Heirarchical Resolution

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Lightning Talk

Search Space

• Consider no drift. Good frequency resolution depends on matching phase of photons at beginning and end of the record. If true frequency is \( f_0 \), the number of cycles in time \( T \) is \( T/f_0 \), so if the hypothesized frequency is \( f = f_0 + \delta f \), \( \delta f \) should be \( o(T^{-1}) \) in order for a photon at the end of the record to be in phase with one at the beginning. The phase error at the end of the record is \( T\delta f \).

\[
10 \text{ days} = 864,000 \text{ sec}, \quad \delta f = T^{-1}. \quad \text{If a 40 Hz range has to be searched, a minimum of } 40 \times 864000 = 34,560,000 \text{ possible frequencies must be examined.}
\]

• Similarly, drift must be resolved within \( o(T^{-2}) \). To search the interval of possible frequency derivatives at this resolution, about 400-500 values must be examined.

• Consequence is that a test statistic must be evaluated \( \sim 10^9 \) values of frequency and its derivative.
Power Versus Computational Cost: Blocking

\[ \text{Power} \propto \theta^2 T \]

\[ f \text{ resolution} \propto T^{-1} \]

\[ \dot{f} \text{ resolution} \propto T^{-2} \]

Calculation of statistic for a single \((f, \dot{f}) \propto T\)

\[ \text{FLOPS} \propto T^4 \]

Partition \(T\) into \(B\) blocks, compute statistic in each block and average:

\[ \text{Power} \propto \frac{\theta^2 T}{\sqrt{B}} \]

\[ \text{FLOPS} \propto \frac{T^4}{B^3} \]
Blocking Vela and Crab

Vela: 318 blocks

Crab: 25 blocks
Hierarchical Search

*Maximize power subject to computational constraint.* Optimal paths through resolution levels found by dynamic programming.
Refinement of search space (local)
Power versus cost

Simulation with 4 different signal strengths