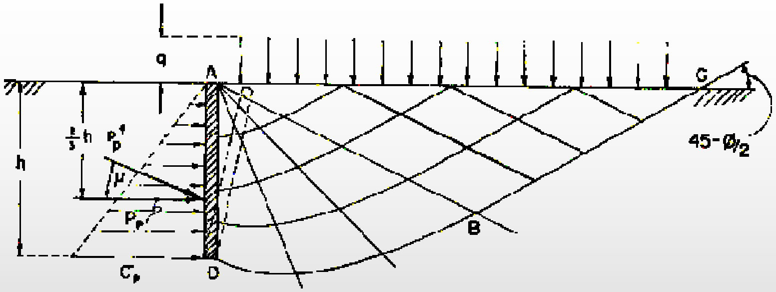


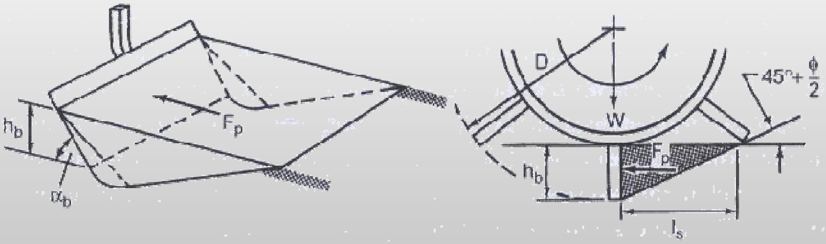
Fig. 69

The value of the force P_p assumed for $\mu = 0$ may be calculated by integrating the pressure σ_p determined by equation (134):

$$P_p = \int_0^h \sigma_p \, dz = \int_0^h (q N_\phi + 2c\sqrt{N_\phi} + \gamma z N_\phi) \, dz$$

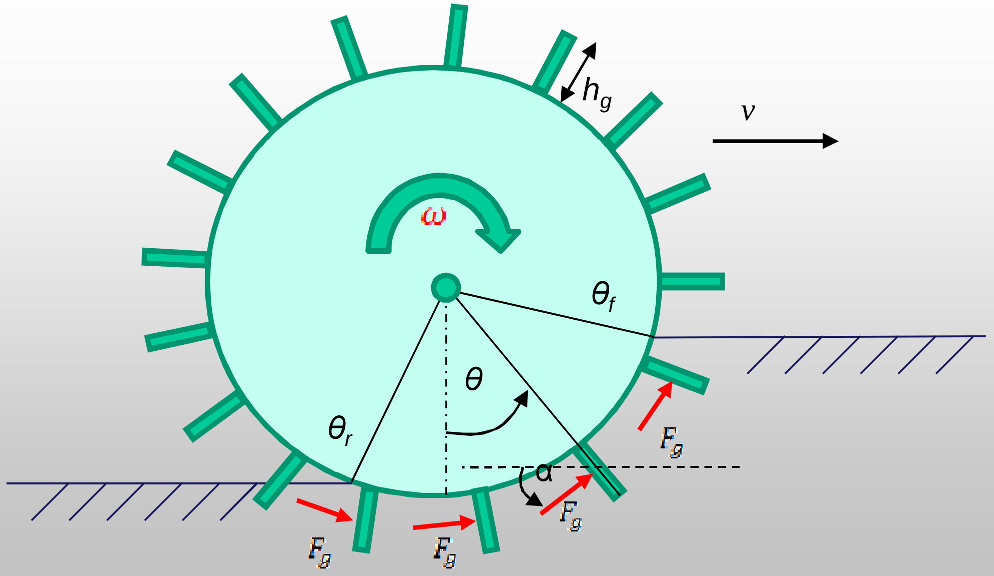
and

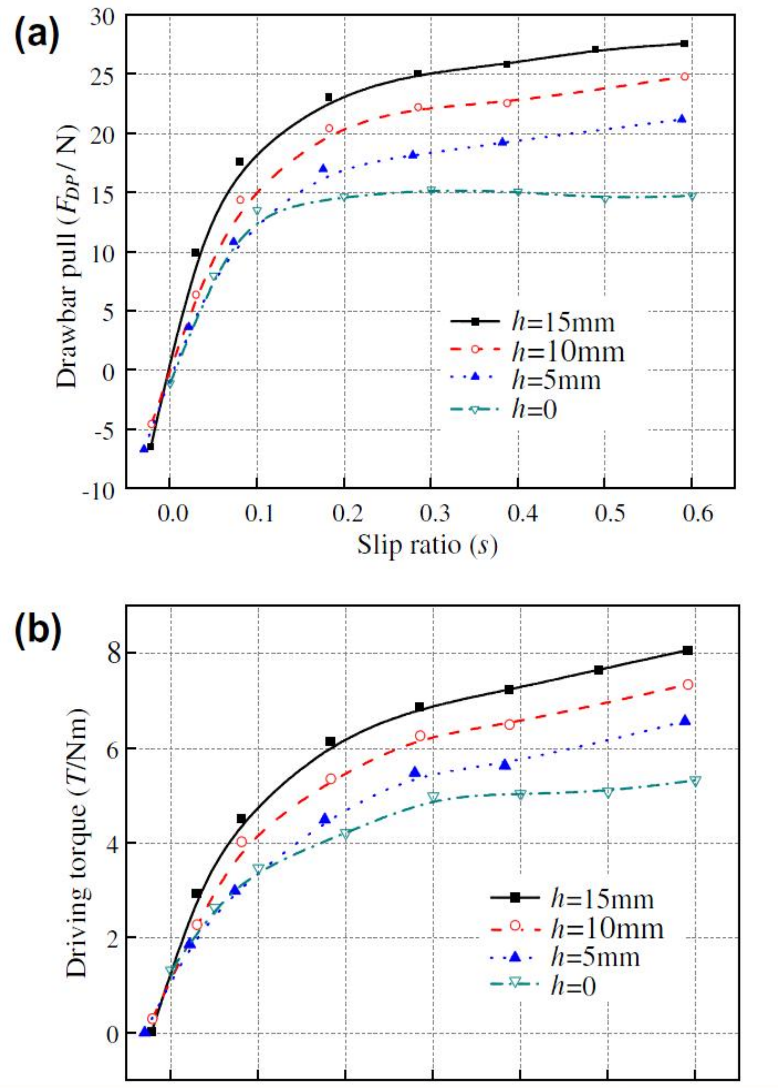
$$P_p = q h N_\phi + 2ch\sqrt{N_\phi} + \frac{1}{2}\gamma h^2 N_\phi .$$



Effect of Grousers

- Grouser effect has also been empirically studied
  - Grouser height, spacing, geometry affect torque, traction, turning performance





Ding L. et al. Journal of Terramechanics 48, 2011, 27-45

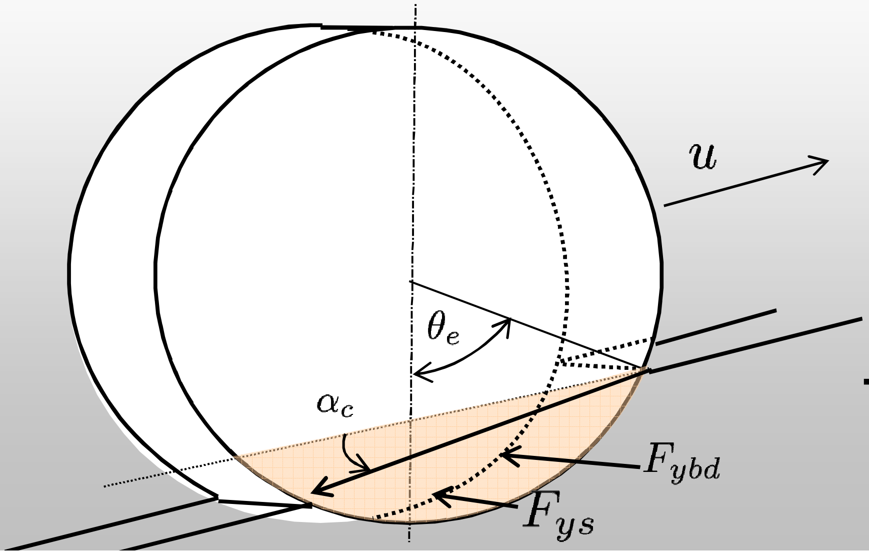
Lateral Forces

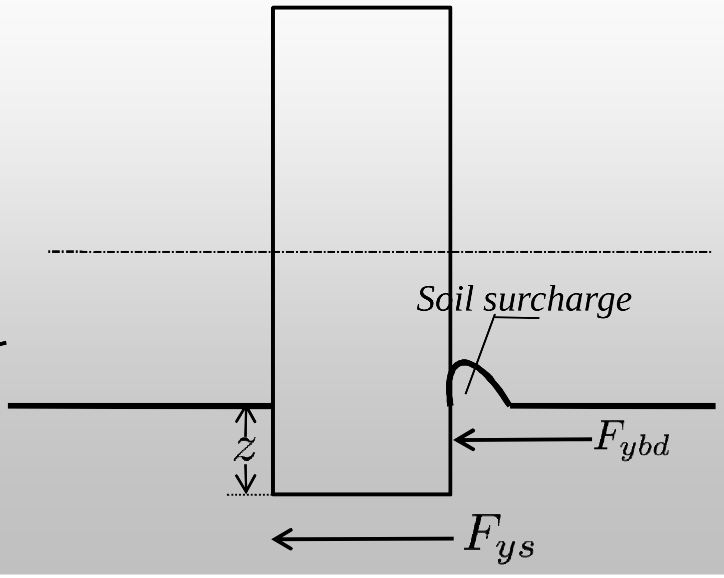
- Lateral forces act on wheel sidewall during turning
  - Forces arise from soil shearing and bulldozing

$$F_y = F_{ys} + F_{ybd}$$

$$F_{ybd} = w\int_{\theta_b}^{\theta_e}(\gamma_s z N_\gamma + cN_c + qN_q)\cos(\delta_f)d\theta$$

