Carbon Cycle:
What do we know?
What else do we want to know?

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February 28 2010
Continuous Carbon Cycling

- Fluxes PgC/yr
- Inventory PgC
- Turnover time = Inventory/Flux

- Atm CO2 --> inventory
- Land: turnover time $10^1$-$10^2$ yrs. Ocean: turnover time $10^2$-$10^3$ yrs
- Difficult (time consuming and expensive) to measure changes in land and ocean inventories. Focus on fluxes
Atm CO$_2$ measurements at Mauna Loa Obs
What We’ve got: The data: Atm CO2 (for now)

- **Discrete surface flasks (~weekly)**
- **Continuous surface (hourly) observatories**
- **Tall towers continuous (hourly)**
- **Aircraft profiles (~weekly)**

GMD Carbon Cycle operates 4 measurement programs. Semi-continuous measurements are made at 4 GMD baseline observatories and from tall towers. Discrete samples from the cooperative air sampling network and aircraft are measured at GMD. Presently, atmospheric carbon dioxide, methane, carbon monoxide, hydrogen, nitrous oxide, sulfur hexafluoride, and the stable isotopes of carbon dioxide and methane are measured. Contact: Dr. Pieter Tans, NOAA ESRL GMD Carbon Cycle, Boulder, Colorado, (303) 497-6678 (pieter.tans@noaa.gov, http://www.cmdl.noaa.gov/ccg).
**CO₂ mixing times in atm:**

~2 wks around lat circle
~3 mo within hemisphere
~1 yr bet’ hemispheres

- NOAA-ESRL
- ~ 100 sites at remote marine locations, bi-weekly flasks, 2m
- Long-term increase
- Seasonal cycle
- N-S gradient
PROBLEMS: Missing Sinks and Missing Sources

• Only half of the CO$_2$ produced by human activities is remaining in the atmosphere
• How well do we know the sources?
• Where are the sinks that are absorbing over 40% of the CO$_2$ that we emit?
  – Land or ocean?
  – Eurasia/North America?
• Why does CO$_2$ buildup vary dramatically with nearly uniform emissions?
• How will CO$_2$ sinks respond to climate change?
Rule: Conservation of Carbon

\[
\frac{\partial C}{\partial t} + \tilde{\mathcal{S}}(C) = S_{z=0} + P_{\text{Chem Prod}}
\]

\[
S = FF + \text{LandUse} + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab})
\]

Separate out background (pre-industrial) from the perturbation (last 200 years) carbon cycle:

\[
C = \overline{C} + C'
\]

“Steady State”:

\[
\tilde{\mathcal{S}}(C) = \overline{(F_{oa} - F_{ao})} + \overline{(F_{ba} - F_{ab})}
\]
Units: $1 \text{ Pg} = 1 \text{ Gt} = 10^{15} \text{ gram}$

\[ C(\text{kgC/m}^3) = \rho(\text{kgAir/m}^3) \times X(\text{moleC/moleAir}) \times (\text{MWt}_C/\text{MWt}_{\text{air}}) \]

\[ \text{MWt}_C = 12 \text{ gm/mole}; \text{MWt}_{\text{air}} = 29 \text{ gm/mole} \]

\[ \text{Area} = \int \text{dxdy} = 5 \times 10^{14} (\text{m}^2) \]

\[ \text{MassAtm} = \int \rho \text{dxdydz} = \frac{100 \text{P}_{mb}}{\text{g}} (\text{kgAir/m}^2) \times \text{Area} \sim 5 \times 10^{18} \text{ kgAir} \]

\[ \text{MassC (300 ppmv)} = \text{MassAtm} \times (300 \times 10^{-6}) \times \left(\frac{12}{29}\right) \]

\[ \sim 600 \times 10^{12} \text{ kg} = 600 \text{ PgC} = 600 \text{ GtC} \]

\[ 1 \text{ PgC} \rightarrow 0.5 \text{ ppmv (mixed _entireAtm)} \]
“Known” and/or Observed

- Sources: fossil fuel emission
- Atmospheric CO2
- Land carbon; incl deforestation
- Ocean carbon
Conservation of Perturbation Carbon in Atm

\[ \frac{\partial C}{\partial t} + \mathcal{Q}(C)_{\text{Atm}_\text{transport+mixing}} = \left. S \right|_{z=0}^{\text{SourcesSinks}} + P_{\text{Chem Prod}} \]

\[ S = FF + \text{LandUse} + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab}) \]
CO2 Emission from Fossil Fuel Combustion & Industrial Processes

