Conceptual Design and Passive Stability of Tethered Platforms

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Towed Bodies

**Definition:** Two or more tethered objects immersed in a moving fluid.

- Aerostats
- Underwater towed vehicles
- Kites
- Towed aircraft
Objectives

Performance Analysis: Conceptual design for tethered body
- Identify important design parameters
- Investigate sensitivities to design parameters
- Determine static stability criteria for bridled wings

Design Methodology: Quickly design a stable efficient system

Key Results:
- Reduced the static design problem to 4 design parameters
- Tether diameter is a stronger design driver than weight
- High effective lift to drag ratios are not advantageous to reach altitude
- Higher wind requires a more costly design
Performance Literature Review

**Tether Static Analysis**

- H. Glauert (1934)
  - Equations for heavy cable used for aerodynamic towing
  - Family of curves used to describe tether shape
- Thews and Landweber (1936)
  - Equations including tangential loading
- Pode (1955)
  - Faired sections
  - Special cable functions

**Equilibrium as a special case in Dynamic Analysis**

- J. DeLaurier
  - Finite differencing to calculate tether shape and height of an aerostat

**Numerical Simulation**

- P. Williams, B. Lansdrop, and W. Ockels.
  - Simulated the tether as rods and masses.
$L_e = \text{lift subtracted by wing weight}$
$\lambda = \text{tether density per unit length}$
$D_t = \text{drag per unit length}$
$V = \text{wind speed}$
$g = \text{gravity}$
$D = \text{drag}$
Analytic Static Equations

\[ D_t = \frac{1}{2} \rho V^2 C_{d_x} d \]

\( C_{d_x} \) = ballistic drag coefficient
\( \rho \) = air density
\( V \) = wind speed
\( d \) = diameter

\[ T(s + ds) \]

\[ T(s) \]

\[ D_t ds \]

\[ \lambda g ds \]
Analytic Static Tether Equations

\[
\frac{d\theta}{ds} = \frac{D_t \sin^2 \theta + \lambda g \cos \theta}{T} \\
\frac{dT}{ds} = \lambda g \sin \theta
\]

\[
\frac{dx}{ds} = \cos \theta \\
\frac{dy}{ds} = \sin \theta
\]

Boundary Conditions:

\[
\begin{align*}
x(0) &= 0 \\
y(0) &= 0 \\
T_x(l_t) &= D \\
T_y(l_t) &= L_e
\end{align*}
\]

There is no known analytic solution

H. Glauert. The form of a heavy cable used for towing a heavy body below an aeroplane.
Physical Dependencies: Static Tether

**Buckingham Pi Theorem:** The number of physical quantities subtracted from the fundamental units gives the minimum number of dimensionless quantities to describe the system.

\[
\begin{align*}
[l_t] &= l & \text{tether length} \\
[h] &= l & \text{altitude} \\
[D_t] &= \frac{m}{t^2} & \text{tether drag per unit length} \\
[\lambda g] &= \frac{m}{t^2} & \text{tether acceleration per length} \\
[L_e] &= \frac{ml}{t^2} & \text{effective lift} \\
[D] &= \frac{ml}{t^2} & \text{drag boundary condition}
\end{align*}
\]
Dimensionless Basis

Unit exponent matrix

\[
A = \begin{bmatrix}
    l & D_t & \lambda g & L_e & D & h \\
    l & 1 & 0 & 0 & 1 & 1 & 1 \\
    t & 0 & -2 & -2 & -2 & -2 & 0 \\
    m & 0 & 1 & 1 & 1 & 1 & 0
\end{bmatrix}
\]

The null space gives the dimensionless basis

\[
Null(A) = \begin{bmatrix}
    0 & -1 & -1 & -1 \\
    -1 & -1 & -1 & 0 \\
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

Gravity force to tether drag ratio

\[
\lambda g \quad \frac{D_t}{D}
\]

Effective lift to tether drag

\[
L_e \quad \frac{L_e}{l_tD_t}
\]

Effective lift to tether drag

\[
D \quad \frac{L_e}{D}
\]

Altitude to tether length

\[
h \quad \frac{h}{l_t}
\]
Design Evaluation: End Condition

Effective Lift to Drag Ratio versus Altitude

\[ \frac{D_t}{\lambda g} = 15.5 \]

\[ \frac{L_e}{D tr_0} = 0.88 \]
Design Evaluation: Tether Drag

Effect of Lift to Tether Drag Ratio on Altitude

\[ \frac{L_e}{D} = 5 \]

\[ \frac{D_t}{\lambda g} = 15.5 \]
Design Evaluation: Tether Weight

Effect of Tether Drag to Tether Weight Ratio on Altitude

\[ \frac{L_e}{D} = 5 \]

\[ \frac{L_e}{D_t r_0} = 0.88 \]
## Scaling

**Dimensionless parameters for static tethered system**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\lambda g}{D_t} )</td>
<td>( \frac{L_e}{l_t D_t} )</td>
</tr>
<tr>
<td>( \frac{L_e}{D} )</td>
<td>( \frac{h}{l_t} )</td>
</tr>
</tbody>
</table>