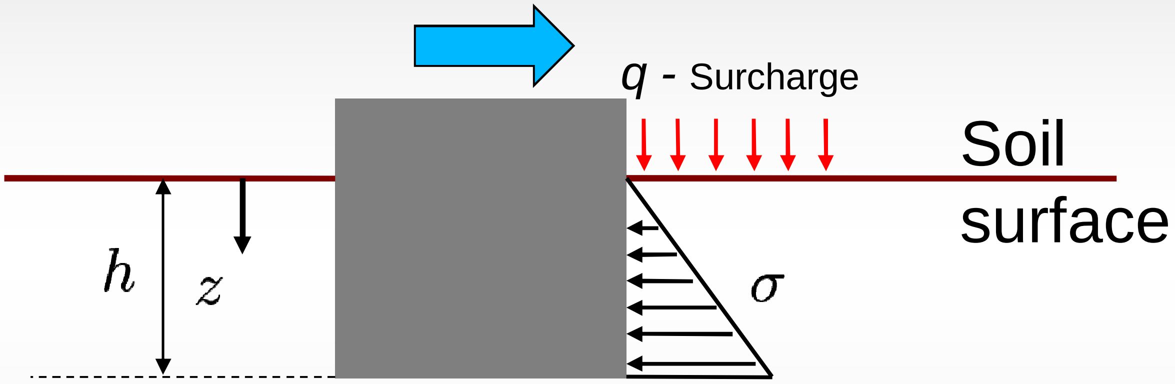
$$F_{ys} = b\int_{\theta_b}^{\theta_e}(c+\sigma_n(\theta)\tan(\phi))\left(1-e^{\frac{-j_y}{k_y}}\right)d\theta$$





Lateral Force - Bulldozing

- Like grouser effect, bulldozing is typically modeled through soil bearing capacity analysis



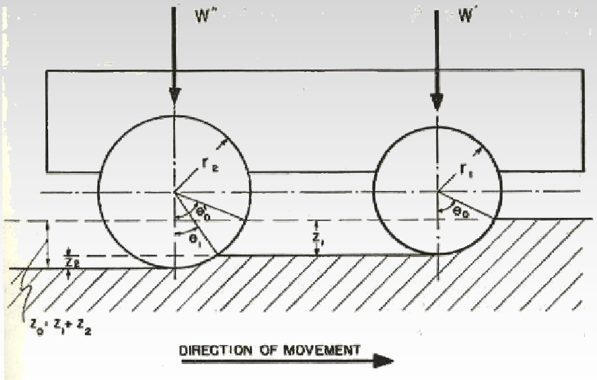
$$\sigma = \gamma z N_\gamma + c N_c + q N_q \text{ [Pa]}$$

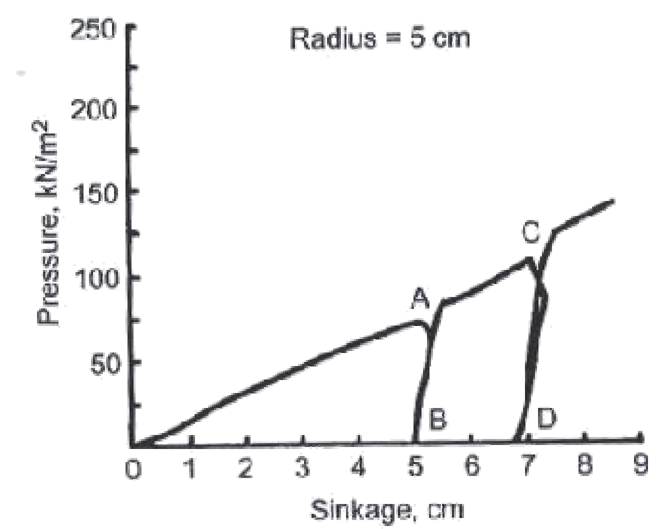
- *N*-factors are function of soil angle of internal friction

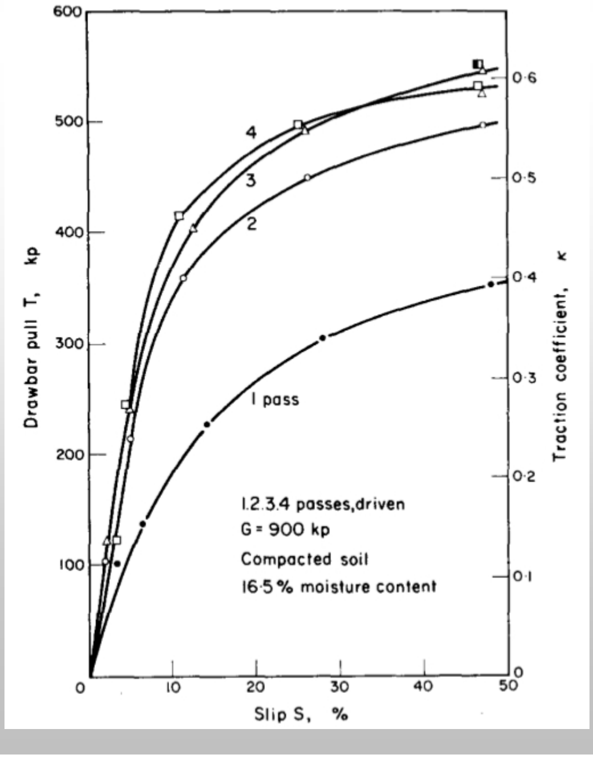
$$N_\gamma = \frac{2(N_q + 1)\tan\phi}{1 + 0.4\sin 4\phi} \qquad N_c = \frac{N_q - 1}{\tan\phi} \qquad N_q = \frac{e^{(1.5\pi - \phi)\tan\phi}}{2\,cos^2(\pi/4 + \phi/2)}$$

Repetitive Loading

- Rover trailing wheels may pass through soil deformed by leading wheels
  - Repetitive loading alters soil behavior
  - Increases compaction (relative density)

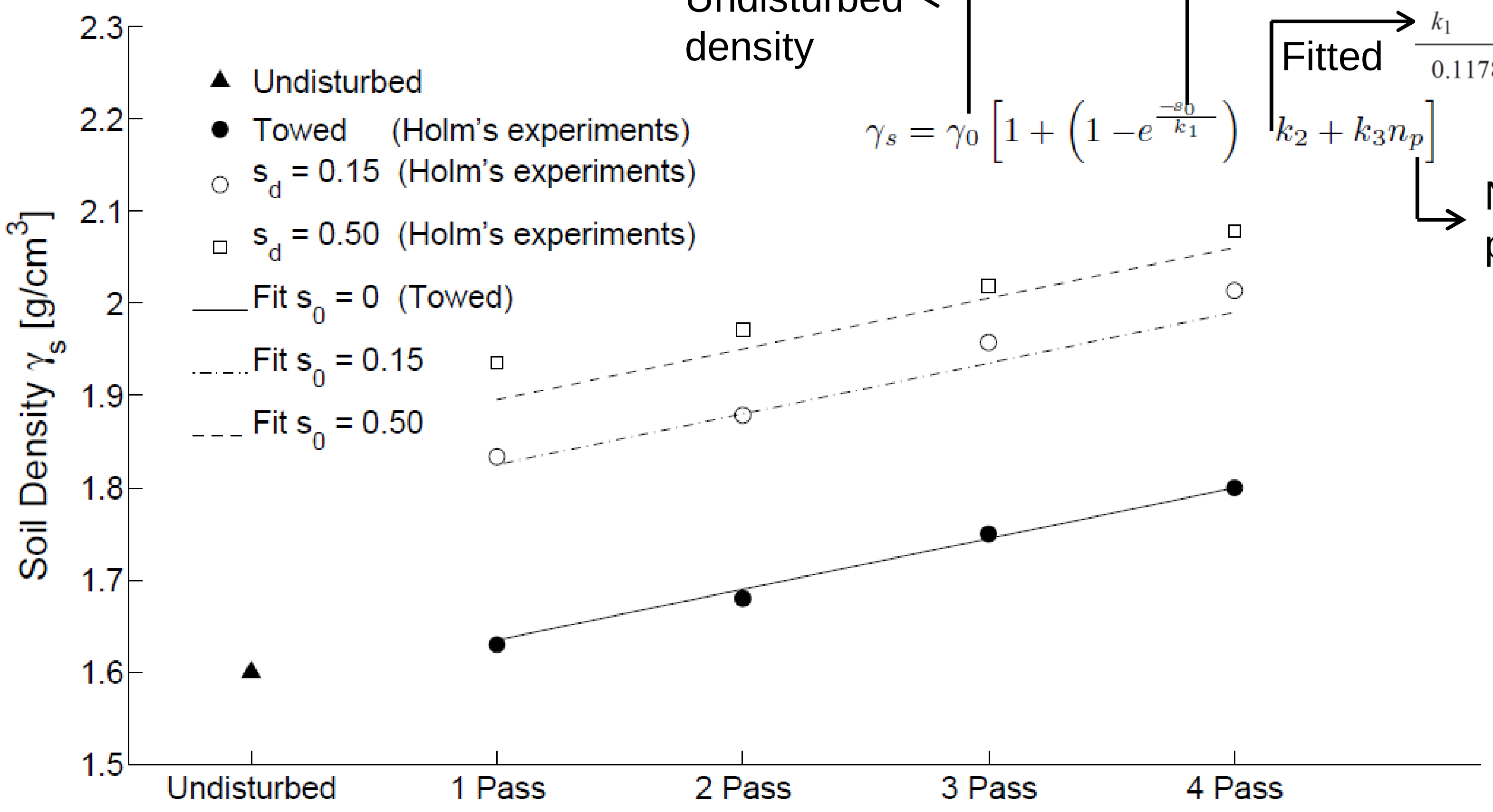






Repetitive Loading

- Multi pass can be modeled by modifying soil parameters according to number and type of passes



$$\gamma_s = \gamma_0 \left[1 + \left(1 - e^{\frac{-s_0}{k_1}}\right) k_2 + k_3 n_p\right]$$

