Low-deltaV Trajectories

to move a small asteroid to a Lagrange point
Summary

- Lagrange points: definition

- Orbits about Lagrange points and invariant manifolds (Low DeltaV Trajectories)

- Why a NEA near EM L1/L2 or SE L2? Potential destination for human missions

- Missions to Lagrange points: overview

- Asteroid Retrieval Mission Study: matching low-thrust with invariant manifolds?
The Circular Restricted 3-Body Problem describes the motion of a massless particle under the gravitational influence of two point masses $m_1$ and $m_2$, called primaries, in circular motion around their common centre of mass.

Equations of motion

\[
\begin{align*}
\dddot{x} - 2\ddot{y} &= \frac{\partial U}{\partial x} \\
\dddot{y} + 2\ddot{x} &= \frac{\partial U}{\partial y} \\
\dddot{z} &= \frac{\partial U}{\partial z}
\end{align*}
\]

\[
U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}
\]

\[
r_1^2 = (x - \mu)^2 + y^2 + z^2 \\
r_2^2 = (x - \mu + 1)^2 + y^2 + z^2
\]

Jacobi constant

\[
C = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - V^2
\]
Surfaces of Hill
boundary for the admissible motion

Case 1: $C > C_1$

Case 2: $C_1 > C > C_2$

Case 3: $C_2 > C > C_3$

Case 4: $C_3 > C > C_4$

Case 5: $C_4 > C$
Lagrange points

SE L1/L2: about 1.5 million km from the Earth

EM L1/L2: about 60,000 km from the Moon

1 AU: about 150 million km

1 LD: 384,403 km

L1, L2, L3

±iω1
center
↑
planar p.o.

±iω2
center
↑
vertical p.o.

±λ
saddle
↑
inv. manif.

Cantor set of 2D tori
Invariant Manifolds

Unstable Invariant Manifold

Stable Invariant Manifold

Interior region

Exterior region

Stable Invariant Manifold

Unstable Invariant Manifold

credit: Romero-Gomez
EM L1/L2 vs SE L2 as targets for human missions

<table>
<thead>
<tr>
<th></th>
<th>EM L1</th>
<th>EM L2</th>
<th>SE L2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔV1</td>
<td>3.0661 km/s</td>
<td>3.0966 km/s</td>
<td>3.1537 km/s</td>
</tr>
<tr>
<td>ΔV2</td>
<td>0.8859 km/s</td>
<td>0.7819 km/s</td>
<td>0.4666 km/s</td>
</tr>
<tr>
<td>ΔVtot</td>
<td>3.9520 km/s</td>
<td>3.8786 km/s</td>
<td>3.6203 km/s</td>
</tr>
<tr>
<td>Transfer</td>
<td>3.8 days</td>
<td>6.2 days</td>
<td>37.6 days</td>
</tr>
</tbody>
</table>

Hohmann transfer from LEO (h = 400 km)

ISEE-3 (International Sun-Earth Explorer-3)

SOHO (Solar and Heliospheric Observatory)

ACE (Advanced Composition Explorer)
GENESIS

WMAP
Wilkinson Microwave Anisotropy Probe

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Other missions to Lagrange points

Wind (SE L1)
Grail (SE L1)
Herschel and Planck (SE L2)
Chang’e 2 (SE L2)
Artemis (EM L1 and L2)
James Webb Space Telescope (SE L2) ?

Trajectory design for Asteroid Retrieval Mission

1) Earth - NEA leg: pure low-thrust
2) NEA - Lagrange point: how?
Invariant Manifolds SE L2

Time of flight: one year per bounce

credit: Pedro Llanos
Stable Manifolds SE L1/L2

Time of flight: one year per bounce

credit: Pedro Llanos
Connection between L1 and L2

Direction to Earth

Moon

L1

ARTEMIS L1 Orbit

L2 to L1 Transfer Orbit

Moon’s Orbit

ARTEMIS L2 Orbit

ARTEMIS-P1
Here on August 25th

Direction of motion

credit: NASA GFSC

09/27/2011

Low-DeltaV Trajectories
Marco Tantardini
Low-DeltaV Trajectories
Marco Tantardini

1991 VG
Earth Distance: 0.79 AU
Sun Distance : 1.064 AU
Sep 27, 2020

2007 UN12
Earth Distance: 0.11 AU
Sun Distance : 0.994 AU
Sep 27, 2020

2009 BD
Earth Distance: 1.183 AU
Sun Distance : 1.101 AU
Sep 27, 2020

2010 UE51
Earth Distance: 1.426 AU
Sun Distance : 0.994 AU
Sep 27, 2020

ssd.jpl.nasa.gov
Combining low-thrust + invariant manifolds has been studied


GTO -> thrust arc -> stable manifold -> halo orbit

- GTO:
  chosen as the starting orbit
- thrust arc:
  solution of an optimal control problem
- stable manifold:
  associated to the final halo
- halo orbit:
  computed through the Richardson method [1980]
IDEA for Asteroid Retrieval

NEA -> thrust arc -> stable manifold -> target Lagrange point (SE L1/L2)

Why study SE L1/L2 first? Because if the NEA is in SE L1/L2 then it can be moved to EM L1/L2 through invariant manifolds (EM CR3BP)

If the final destination is a Lagrange point, low-thrust + invariant manifolds might be more energy efficient than pure low-thrust. Thus, for a selected NEA, we might need smaller SEP.