**Sub-scale Testing:** Systems with the same dimensionless parameters will have the same normalized performance  
- Reduce development cost of prototype  
- Reduce development time  
- Increase safety during testing phases
# Testing Scaling Parameters

<table>
<thead>
<tr>
<th></th>
<th>Full Scale</th>
<th>Sub-Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tether length (m)</td>
<td>30</td>
<td>12.7</td>
</tr>
<tr>
<td>Wind Speed (m/s)</td>
<td>5</td>
<td>3.2</td>
</tr>
<tr>
<td>Wing + Sensor mass (kg)</td>
<td>1.02</td>
<td>0.091</td>
</tr>
<tr>
<td>Wing Area (m^2)</td>
<td>2</td>
<td>0.42</td>
</tr>
</tbody>
</table>
Testing Scaling Parameters

1.5 mm
1.4 g/m

3 mm
6.6 g/m
Scaled Testing
Measurements

Board designed by Geoff Bower

GPS
Differential Pressure
Accelerometer
Barometric Pressure
Rate Gyros
Pic Processor
Micro SD card

Kestrel Anemometer

Hand held scale
### Results: Static Approximations

<table>
<thead>
<tr>
<th></th>
<th>Full Scale Kite</th>
<th>Sub-Scale Kite</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Height</strong></td>
<td>$\frac{h}{l_t}$</td>
<td>$0.98 \pm 0.03$</td>
</tr>
<tr>
<td><strong>Estimated Tension</strong></td>
<td>$\frac{T}{\lambda g l_t}$</td>
<td>38.9</td>
</tr>
<tr>
<td><strong>Average Wind Speed</strong></td>
<td>$\frac{V}{\bar{V}}$</td>
<td>1</td>
</tr>
</tbody>
</table>

Data was taken from periods of time with the necessary average wind speed
Preliminary Results: Frequency Content

**Full Scale Kite Frequency Content**
for Body-Y Angular Rates

Peaks centered at 7 and 15

**Sub-Scale Kite Frequency Content**
for Body-Y Angular Rates

Peaks centered at 8 and 14.5
Current Single Line Altitude Record

The current record holding kite built and flown by Richard Synergy.

*Millibar Messenger*

**Area:** 25 m²  
**Max Wind Speed:** 12 m/s  
**Tether:** 270 pound test woven Kevlar line 3/32 inch in diameter

**Current Record:** 13600 ft above ground

**Goal:** 5000m or 16400 ft above ground
Design Method

Performance Based Design

**Objective:** Reach the target altitude while minimizing cost

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{subject to} & \quad H(x) \leq 0 \quad \text{nonlinear constraints} \\
& \quad x \leq u \quad \text{upper bound} \\
& \quad x \geq l \quad \text{lower bound}.
\end{align*}
\]

**Variables (x)**
- Tether Length
- Tether Diameter
- Tether Material
- Aspect Ratio
- Wing Area
- Lift Coefficient
- Spar Radius

**Constraints H(x)**
- Achievable altitude \( >5000 \text{ m} \)
- Tether tension \(<\) material strength
- Bending loads \(<\) Spar bending strength

**Cost Function f(x)**
Estimated prices of
- Tether
- Spar cost
- Wing material
Achievable Altitude

Altitude is calculated by numerically integrating the static tether equations

\[
\begin{align*}
\frac{1}{\lambda g} \frac{d\theta}{ds} &= \frac{D_t}{\lambda g} \sin^2 \theta + \cos \theta \\
\frac{1}{\lambda g} \frac{dT}{ds} &= \sin \theta \\
\frac{dx}{ds} &= \cos \theta \\
\frac{dy}{ds} &= \sin \theta
\end{align*}
\]

\[x(0) = 0\]
\[y(0) = 0\]
\[T_x(l_t) = D\]
\[T_y(l_t) = L_e\]

T(s) must be less than the tether material strength for all s

Calculate the Boundary conditions
- Weight model
- Aerodynamic model
Wing Model

- **Lift determined by area and $C_L$**
  - Wind Speed = 12 m/s
  - Safety factor included

- **Fixed planform design variables**
  - Sweep = 45°
  - Taper Ratio = 0.06
  - Crossbar intersection at half the semi-span

- **Moments calculated about the neutral axis**
  - Carbon composite tubes
  - Spars have uniform radius
  - Mass wing area assumed to be the mass of ripstop nylon
Cost Function

**Tether Prices**

- Price per meter vs. Diameter (mm)
- Graphs for Spectra, Nylon, Kevlar, Steel

**Spar Prices**

- Price per 1.5 m vs. Graphite Tube Radius (mm)
- Graphs for Prices

**Total cost includes**

- Tether cost as a function of diameter and length
- Spar cost as a function of tube radius and length
- Wing material cost as a function of area
Performance Results