Undisturbed density

Wheel slip of previous pass

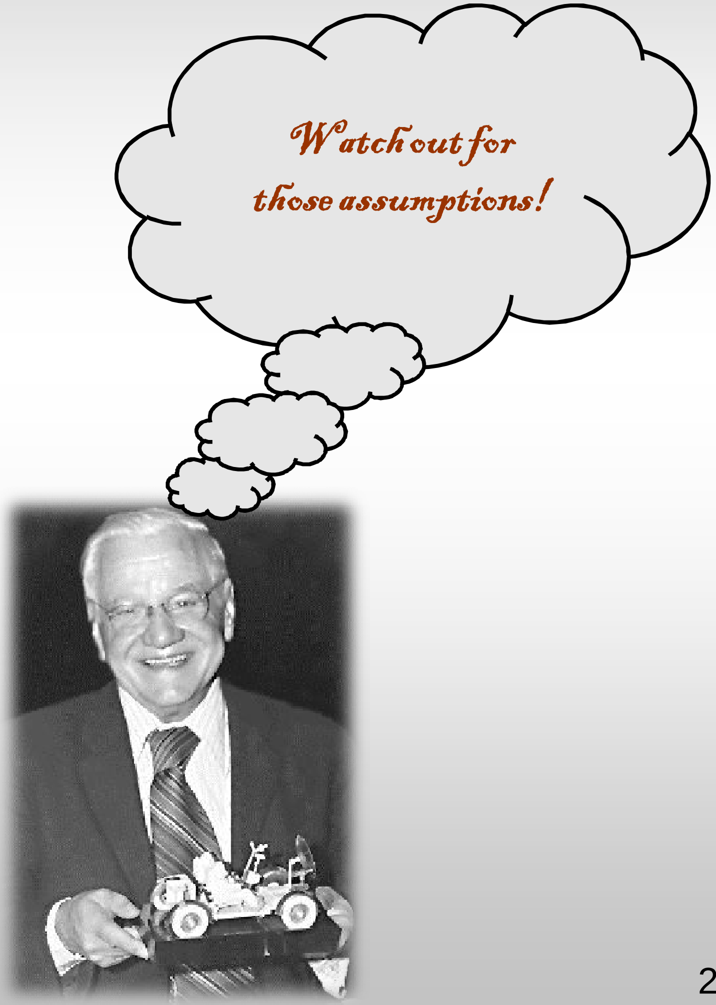
Fitted

k_1	k_2	k_3
0.1178	0.1672	0.0348

Number of passes

Terramechanics Principles

- Fundamental relations
  - Pressure-sinkage
  - Shear stress-shear displacement
    - Wheel slip
- Other effects
  - Grousers/lugs
  - Lateral forces
  - Repetitive loading
- Limitations
  - Inhomogeneity
  - Scale effects
  - Slipping and sinking



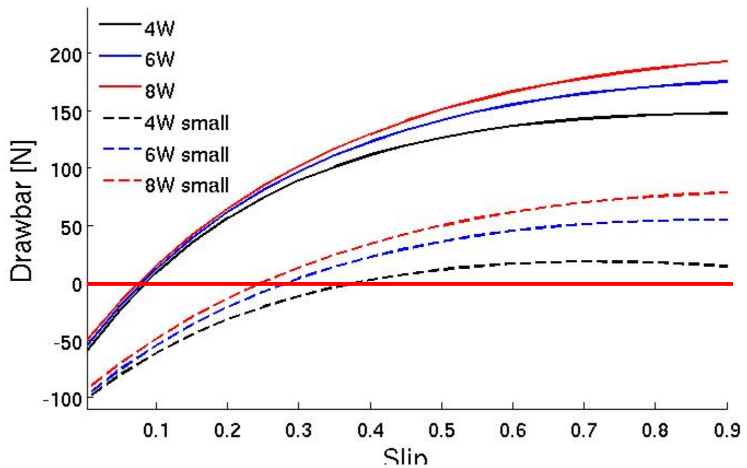
M.G. Bekker

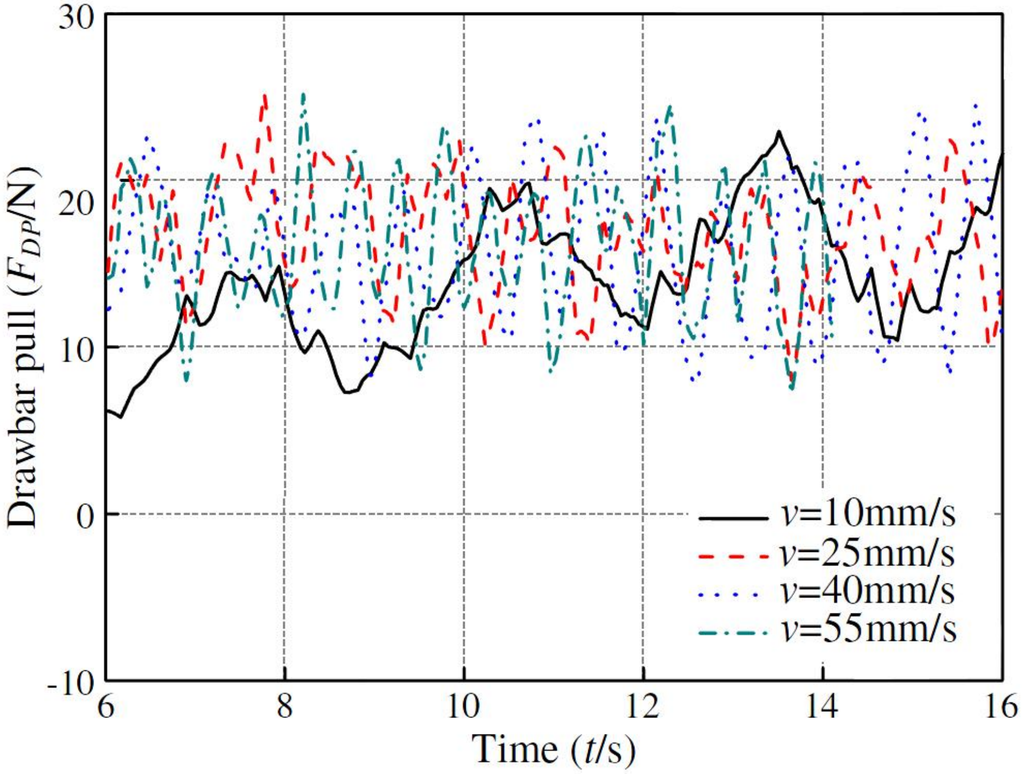
Classical Model Limitations

- Terramechanics developed in context of large vehicles, for design trade space analysis
  - Would like to apply to smaller, lighter systems, for dynamic sim

- Key limitations
  - Effect of terrain inhomogeneity
    - Soil condition dependence
      - Layering, relative density, moisture content
  - Scale effects
    - Parameter scale dependence (non-intrinsic soil properties)
  - Effects related to slipping and sinking
    - Slip ratio definition
    - Rate dependence

Terrain Inhomogeneity (1)





Classical Model Limitations

- Terramechanics developed in context of large vehicles, for design trade space analysis
  - Would like to apply to smaller, lighter systems, for dynamic sim

- Key limitations
  - Effect of terrain inhomogeneity
    - Soil condition dependence
      - Layering, relative density, moisture content
  - Scale effects
    - Parameter scale dependence (non-intrinsic soil properties)
  - Effects related to slipping and sinking
    - Slip ratio definition
    - Rate dependence

Terrain Inhomogeneity

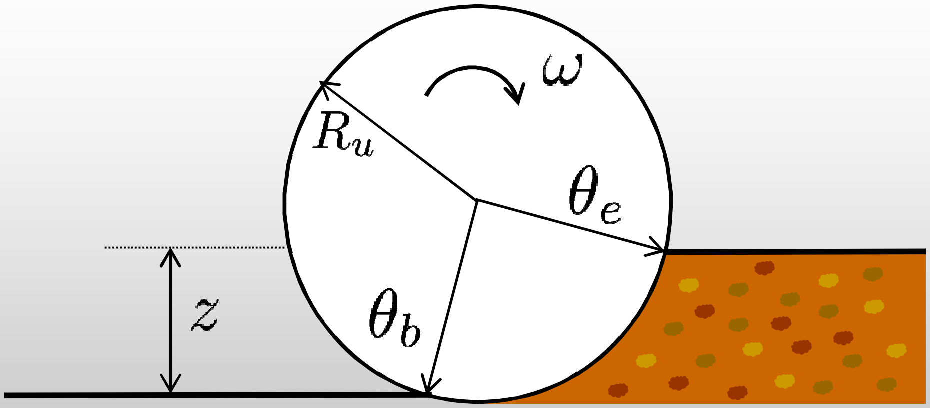
- Pressure-sinkage relation characterizes wide range of terrains with single equation
    - Loose, granular soils, crusty materials, clay

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi \right) z^n$$

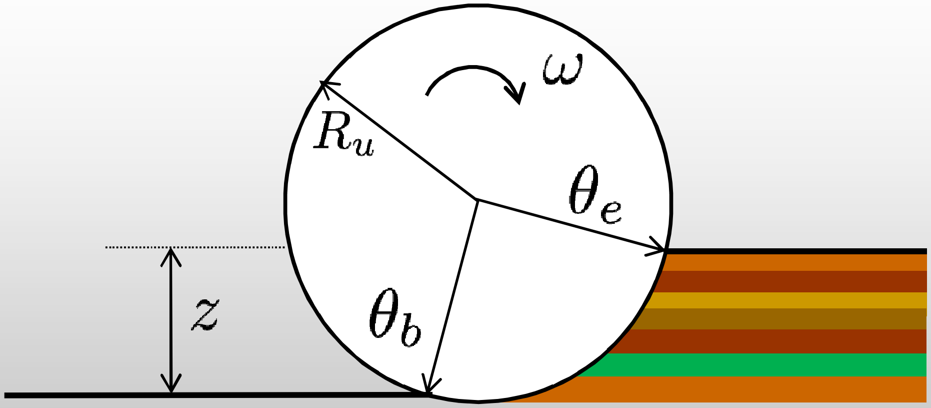
- Observations: significant experimental variation wrt soil condition
    - Layering
    - Relative density
    - Moisture content

Terrain Inhomogeneity (1)

- Bekker theory assumes homogenous soil
  - Soil is often layered, inhomogeneous
- Lack of analytical formulations for pressure-sinkage, shear stress-shear deformation



Inhomogeneous



Layered

Terrain Inhomogeneity (1)

