Space-based gravitational wave interferometry with optical frequency comb

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M. Tinto and N. Yu, "Time-Delay Interferometry with Optical Frequency Comb", *Phys. Rev. D*, **92**, 042002 (2015).

KISS Workshop: Optical Frequency Combs
for Space Applications.
11/02-05/2015

Introduction

- The detection of gravitational radiation will allow us to make astronomical observations otherwise unobtainable within the electromagnetic spectrum.
- Currently operating ground-based interferometers are expected to make the first detection before the end of this decade.
- Space-based interferometers operate in the mHz frequency band, complementary to that of ground-based detectors and significantly richer of gravitational wave signals (both in number and strength).
- Although the European eLISA mission is not expected to be launched before 2034, it is possible that a cheaper/smaller version of it might become operational by the middle of the next decade due to the significant cost-reductions experienced by both spacecraft and launching vehicles.

M. Tinto, D. DeBra, S. Buchman, S. Tilley, "gLISA: geosynchronous Laser Interferometer Space Antenna concepts with off-the-shelf satellites", *Review of Scientific Instruments*, **86**, 014501 (2015).

Introduction (cont.)

- No matter which mission will eventually fly, its triangular configuration will not be exactly equilateral!
- Unequal (and changing in time!) arm-lengths, together with large relative laser frequency shifts (due to inter-spacecraft relative velocities) will make the cancellation of the lasers and microwave phase noises a challenging task.
- Use of the optical frequency comb technique for generating RF signals (coherent to their onboard lasers) significantly simplifies the onboard interferometry architecture and makes it more robust to subsystems failure.

What are Gravitational Waves?

- Gravitational waves are propagating variations of space-time.
- They are consequence of the Equivalence Principle and of the fact that in Nature nothing propagates at a speed faster than the speed of light.
- Their existence was first proved by accurately monitoring the change in the orbital period of a binary system containing a Pulsar (PSR 1913+16) (Hulse & Taylor, Nobel Prize, 1993)
- We can attempt to detect them by monitoring relative changes in the frequency of coherent electromagnetic signals that propagate through space.
- Since the relative frequency changes, $\Delta\nu/\nu_0$, induced by a gravitational wave are proportional to the amplitude of the wave itself, **h**, any detector design must be built with a well defined frequency stability requirement.
- All the instrumental frequency noises must be kept below a level determined by a characteristic wave amplitude whose magnitude has been predicted by theorists!

Useful Formulas

Amplitude:

$$h^{TT}_{ij} = \frac{2}{r}\frac{d^2}{dt^2}\int \rho x_i x_j d^3x \Big|^{TT} \Rightarrow h \sim \phi_{in}\phi_{out}$$

$$h_+(t), h_\times(t)$$

Energy Radiated (Luminosity):

$$L = \frac{1}{5}\sum_{i,j} \dot{h}^{TT}_{ij}\dot{h}^{TT}_{ij} \propto (hf)^2$$

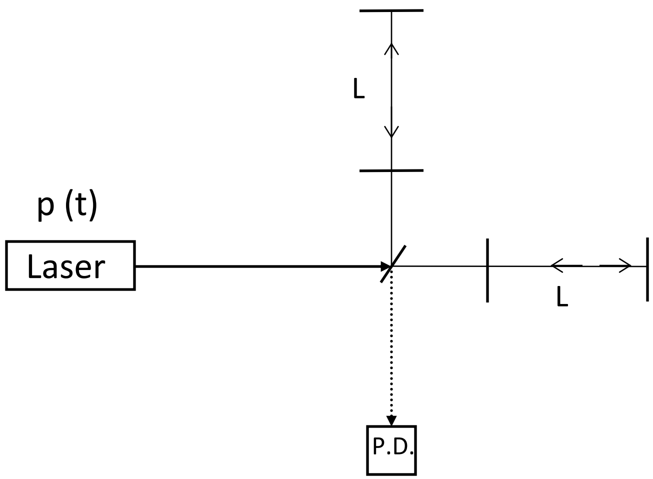
Why we want to study GW in Space?

- For a given energy-flux, at lower frequencies the strength of the signal is larger.
- To avoid seismic noise, which affects ground-based detectors!
- The number of known sources radiating in the mHz frequency band is very large.