- Liquids (~36%)
- Solids (~35%)
- Gas (~20%)
- Cement production (~3%)
- Flaring at wells (<1%)
- Bunker fuels (~4%)
- Others (<1%)

CO2 Emission Growth Rate:
- 1990-1999: 1.1% per year
- 2000-2004: >3% per year
Shifting Emission Sources

- USA and EU contributed >50% of cumulative CO$_2$ emission since 1751
- China leads recent growth in emissions
- 2004: China emissions close to US emissions

Raupach, Michael R. et al. (2007)
Fossil Fuel CO2 Emissions

1994-1997
Fossil Fuel CO2 Emissions 1994-1997 6.649 PgC/yr

2001-2004
Fossil Fuel CO2 Emissions 2001-2004 7.504 PgC/yr

6.6 PgC/yr

7.5 PgC/yr
CO₂ is a long-distance traveller in the atmosphere

- NOAA-ESRL
- ~ 100 sites at remote marine locations, bi-weekly flasks, 2m
- Long-term increase
- Seasonal cycle
- N-S gradient
Atm CO2 Obs: (1) Time series at stations

Mixing time between hemispheres ~ 1 year:

N-S gradient increases as FF emission (~NH source) increases
Atmospheric CO$_2$ Signature of Ecosystem C Exchange: Seasonal Cycle

- Mixing time within a hemisphere ~ 3 months
- MLO seasonal cycle integrates NH vegetation dynamics
- Amplitude of atmospheric CO$_2$ seasonal cycle increases poleward: telecoping of growing season and greater asynchronicity bet’ fluxes

- Growing season net flux ~15-20% of annual NPP
(2) Tall Towers: Diurnal CO2 is highly variable in boundary layer (<500m)

- Local vertical mixing time- ~ minutes to hours
- Diurnal cycle of photosynthesis and respiration
  \( \rightarrow > 60 \text{ ppmv (20\%)} \) diurnal cycle near surface
- Varying heights of the planetary boundary layer (varying mixing volumes)
(3a) Occasional Aircraft Campaigns
(3b) “Regular” Aircraft Monitoring: Vertical Profiles (free troposphere)
Conservation of Perturbation Carbon in Atm

\[
\frac{\partial C}{\partial t} + \nabla \cdot (C) = S \bigg|_{z=0}^{\text{Sources/Sinks}} + P \bigg|_{\text{Chem Prod}}
\]

\[
S = FF + \text{LandUse} + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab})
\]
Terrestrial Carbon Cycle

- Growth, mortality, decay
- **GPP**: Gross Primary Productivity (climate, \( \text{CO}_2 \), soil \( \text{H}_2\text{O} \), resource limitation)
- **Ra**: Autotrophic respiration (T, live mass, …)
- **Rh**: Heterotrophic respiration: Decay (T, soil \( \text{H}_2\text{O} \), …)
- NPP = GPP - Ra

\[
\text{NPP} = 120 \text{ PgC/yr} - 60 \text{ PgC/yr} = 60 \text{ PgC/yr}
\]
Impact on Atmospheric CO$_2$

- Seasonal asynchrony between photosynthesis and decomposition
  - net fluxes of CO$_2$ to and from atm
  - seasonal cycle of CO$_2$ in atm
- Annual imbalance $\rightarrow$ carbon source/sink
(1) History of Ecological Measurements

Veg Type(x,y) \rightarrow \text{annual mean NPP}(x,y)
(2) Satellite Greenness index: NDVI

Seasonality of NPP
Seasonality of Respiration not well-defined
Net flux not well-defined at every location
Tough to estimate
• deforested area
• Carbon inventory before deforestation
• Fate of removed carbon
• Fate of litter and soil carbon
Tough to discriminate atm CO2 signature
FluxNet Towers