- Pressure-sinkage relations

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

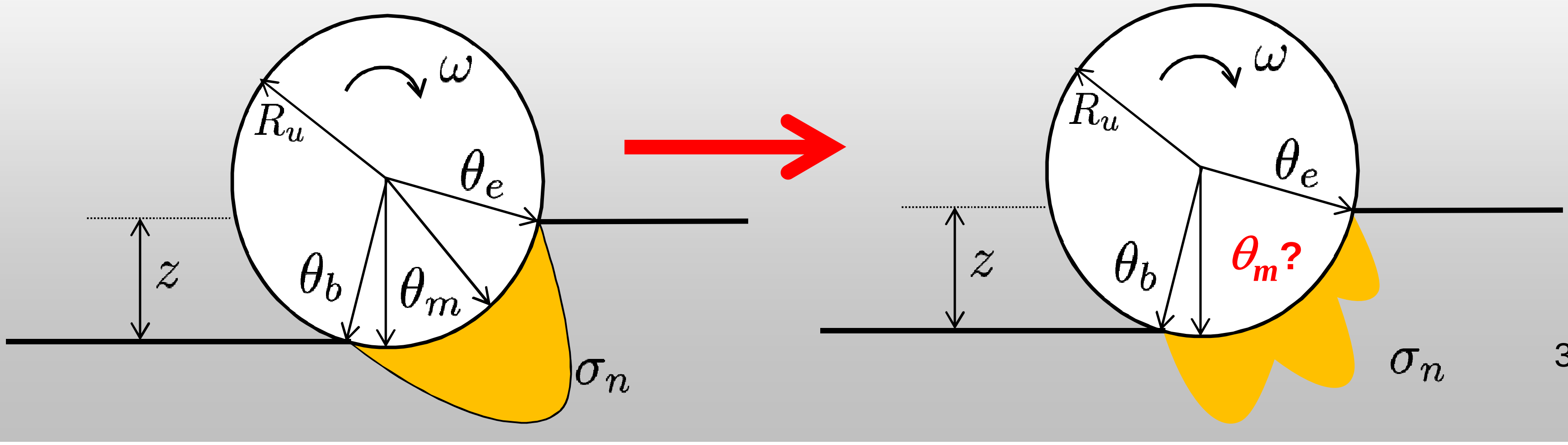
$$\sigma_{nf} = \left(\frac{k_c}{b} + k_\phi\right)\left[R\left(\cos(\theta) - \cos(\theta_e)\right)\right]^n \qquad \boxed{\theta_m} < \theta \leq \theta_e$$

$$\sigma_{nr} = \left(\frac{k_c}{b} + k_\phi\right)\left[R\left(\cos\left(\theta_e - \left(\frac{\theta - \theta_r}{\theta_N - \theta_r}\right)(\theta_e - \theta_N)\right) - \cos(\theta_e)\right)\right]^n \qquad \theta_b \leq \theta \leq \boxed{\theta_m}$$

How to define?

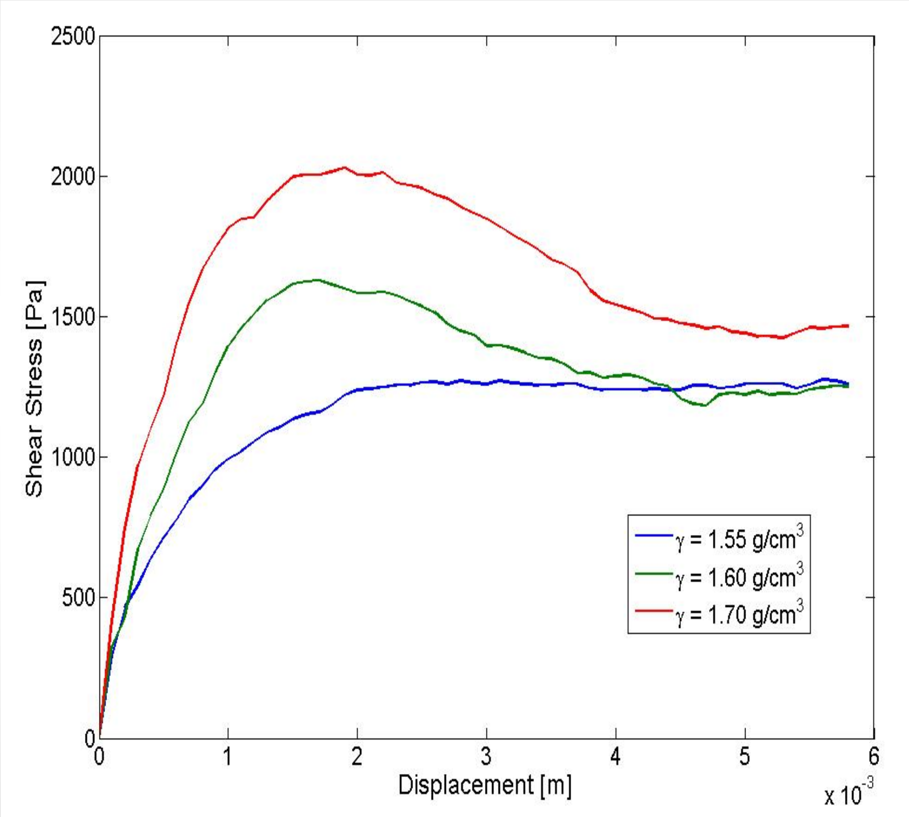
- Shear stress-shear displacement

$$\tau_x(\theta) = \tau_{max}\left(1 - e^{\frac{-j_x}{k_x}}\right) \qquad \tau_{max} = c + \sigma_n(\theta)\tan\phi$$



Terrain Inhomogeneity (2)

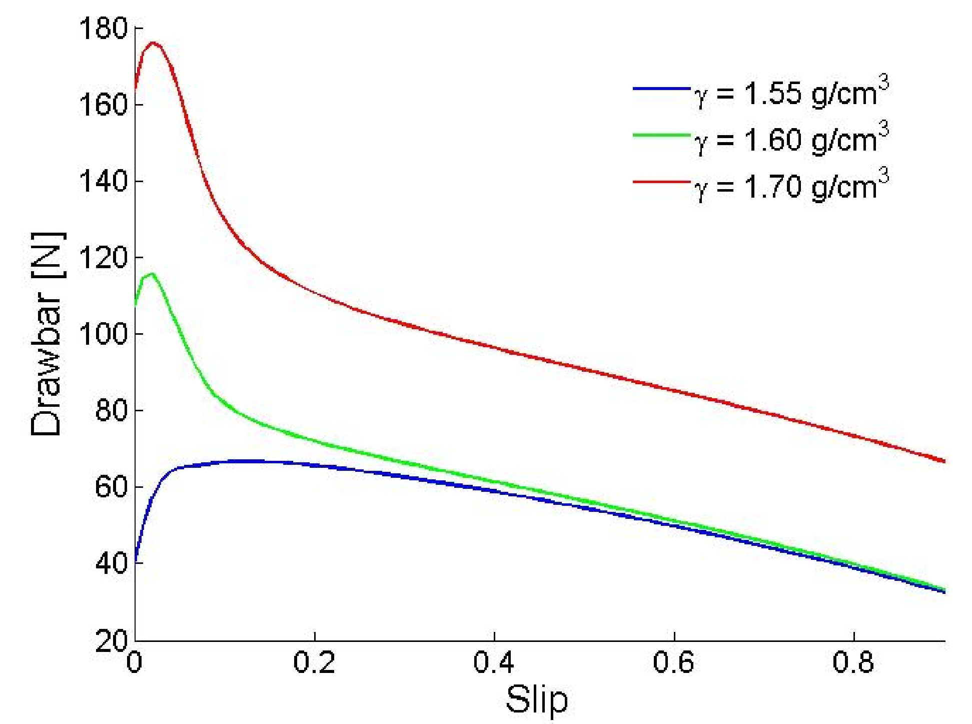
- Bekker theory (generally) ignores soil state
- Large vehicles tend to compact terrain to dense state upon passage
  - For small rovers, weight is insufficient to compact soil
- Relative density can strongly influence shear stress at interface
  - Strong influence on thrust
  - Strong influence on torque during digging/scooping



Shear box test of MMS

Terrain Inhomogeneity (2)

- Bekker theory (generally) ignores soil state
- Large vehicles tend to compact terrain to dense state upon passage
  - For small rovers, weight is insufficient to compact soil
- Relative density can strongly influence shear stress at interface
  - Strong influence on thrust
  - Strong influence on torque during digging/scooping



Terrain Inhomogeneity

- Questions (Solutions?)
  - How to compute sinkage in inhomogeneous soil?
    - Express sinkage in integral form (layered)?

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

    - Effective parameters for mixed soils?
  - How to compute failure of layered (crusty) soil?

$$\tau_{max} = c + \sigma_n(\theta) \tan\phi$$

    - Piecewise formulation?
      - Smoothness of stress distribution?
  - How to represent parameters?
    - Intervals? Distributions?
    - State dependent? (For all soils, or only some?)
  - How to represent governing equations?
    - Deterministic? Stochastic?

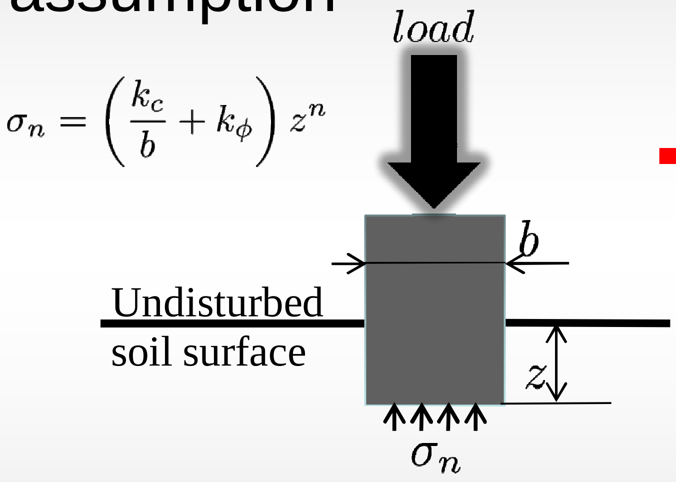
Classical Model Limitations

- Terramechanics developed in context of large vehicles, for design trade space analysis
  - Would like to apply to smaller, lighter systems, for dynamic sim