MATLAB’s SQP based optimizer fmincon

Run Parameters

- **Wind Speed**: 12 m/s
- **Achievable Altitude**: 5000 m
- **Maximum Lift Coefficient**: 0.675
- **Spar Material Yield Stress**: 300 MPa
- **Spar Material Density**: 1000 Kg/m³
- **Wing Material Density**: 0.1 Kg/m³
- **Taper Ratio**: 0.06
- **Sweep**: 45°
- **Safety Factor**: 10

* Price for zylon is uncertain due to limited pricing information of thin pure zylon ropes.
### Performance Results

#### Bar Chart
- **Zylon**: $24,557
- **Spectra**
- **Technora**
- **Kevlar**

#### Table

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$24,557</td>
</tr>
<tr>
<td>Wing Area $S$</td>
<td>24.7 m²</td>
</tr>
<tr>
<td>Aspect Ratio $AR$</td>
<td>3.5</td>
</tr>
<tr>
<td>Tether Diameter $d$</td>
<td>1.8 mm</td>
</tr>
<tr>
<td>Tether Length $l_t$</td>
<td>8054 m</td>
</tr>
<tr>
<td>Lift Coefficient $C_L$</td>
<td>0.675</td>
</tr>
<tr>
<td>Wing Mass</td>
<td>36.5 Kg</td>
</tr>
<tr>
<td>Leading Edge Spar</td>
<td>48 mm</td>
</tr>
<tr>
<td>Cross Bar</td>
<td>57 mm</td>
</tr>
<tr>
<td>Lift to Drag Ratio</td>
<td>16</td>
</tr>
<tr>
<td>Effective Lift to Drag</td>
<td>9.5</td>
</tr>
<tr>
<td>Maximum Tension</td>
<td>1041 N</td>
</tr>
<tr>
<td>Tether Mass</td>
<td>18.9 Kg</td>
</tr>
</tbody>
</table>
# Detailed Performance Results

<table>
<thead>
<tr>
<th></th>
<th>Zylon</th>
<th>Spectra</th>
<th>Technora</th>
<th>Kevlar</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost</strong></td>
<td>24557</td>
<td>27708</td>
<td>27170</td>
<td>25963</td>
</tr>
<tr>
<td><strong>Wing Area S</strong></td>
<td>24.7 m²</td>
<td>23.8 m²</td>
<td>22.6 m²</td>
<td>21.3 m²</td>
</tr>
<tr>
<td><strong>Aspect Ratio AR</strong></td>
<td>3.5</td>
<td>2.9</td>
<td>2.9</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>Tether Diameter d</strong></td>
<td>1.8 mm</td>
<td>3.0 mm</td>
<td>2.9 mm</td>
<td>3.2 mm</td>
</tr>
<tr>
<td><strong>Tether Length l_t</strong></td>
<td>8054 m</td>
<td>9603 m</td>
<td>9514 m</td>
<td>9794 m</td>
</tr>
<tr>
<td><strong>Lift Coefficient C_l</strong></td>
<td>0.675</td>
<td>0.675</td>
<td>0.675</td>
<td>0.675</td>
</tr>
<tr>
<td><strong>Wing Mass</strong></td>
<td>36.5 Kg</td>
<td>34.0 Kg</td>
<td>32.0 Kg</td>
<td>28.7 Kg</td>
</tr>
<tr>
<td><strong>Leading Edge Spar</strong></td>
<td>48 mm</td>
<td>47 mm</td>
<td>46 mm</td>
<td>46 mm</td>
</tr>
<tr>
<td><strong>Cross Bar</strong></td>
<td>57 mm</td>
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<td>55 mm</td>
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<td><strong>Lift to Drag Ratio</strong></td>
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<td>13.6</td>
<td>13</td>
</tr>
<tr>
<td><strong>Effective Lift to Drag</strong></td>
<td>9.5</td>
<td>8.3</td>
<td>8.3</td>
<td>8</td>
</tr>
<tr>
<td><strong>Maximum Tension</strong></td>
<td>1041 N</td>
<td>1030 N</td>
<td>984 N</td>
<td>937 N</td>
</tr>
<tr>
<td><strong>Tether Mass</strong></td>
<td>18.9 Kg</td>
<td>51.4 Kg</td>
<td>53.0 Kg</td>
<td>61.2 Kg</td>
</tr>
</tbody>
</table>
Scaled Design

Subscale Flight

- IMU, GPS and air pressure sensors (50g)
- Average wind speed 4 m/s
- 20 lb test spectra
Experimental Results

Target Altitude: 485 m
Max Altitude: 460 m
Experimental Results

Max Altitude: 460 m

Ascending
Wind Profiles

Sounding Data for Oakland in June

- Temporal Average
- Measured
Wind Speed Profile

1. Assume a constant wind profile
2. Choose the reference wind speed

**Data:** Oakland sounding records for January 2010

- Collect every 12 hours
- 64 samples total