Problem: Searching for long-lived, weak chirps in noise

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Examples from gravitational-wave searches

GW source: rapidly rotating neutron star with small, quadrupolar “mountain” (order 1 cm high)

GW source: inspiraling binary of two black holes
“Gravitational-wave pulsar” searches

Order of magnitude values

\[ f \sim 300 \text{ Hz} \]

\[ T_{obs} \sim 1 \text{ yr} \sim 3 \times 10^7 \text{ s} \]

\[ \Rightarrow N_{cyc} \sim 10^{10} \]

\[ \frac{h}{n} \sim 10^{-4} \]

at any instant, so with matched filtering:

\[ \text{SNR} \sim \frac{h}{n} \sqrt{N_{cyc}} \sim 10 \]

Actually, blind all-sky searches are looking for signals with SNR \( \sim 20-25 \).
Parameter space and # of Templates

• Typically ~5-6 parameters: \( \theta, \varphi, f_0, f_0, f_0 \)
  (plus 4 others that are trivially searched over)

For all-sky searches for unknown, fast, young pulsar, number of templates to cover search space is

\[ \sim 10^{22} \] This is # of FFTs
Comments

• To my knowledge, best current methods are semi-coherent and hierarchical (see talk by J. Rice)

• Observation: when template has decent (not great) overlap with imbedded signal, most of the SNR is “localized” in t-f space.
How 2 different chirping waveforms “interfere with” each other:

\[ \int h_1(t) h_2(t) \, dt \] integral dominated by contribution from short time around crossing of \( f_1(t) \) and \( f_2(t) \)

\[ SNR^2 \propto (\delta f)^{-1/2} \]

t-f tracks for 2 merging NS binaries at different z
1\textsuperscript{st} (simpler) version of proposed problem

1) Parameters are just: \( f_0, f_0, f_0, f_0 \)

with

\[
h = A \sin \varphi(t) \quad \varphi(t) = f_0 t + \frac{1}{2} f_0 t^2 + \frac{1}{6} f_0 t^3 + \frac{1}{24} f_0 t^4
\]

and \( A, T_{obs} \) adjusted to make simplest (grid of templates) matched-filtering search intractable.
2nd (harder) version of proposed problem

1) Few Parameters (~5): $\theta^i$

with

\[ h = A \sin 2\pi \int f(t') \, dt' \]
\[ \frac{df}{dt} = F(f, \theta^i) \]
\[ A = A(t, f(t), \theta^i) \]

with both $f, A$ slowly varying, and $T_{obs}$ adjusted to make simplest (grid of templates) matched-filtering search intractable.