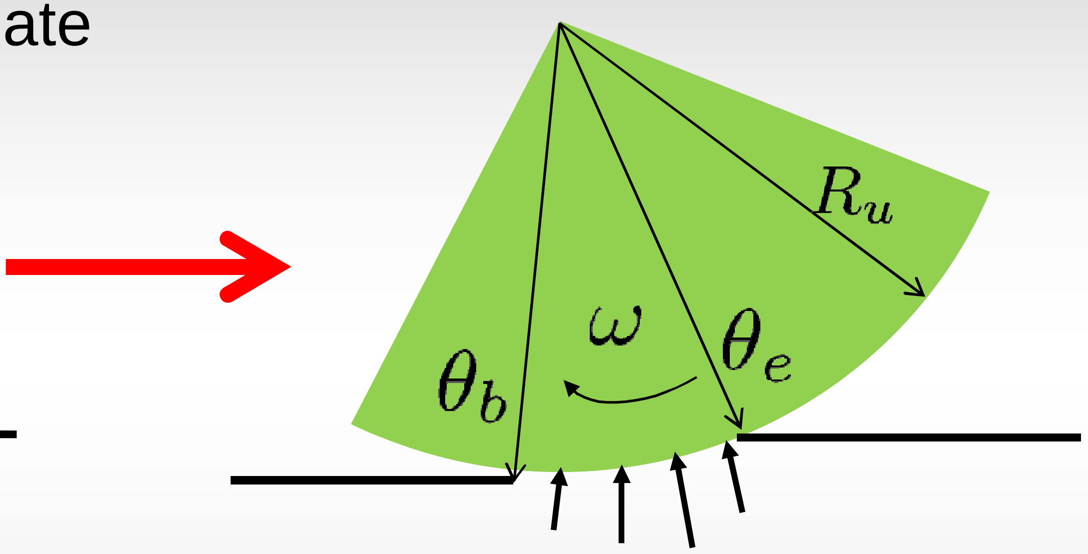
- Key limitations
  - Effect of terrain inhomogeneity
    - Soil condition dependence
      - Layering, relative density, moisture content
  - Scale effects
    - Parameter scale dependence (non-intrinsic soil properties)
  - Effects related to slipping and sinking
    - Slip ratio definition
    - Rate dependence

Scale Effects (1)

- Pressure-sinkage relations developed under flat plate assumption



$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

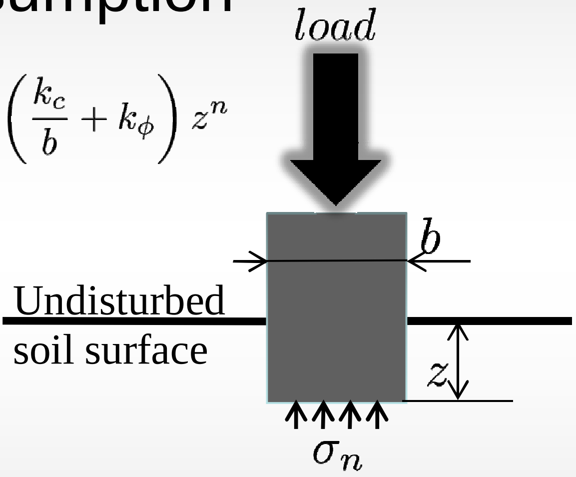


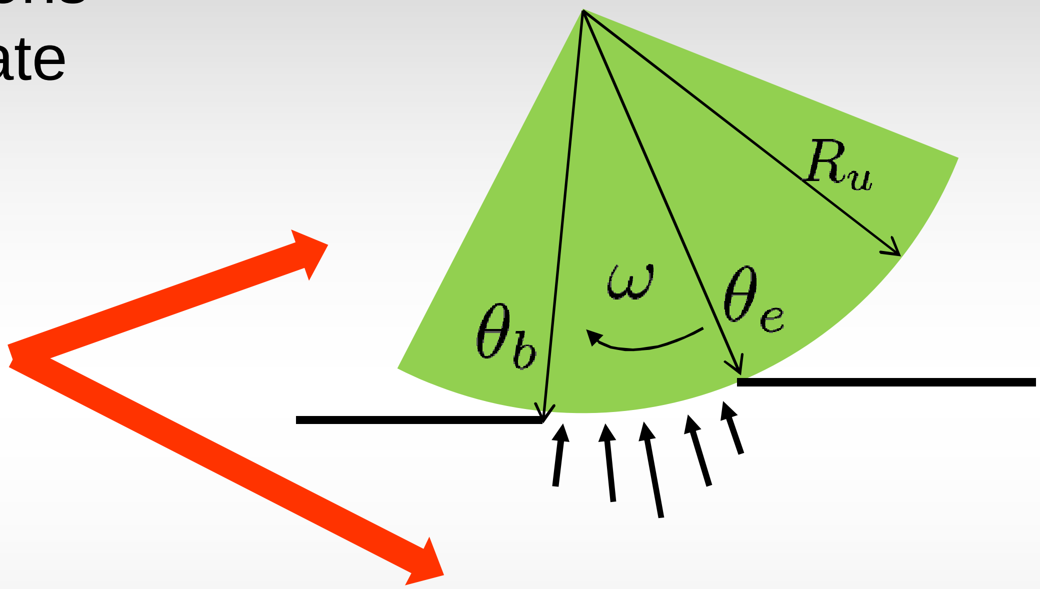
- Reasonable for large vehicles
  - Uniform stress distribution at interface

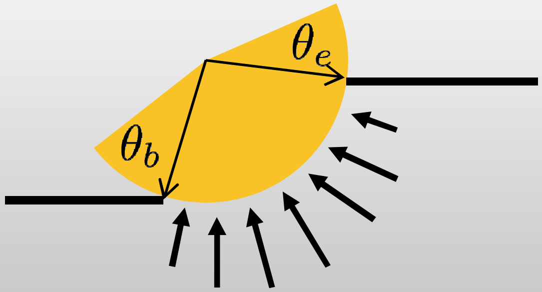
Scale Effects (1)

- Pressure-sinkage relations developed under flat plate assumption

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$





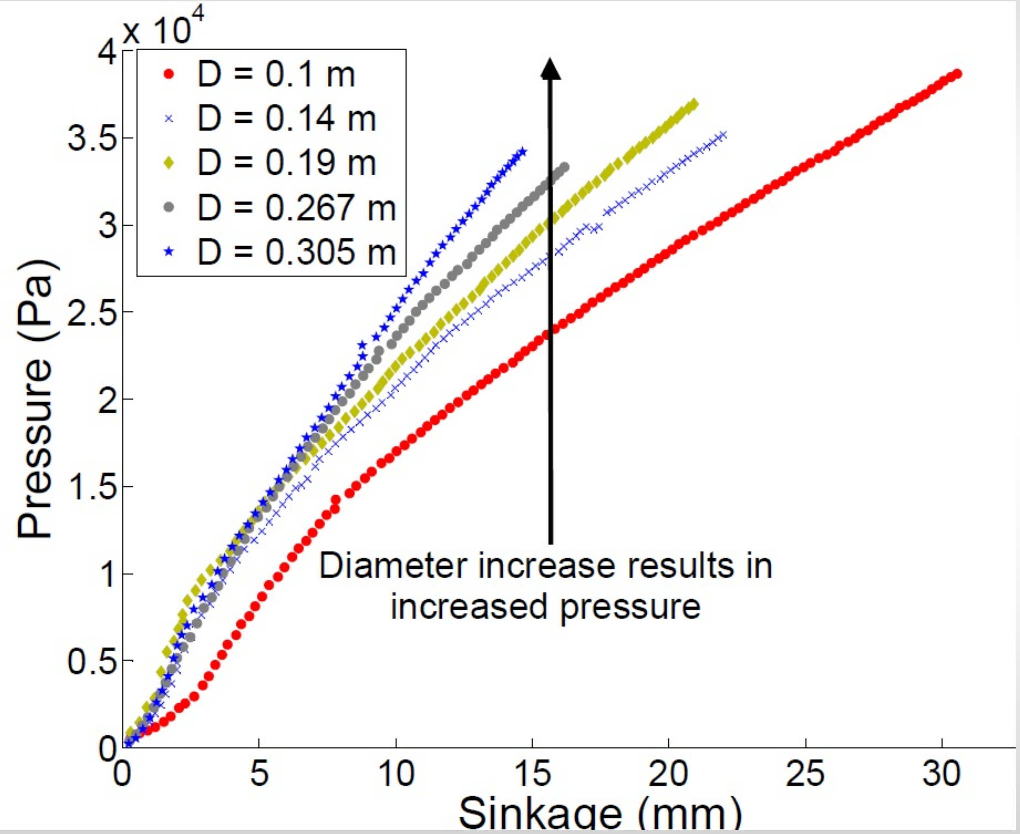


- What about for small vehicle, with high wheel curvatures?
  - Stress distribution at interface non-uniform
  - Component of normal stress balances load

Scale Effects (1)

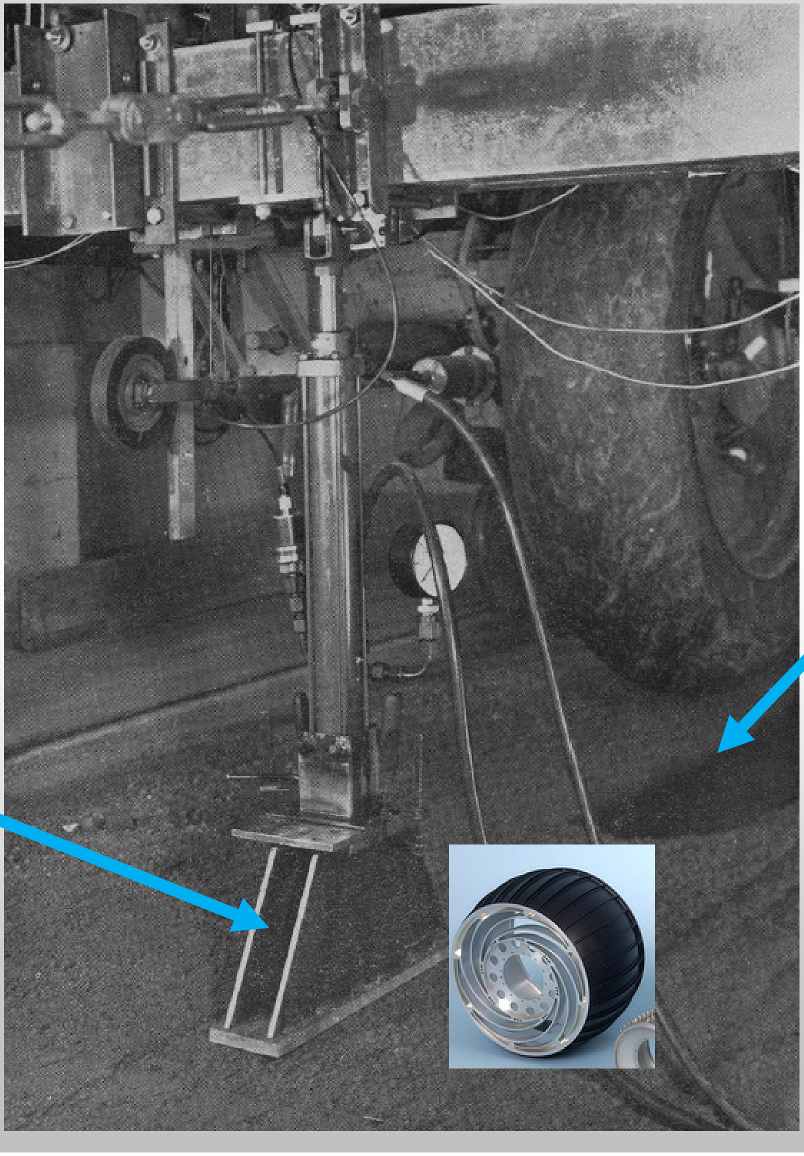
- Result: Poor prediction of sinkage
- Why is this?
  - Intrinsic parameters not really intrinsic