Vertical CO2 flux: vertical velocity ($w$), CO2 ($c$)

$$wc = (\bar{w} + w')(\bar{c} + c')$$

$$= \bar{w}\bar{c} + w'\bar{c} + \bar{w}c' + w'c'$$

$$\bar{wc} = \bar{w}\bar{c} + w'c'$$
• Plot-scale measurement of carbon storage, age structure, growth rates: 170,000 plots in US!
• Allows assessment of decadal trends in carbon storage
Conservation of Perturbation Carbon in Atm

\[
\frac{\partial C}{\partial t} + \nabla (C)_{Atm\_transport+mixing} = S|_{z=0} + P_{Chem\Prod} + SourcesSinks
\]

\[
S = FF + LandUse + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab})
\]
Ocean C from the Atm’s Perspective:

$$\text{DIC} = \text{CO}_2^* + \text{HCO}_3^- + \text{CO}_3^= \quad 1-2\% \quad 80-90\%$$

$$F_{oa} - F_{ao} = k_{\text{GasExchRate}} \left( \frac{\text{CO}_2^*_{\text{sfc.ocn}}}{\text{molec/m}^3} \right) - \beta_{\text{solubility}} \times p\text{CO}_2_{\text{atm.sfc}}$$
Productivity is possible when upwelling brings:

- Nutrients from below to euphotic zone
- Cold water

Small Flux, small inventory of organic C
But alters DIC(z)
(1a) Time series: Hawaii (ALOHA) and Bermuda (BATS)

Carbonate ion decreasing: Tougher to precipitate

pH has decreased by 0.04 in 20 years - carbonate more soluble

Surface ocean pCO$_2$ increasing; follows the atmospheric record at Mauna Loa
Air-sea difference in pCO2 is highly variable:

>100 μatm

Chavez et al.
Research Cruises + Ships of Opportunity: Air sea difference in pCO2

Jan-Mar Obs

Jul-Sept Obs

No. Months with At least One Obs

0 1 2 3 4 5 6 7 8 9 10 11 12
Number of Months

Year
(2) Air-Sea Fluxes of CO2

\[ \Delta F_{ocn} = F_{oa} - F_{ao} \]

\[ = k \times \left( \frac{CO2_{sfc\_ocn}}{\text{moleC} / \text{m}^3} \right) - \beta \times pCO2_{atm\_sfc} \]

Where:
- \( k \) is the gas exchange rate (m/s)
- \( \beta \) is the solubility (T)
- \( pCO2_{atm\_sfc} \) is the partial pressure of CO2 at the surface
(3) International Research Campaigns: JGOFS/WOCE global survey (1980s and 1990s)

- Improved analytical techniques for inorganic carbon and alkalinity (±1-3 µmol/kg or 0.05 to 0.15%)
- Certified Reference Materials
- Data management, quality control, & public data access
(3) Campaigns:
Total Dissolved Inorganic Carbon (DIC)

Conveyor Belt Transport of DIC:
- Southward in Atlantic
- Northward in Pacific

Ocn currents ~ cm/s
Time scale ~ $10^3$ yr

Biology and DIC:
- Depletion near sfc
- Enrichment at Depth
(3) Campaigns: Penetration of Anthropogenic Carbon into Ocean Interior

Sabine et al. 2004
(4) Autonomous Platforms: Profiling Floats --> Biology
SUMMARY

\[
\frac{\partial C}{\partial t} + \left( \Sigma(C) \right)_{Atm\_transport+mixing} = \left. S \right|_{z=0}^{SourcesSinks}
\]

\[
S = FF + \text{LandUse} + \left( F_{oa} - F_{ao} \right) + \left( F_{ba} - F_{ab} \right)
\]

**FF**: mainly NH, relatively aseasonal

**LandUse**: mainly source tropics, sink in mid-latitudes

**Ocean**: outgassing in equatorial oceans, absorption at mid-hi latitudes (in summer. Not sure about winter)

**Vegetation and soils**: annual mean fluxes~0 locally, fluxes have large diurnal (~100 ppmv) and seasonal ranges (~30 ppmv)
How to harmonize the diverse data?

- Spatial scales
- Observing frequency
- Observing periods - varying meteorology and climate variability
- Incomplete suite - e.g. respiration
Atm Carbon Models

\[
\frac{\partial C}{\partial t} + \mathcal{S}(C) = S_{z=0} \quad \text{Atm}_\text{transport+mixing} \quad \text{SourcesSinks}
\]

Other ways to write the same equation

\[
X_b(t_{i+1}) = M \left( X_a(t_i) \right) \quad \text{forecast} \quad \text{model} \quad \text{analysis}
\]

\[
X_{i+1} = \Phi(X_i) + G(u) \quad \text{conc, fluxes, parameters}
\]

\[
X = \text{conc} \quad \text{flaxes}
\]
An Atm Carbon Cycle Model

What we’ve got:

• Sources/Sinks $S$ known approximately or not well constrained
• $C_{obs}$ (actually mixing ratios $X_{obs}$) biweekly, at ~100 stations near the surface
• “Decent” transport model (winds, turbulent mixing)