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$



Diameter increase results in increased pressure

Scale Effects (1)



Tire imprint

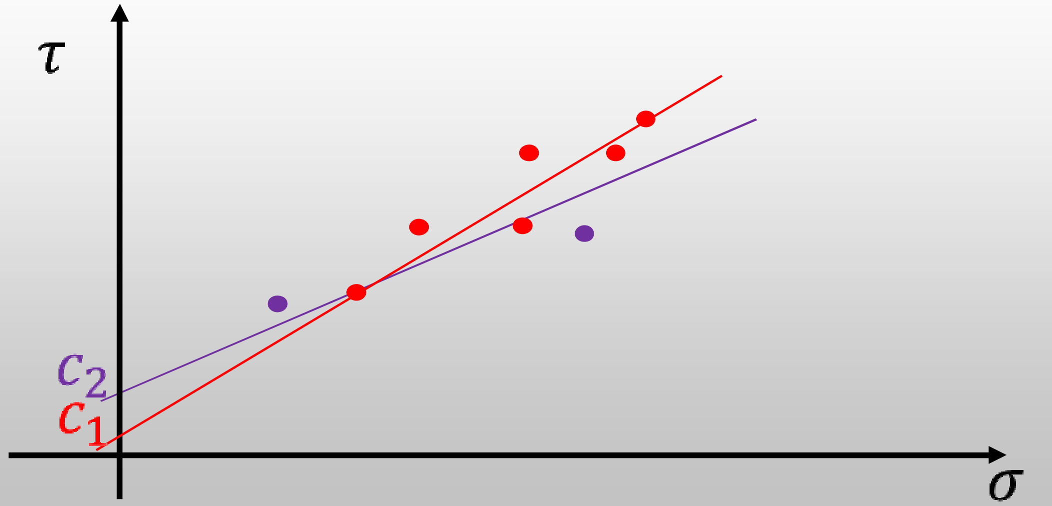
Bevameter plate

Scale Effects (2)

- Soil shear failure is governed by soil cohesion and internal friction angle

$$\tau_{max} = c + \sigma_n(\theta)\tan\phi$$

- Cohesion often measured at high normal stress
  - At low normal loads, effect of cohesion can dominate

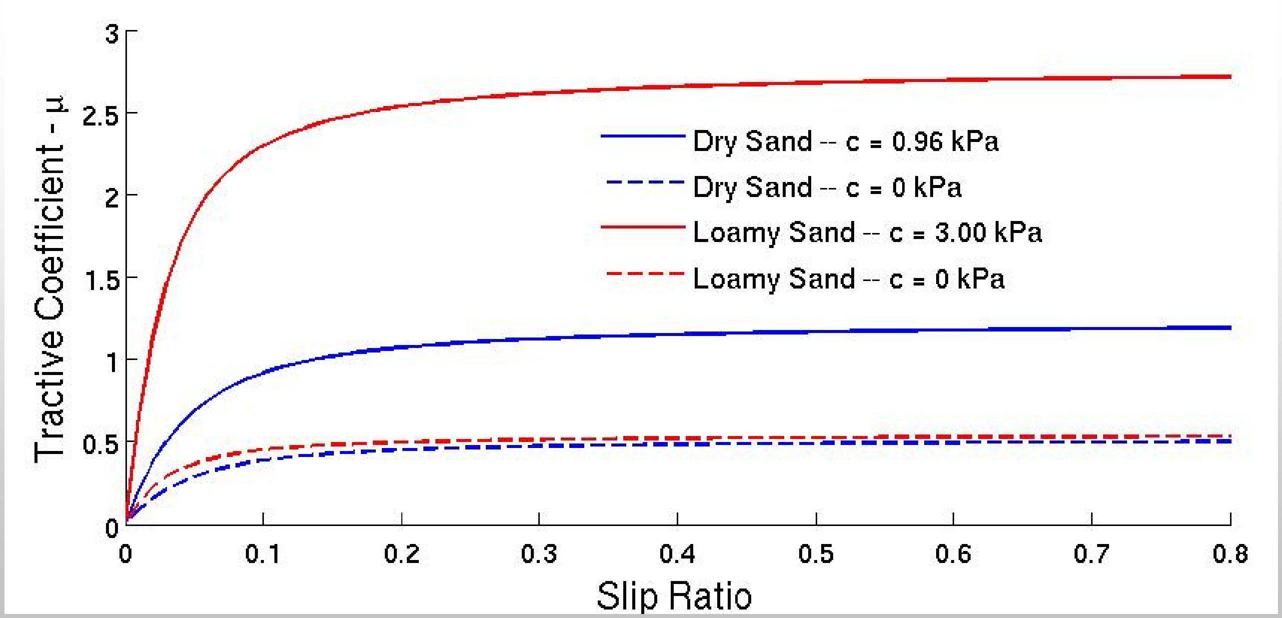


Scale Effects (2)

- Soil shear failure is governed by soil cohesion and internal friction angle

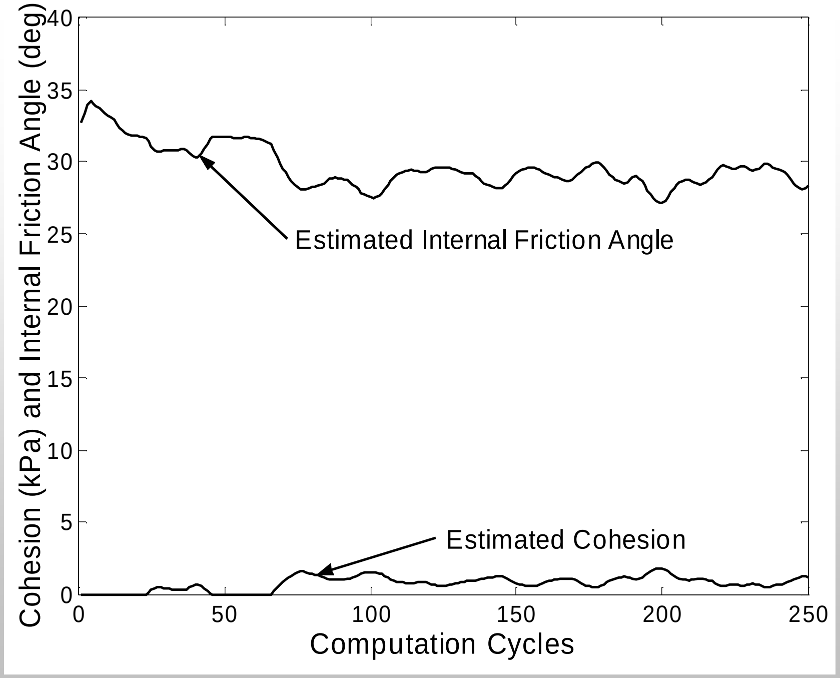
$$\tau_{max} = c + \sigma_n(\theta) \tan \phi$$

- Cohesion often measured at high normal stress
  - At low normal loads, effect of cohesion can dominate



Scale Effects

- Questions (Solutions?)
  - Can we formulate terramechanics relations with intrinsic parameters?
    - Consistent results across scales
  - Can we develop in situ measurement/estimation procedures for parameter estimation?
  - Can we develop lab test devices/procedures for measurement at low normal stress?



Classical Model Limitations

- Terramechanics developed in context of large vehicles, for design trade space analysis
  - Would like to apply to smaller, lighter systems, for dynamic sim

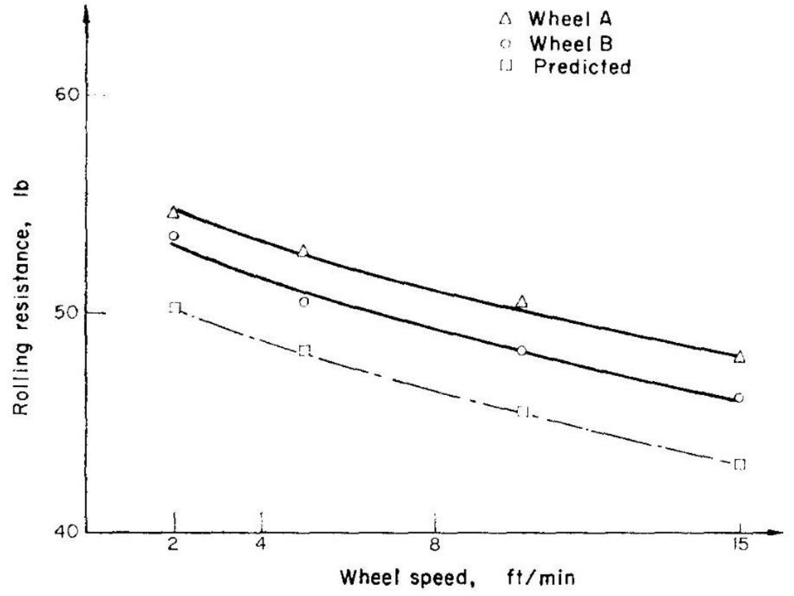
- Key limitations
  - Effect of terrain inhomogeneity
    - Soil condition dependence
      - Layering, relative density, moisture content
  - Scale effects
    - Parameter scale dependence (non-intrinsic soil properties)
  - Effects related to slipping and sinking
    - Slip ratio definition
    - Rate dependence

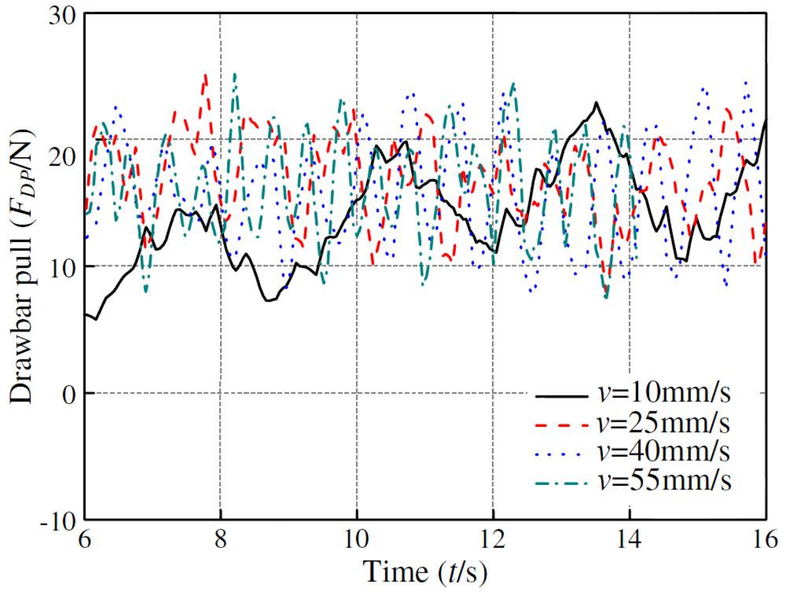
Slipping and Sinking (1)

- Terramechanics models are not rate dependent

- Studies on large wheels show that at higher velocity[1,2]:
  - Sinkage decreases
  - Traction improves

- Experiments[3] on small wheels have suggested little influence

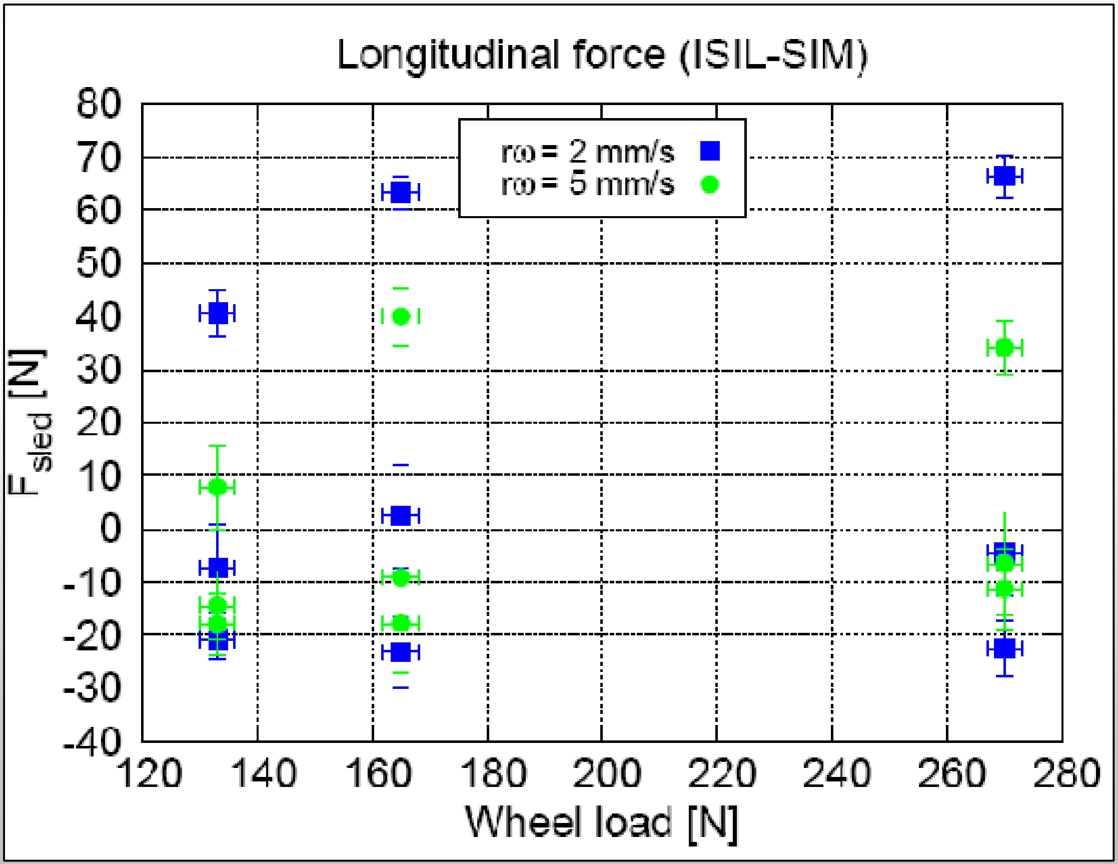




1. Shmulevic I. et al./Journal of Terramechanics 35,1998, 189-207
2. Pope R.G./ Journal of Terramechancis 8(1), 1971, 51-58
3. Ding L. et al./Journal of Terramechanics 48, 2011, 27-45

Slipping and Sinking (1)

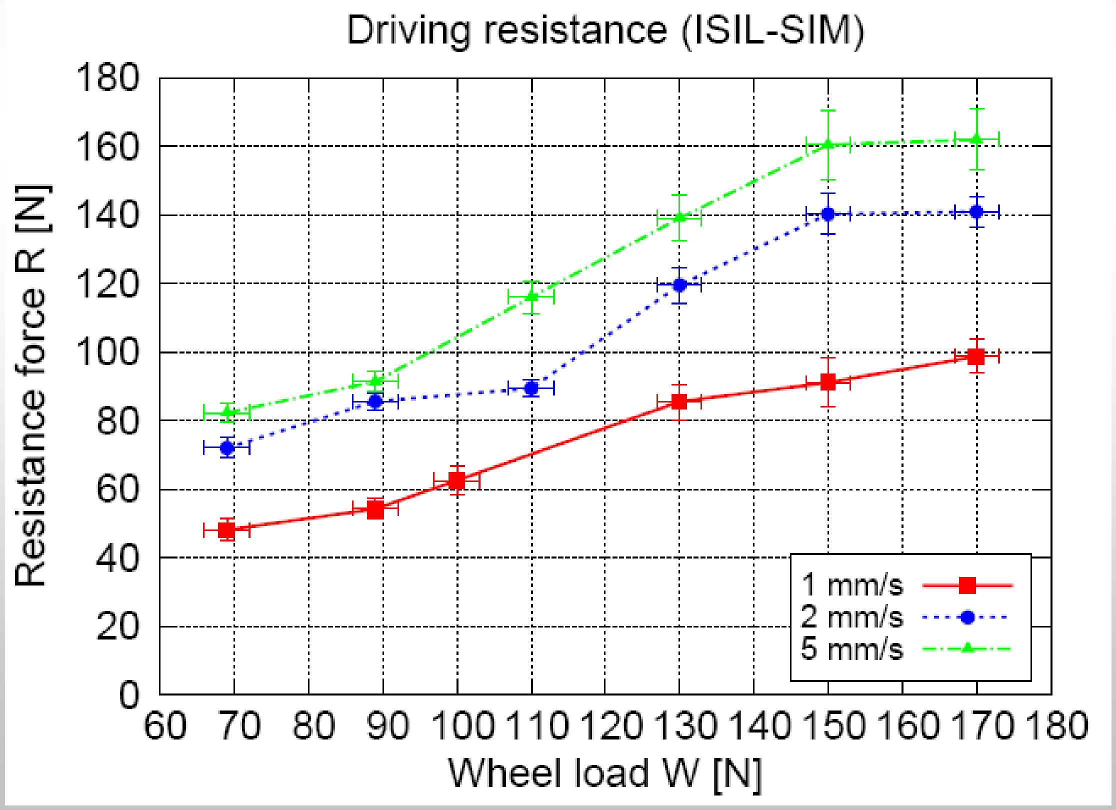
- Experiments with MER wheels have shown significant velocity effect
  - Plot of thrust force vs. vertical wheel load



- Vertical array of same-color data points: slip increasing top to bottom (83 %, 92 %, 98 %)

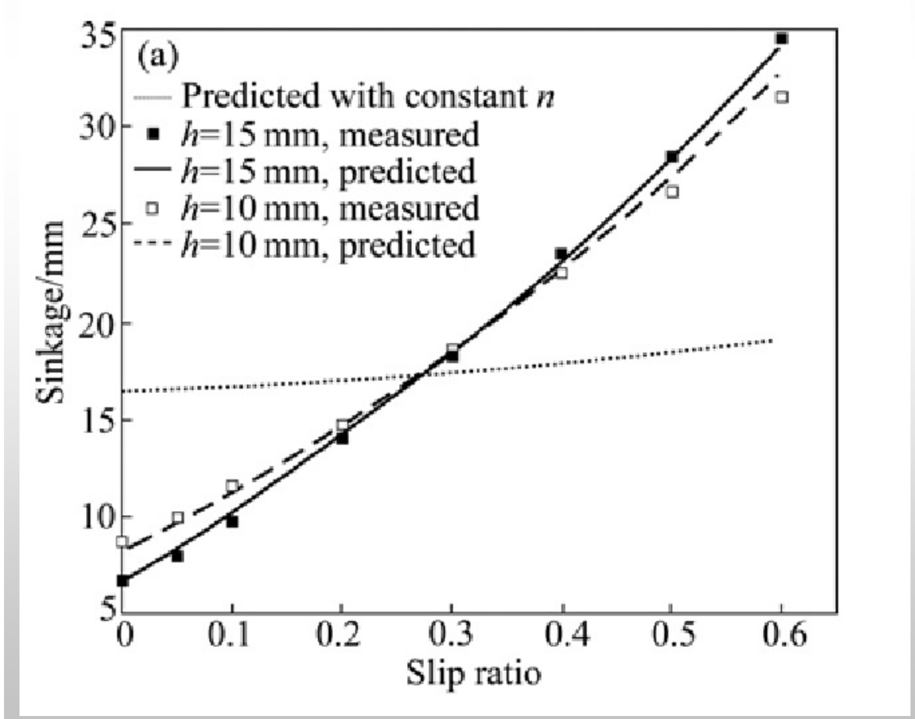
Slipping and Sinking (1)