What we want:

• Where has the fossil fuel CO2 gone?  {Better estimates of the magnitude and distribution of $S$ (e.g. land exchange)}
• How did the fossil fuel CO2 get there?  {improved understanding and representation of processes, e.g.
  • $F_{ab} = LUE \times \text{AvailableLight}$; $F_{ba} = \exp(\alpha T)$;
• Protocol verification: What are the strengths of local/regional emissions and sinks?

\[
\frac{\partial C}{\partial t} + \nabla \cdot \langle C \rangle = \left. S \right|_{z=0} \\
S = FF + \text{LandUse} + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab})
\]
What we’ve got: (1) The Model: NCAR climate model

Source: Fossil fuel combustion (6 PgC/y)

$C(x, y, z)$ at steady state

**CO$_2$ Release from Fossil Fuel Combustion**

- **Surface**
- **Pressure (mb)**
- **Latitude**
- **Zonal mean**

- **Fossil Fuel Concentration (ppmv)**
  - as simulated by NCAR CCM
  - Annual Mean: Year 3; Surface

- **Fossil Fuel Concentration (ppmv)**
  - as simulated by NCAR CCM
  - Zonal Mean: Year 3
What We’ve got (2): The data: Atm CO2 (for now)

- **Discrete surface flasks** (~weekly)
- **Continuous surface** (hourly) observatories
- **Tall towers** continuous (hourly)
- **Aircraft profiles** (~weekly)
Difficult to maintain an international monitoring network
What We’ve Got: (3) The Flux Priors

\[
\frac{\partial C}{\partial t} + \text{Atm}_\text{transport+mixing} + \text{SourcesSinks} = S|_{z=0}
\]

\[
S = FF^\text{"well-known"} + \text{LandUse} + (F_{oa} - F_{ao}) + (F_{ba} - F_{ab})
\]

should net land flux \((F_{ba} - F_{ab})\) be prop to \(F_{ab}\)?
Example I: A Simpler Model - reduce 3D atm to 2 hemisphere
Example I: Interhemispheric Mixing: Two-Box Model, everything is perfect.

\[
\frac{\partial M_N}{\partial t} = - \frac{M_N - M_S}{\tau} + S_N
\]

\[
\frac{\partial M_S}{\partial t} = \frac{M_N - M_S}{\tau} + S_S
\]
Example 1: Interhemispheric Mixing: 
Two-Box Model, everything is perfect.

\[
\frac{\partial M_N}{\partial t} = - \frac{M_N - M_S}{\tau} + S_N
\]

\[
\frac{\partial M_S}{\partial t} = + \frac{M_N - M_S}{\tau} + S_S
\]

\[
\frac{\partial (M_N - M_S)}{\partial t} = -2 \frac{M_N - M_S}{\tau} + (S_N - S_S) = 0 \text{ @ SteadyState}
\]

\[
\tau = 2 \frac{M_N - M_S}{S_N - S_S}
\]

Interhemispheric exchange time $\tau$ determined from inert tracers (e.g. CFC, with $S_S=0$): $\sim 1-2$ years
Ex I: 2-Box Model Applied to the Carbon Cycle

\[ M_N - M_S = \frac{\tau}{2} (S_N - S_S) \]

Consider the case \( S_N = 6 \text{ PgC/yr}; S_S = 0 \)

\( \tau = 1 \text{ yr} \)

\( \rightarrow M_N - M_S = 3 \text{ PgC} \)

Recall 1 PgC \( \rightarrow 0.5 \text{ ppmv} \) if mixed in entire atm.

1 PgC \( \rightarrow 1 \text{ ppmv} \) if mixed in a hemisphere.

\( \rightarrow X_N^{\text{column}} - X_S^{\text{column}} = 3 \text{ ppmv} \)

Guess (3D model) surface gradient 1.5x column mean gradient

\( \rightarrow X_N^{sfc} - X_S^{sfc} = 4.5 \text{ ppmv} \)

Aircraft, Hippo: obs of vertical profile
**Ex I: 2-Box Model Applied to the Carbon Cycle**

**Forward problem:** If 100% FF CO2 remained in atm

\[ M_N - M_S = \frac{\tau}{2} (S_N - S_S) \]

\[ S_N = 6 \text{ PgC/yr}; \ S_S = 0 \]

\[ \tau = 1 \text{ yr} \]

\[ \rightarrow M_N - M_S = 3 \text{ PgC} \]

\[ \rightarrow X_N^{sfc} - X_S^{sfc} = 4.5 \text{ ppmv} \]

But \( (X_N^{sfc} - X_S^{sfc})_{obs} = 2.5 \text{ ppmv} \)

---

**Obs \( \rightarrow \) only 50% of FF CO2 remains in atm**

\[ \frac{\partial (M_N + M_S)}{\partial t} = S_N + S_S = \text{sources} - \text{sinks} \]

\[ \frac{\partial (M_N + M_S)}{\partial t}_{obs} = 3 \text{ PgC/yr} \]

**soures = 6 PgC/yr**

\[ \rightarrow \text{Sinks}_N + \text{Sinks}_S = 3 \text{ PgC/yr} \]
Ex I: 2-Box Model Applied to the Carbon Cycle

Inverse problem

Model: \[ M_N - M_S = \frac{\tau}{2} (S_N - S_S) \]

Given: \((X_N^{sfc} - X_S^{sfc})_{obs} = 2.5 \text{ ppmv}\)

\[ \rightarrow (X_N^{column} - X_S^{column})_{obs} = 1.7 \text{ ppmv} \]

\[ \rightarrow M_N - M_S = 1.7 \text{ PgC} \]

Invert model \[ \rightarrow S_N - S_S = 2 \frac{M_N - M_S}{\tau} = 3.4 \text{ PgC/yr} \]

\((\text{sources}_N - \sin ks_N) - (\text{sources}_S - \sin ks_S) = 3.4 \text{ PgC/yr}\)

\((6 \text{ PgC/yr} - \sin ks_N) - (0 - \sin ks_S) = 3.4 \text{ PgC/yr}\)

\[ \rightarrow \text{sinks}_N - \sin ks_S = 2.6 \text{ PgC/yr} \]

Obs Carbon Budget

\[ \text{Sink}_{N} + \text{Sink}_{S} = 3 \text{ PgC/yr} \]
Where are the Carbon Sinks?

**Budget**

\[\text{sinks}_N + \text{sink}_S = +3 \text{ PgC/yr}\]

**Gradient**

\[\text{sinks}_N - \text{sink}_S = 2.6 \text{ PgC/yr}\]

\[\implies \text{sinks}_N = 2.8 \text{PgC/yr}; \quad \text{sink}_S = 0.2 \text{ PgC/yr}\]

Northern sinks > Southern Sinks !!!!!!!
Example II: Perfect 3D atm circulation model.  
Steady state

(1) Forward Step

• Premise: Atm CO$_2$ = linear combination of response to each source or sink

1.0 Divide surface into “basis regions”

1.1: Specify unitary source (e.g. 1 PgC/year) each year from each region

1.2: Simulate atm CO$_2$ “basis” response with atm general circulation model

1.3 Reconstruct fluxes and concentrations: unknown source strength $\mu_k$

\[
\begin{align*}
\vec{S}_k(x, y) & \quad \text{transport model} \quad \vec{C}_k(x, y, z, t) \\
S &= \sum_{k\text{-regions unknown}} \mu_k \times \vec{S}_k(x, y) \\
c_{\text{model}}(x, y, z) &= \sum_k \mu_k \times \vec{C}_k(x, y, z)
\end{align*}
\]
Ex II: (Step 2) Bayesian Inversion: perfect circulation model

**Inversion:** Seek the optimal source/sink combination \( \{ \mu_k \} \) to match atmospheric CO\(_2\) data:

\[
J = \sum_{\text{stn}} \frac{[C_{\text{obs}}(\text{stn}) - \sum_{k-\text{regions}} \mu_k \times \hat{c}_k(\text{stn})]^2}{\sigma_{\text{stn}}^2} + \sum_{k-\text{regions}} \frac{[\mu_k - \mu_k^{\text{prior}}]^2}{[\sigma_k^{\text{prior}}]^2}
\]

**Obs. Network –**
- mainly remote marine locations

Trying to infer information over land
Undetermined; non-unique solutions; prior estimates of source/sinks as additional constraints
Ex IIa: Posterior from many “perfect” circulation models

$\mu_k^{\text{prior}} \pm \sigma_k^{\text{prior}}$

Model m:

$\{\mu_{mk}^{\text{posterior}} \pm \sigma_{mk}^{\text{posterior}}\}$

- Mean, std_dev ($\mu_{mk}^{\text{posterior}}$)
- Mean ($\sigma_{mk}^{\text{posterior}}$)