- Experiments with MER wheels have shown significant velocity effect
  - Resistance from blocked RF wheel vs wheel load and drag velocity



Slipping and Sinking (2)

- Terramechanics theory is not well suited for modeling motion with high slippage
  - No model of material transport
  - No temporal dependence



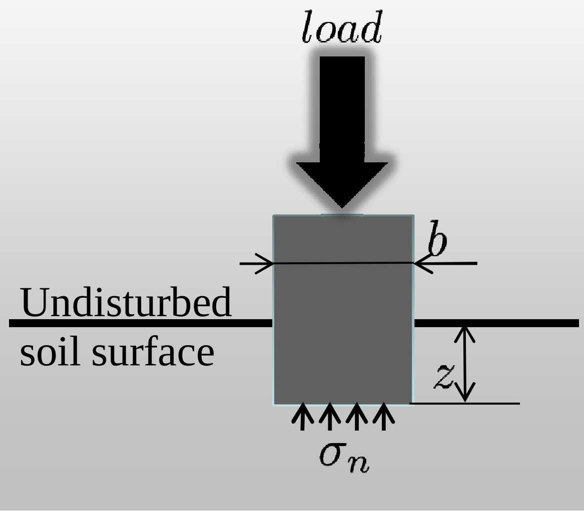
Opportunity
Maneuvers
out of
Sand Trap

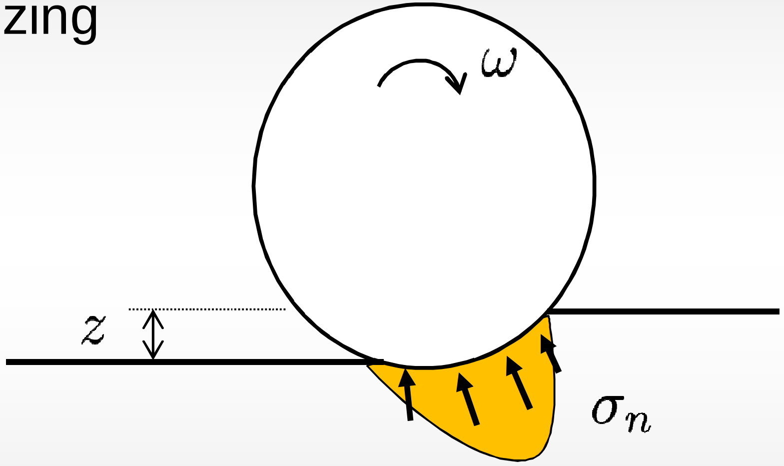
Slipping and Sinking (3)

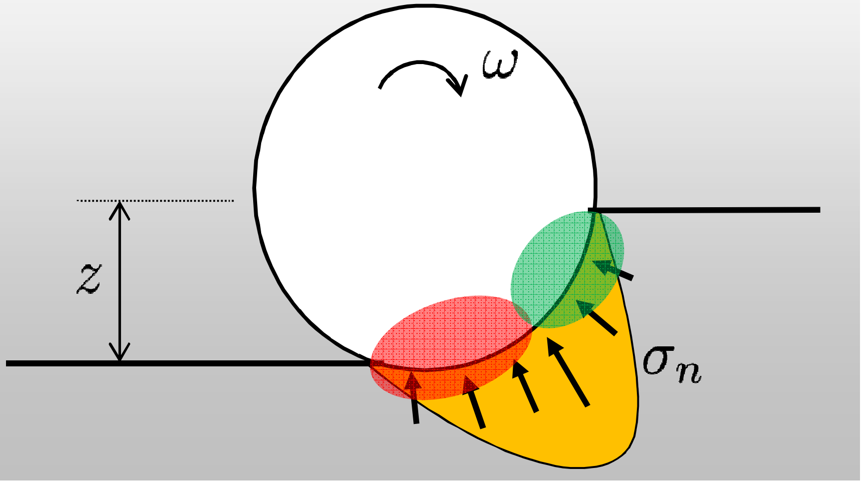
- Terramechanics theory is not well suited for modeling motion with high sinkage
  - Compaction resistance vs. bulldozing
  - “Flattening“ soil vs. “shoving“ soil

$$F_x = bR\int_{\theta_b}^{\theta_e} \tau_x(\theta)\cos(\theta) - \sigma_n(\theta)\sin(\theta)d\theta$$

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$



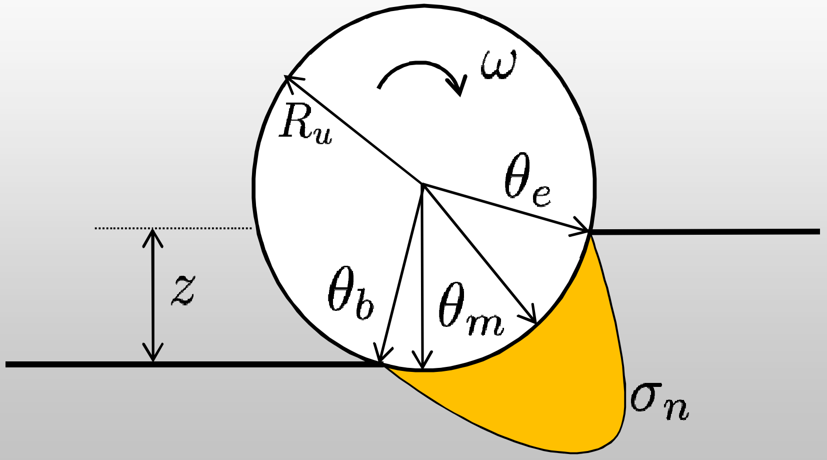




Slipping and Sinking (4)

- Slip ratio defines relative velocity between wheel and soil
  - Dictates shear stress, deformation

$$s_d = 1 - \frac{u}{\omega R}$$



Slipping and Sinking (4)

- Problems with slip ratio
  - Undefined at zero angular velocity
    - Issue for simulation
  - Transition from positive to negative not handled by theory
    - Can occur during free rolling

$$s_d = 1 - \frac{u}{\omega R}$$

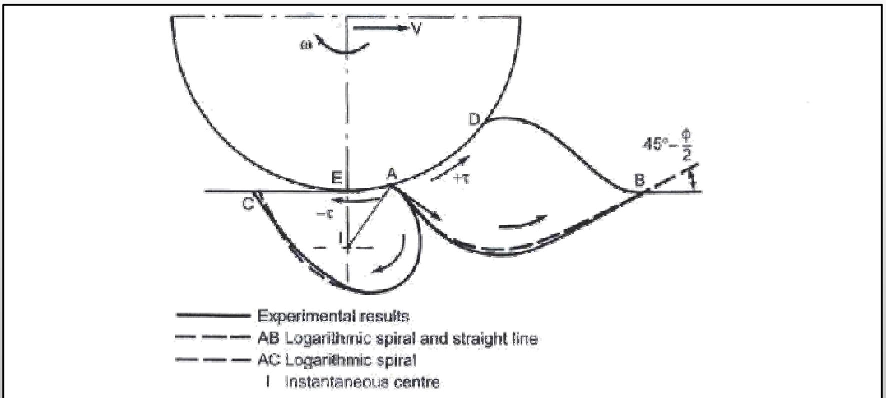


Figure 2.18: Flow patterns and bow wave under the action of a towed rigid wheel in sand

Slipping and Sinking (4)

- Problems with slip ratio
  - Undefined at zero angular velocity
    - Issue for simulation
  - Transition from positive to negative not handled by theory
    - Can occur during free rolling

$$s_d = 1 - \frac{u}{\omega R}$$

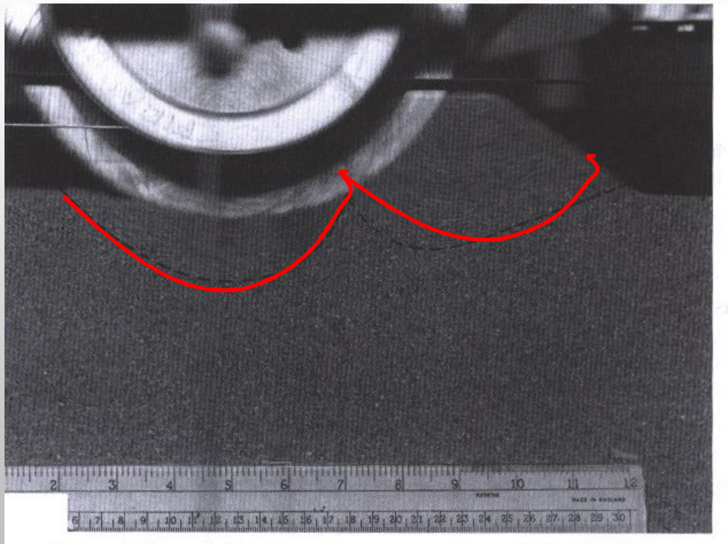


Figure 1.11: Soil flow patterns under a driven rigid wheel in sand

Slipping and Sinking

- Questions (Solutions?)
  - How to model rate dependence?
    - Effect on motion resistance, thrust
    - Momentum formulation of terramechanics relations?
  - How to model temporal effects?
    - Effect on sinkage
    - Model material transport based on grouser geometry?
      - For some soils? All?
  - How to model motion resistance due to high sinkage?
    - Piecewise formulation?
  - “Unified” model of wheel slip?
    - Analysis of particle motion under wheels

Conclusions

- Fundamental limitations of terramechanics modeling
    - Effect of terrain inhomogeneity
        - Soil condition dependence
            - Layering, relative density, moisture content
    - Scale effects
        - Parameter scale dependence (non-intrinsic soil properties)
    - Effects related to slipping and sinking
        - Slip ratio definition
        - Rate dependence
- Issues affect computation, simulation
- Tradeoff between generality and accuracy
- Tradeoff between measurement burden and accuracy