Little innovation in tropics, Africa
Great innovation in S. Ocean

Gurney et al. Nature 2005
Separate atm and oceanic inferences of air-sea CO2 flux
Variation 1: monthly and interannual

Each region k: Pulse release into atm --> concentration for 24 months

\[ \tilde{s}_k(x, y, t_0^{1\text{month}}) \overset{\text{transport}}{\rightarrow} \hat{c}_k(x, y, z, t); \quad t = t_0, t_1, \ldots, t_{24} \]

Concentration at any time is the cumulative result of emission over the past 24 months

\[ C_{\text{model}}(x, y, z, t) = \sum_{k\text{-regions}} \int_{t-24}^{t} \mu_k(t') \cdot \hat{c}_k(x, y, t,t') \cdot dt' \]

Find \( \mu_k(t) \) that minimize \( J \):

\[ J = \sum_{t} \left\{ \sum_{stn} \frac{[C_{\text{obs}}(stn, t) - C_{\text{model}}(stn, t)]^2}{\sigma_{stn}^2} \right\} + \sum_{k\text{-regions}} \left\{ \frac{[\mu_k(t) - \mu_k^{\text{prior}}(t)]^2}{[\sigma_k^{\text{prior}}]^2} \right\} \]
Combine global surface network, transport models & linear inverse to resolve surface CO$_2$ fluxes at regional & monthly scales

Miller et al. J. Geophys. Res. 2007
Variations 2: don’t like regions and/or prior flux pattern for each region

- Ignore flux strength priors in cost function --> non-unique mathematical solution may not be physically realistic (e.g. Fan et al.)

- Solve for source strength for each grid box (Kaminiski et al.): unconstrained

- Geostatics: estimate flux correlation lengths (Rodenbeck, Michalak et al)

\[ J = \sum_t \{ \sum_{stn} \frac{(C_{obs}(stn,t) - C_{mod el}(stn,t))^2}{\sigma_{stn}^2} \}
+ \sum_{k-regions} \frac{[\mu_k(t) - \mu_k^{prior}(t)]^2}{[\sigma_k^{prior}]^2} \} \]
Inversions using Satellite CO2: gaps in space and time

• Old-fashioned inversion (tracer model, min cost function J) for surface fluxes (source strengths for K regions) as before (Rayner and O’Brien):  
  \[ c_{\text{obs}}(\text{stn}, t) \rightarrow c_{\text{sat}}(x_{\text{overpass}}, y_{\text{overpass}}, t_{\text{overpass}}) \]

• 2-step: (1) Assimilate AIRS radiance at top of atm into weather prediction model --> full 4D CO2(x,y,z,t) (Engelen et al. 2009). (2) old fashioned inversion for surface fluxes (Chevallier et al. 2009)

• Estimation of C_model (transport model) and min(J) uncoupled
CO2 Data Assimilation: 2 General Classes

- **Time-dependent inversion - {Kalman filter}**
  
  At time $t_n$:
  - Find $\text{CO}_2_{\text{analysis}}(t_n)$ by $\min(J)$: $J$ includes flux parameters(regions), $\text{CO}_2_{\text{obs}}(t_n)$, $\text{CO}_2_{\text{forecast}}(t_n)$
  - update surface fluxes $\mu$ for regions
  - model: update $\text{CO}_2_{\text{forecast}}(t_{n+1})$ (e.g. CarbonTracker)

- **Variational Approach - Find $\mu$ that minimizes $J$.**
  Use adjoint model ($\text{CO}_2(t_{n-1})$) to efficiently calculate $\frac{\partial J}{\partial \mu}$ (e.g. Baker)
Challenge: Transport Model

- Transport model needed to connect surface fluxes to CO2

- NCEP/ECMWF analysis --> “best approx” to real atm circulation every 3 or 6 hours.

- NCEP/ECMWF Reanalysis: retrospective assimilation of all weather data using the same general circulation model (uniform model physics through time). Treated as “known”, with zero uncertainty.

- NCEP/ECMWF Reanalysis: average of large ensemble of atm circulations; the average circulation is never realized. Yields T and humidity profiles after mixing, and transport model needs to reconstruct convective mixing. Spread of circulation ensemble not utilized
Following numerical prediction prediction:
Every 6 hours:

- Assimilate raw weather observations $(u,v,T,q,P_s)$ + AIRS CO2
- $\text{CO}_2(x,y,z,t)$
- Estimate surface flux from conservation equation (e.g. estimate evaporation from the full 4D water vapor field)