Statement of the problems

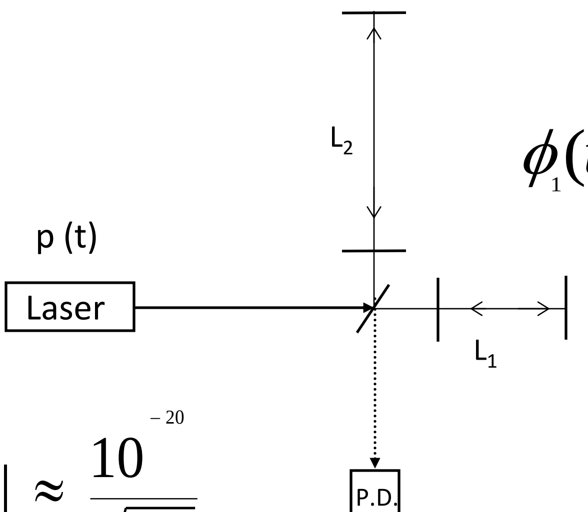
p (t) = Laser phase fluctuations

$$\frac{1}{2\pi\nu_0}\frac{dp(t)}{dt} \equiv \left[\frac{\Delta\nu(t)}{\nu_0}\right]_{Laser} = C(t)$$



$$\phi_1(t) = h_1(t) + p(t - 2L_1) + n_1(t)$$

$$\phi_2(t) = h_2(t) + p(t - 2L_2) + n_2(t)$$

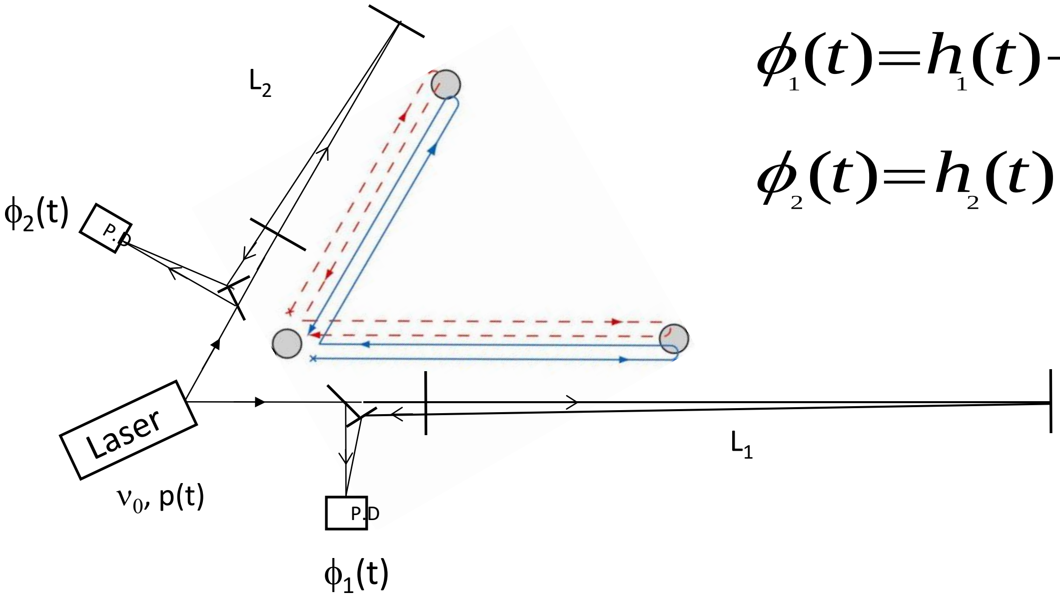


$$\phi_1(t) - \phi_2(t) \Rightarrow p(t - 2L_1) - p(t - 2L_2) \cong 2\frac{dp}{dt}\varepsilon L_1$$

$$|\tilde{h}(f)| \approx \frac{10^{-20}}{\sqrt{Hz}}$$

$$|\tilde{C}(f)| \approx \frac{10^{-13}}{\sqrt{Hz}}\ ,\ \varepsilon \cong 3\times 10^{-2} \Rightarrow \frac{5\times 10^{-16}}{\sqrt{Hz}}$$

Time-delay Interferometry (TDI)



$$\phi_1(t) = h_1(t) + p(t - 2L_1) - p(t) + n_1(t)$$

$$\phi_2(t) = h_2(t) + p(t - 2L_2) - p(t) + n_2(t)$$

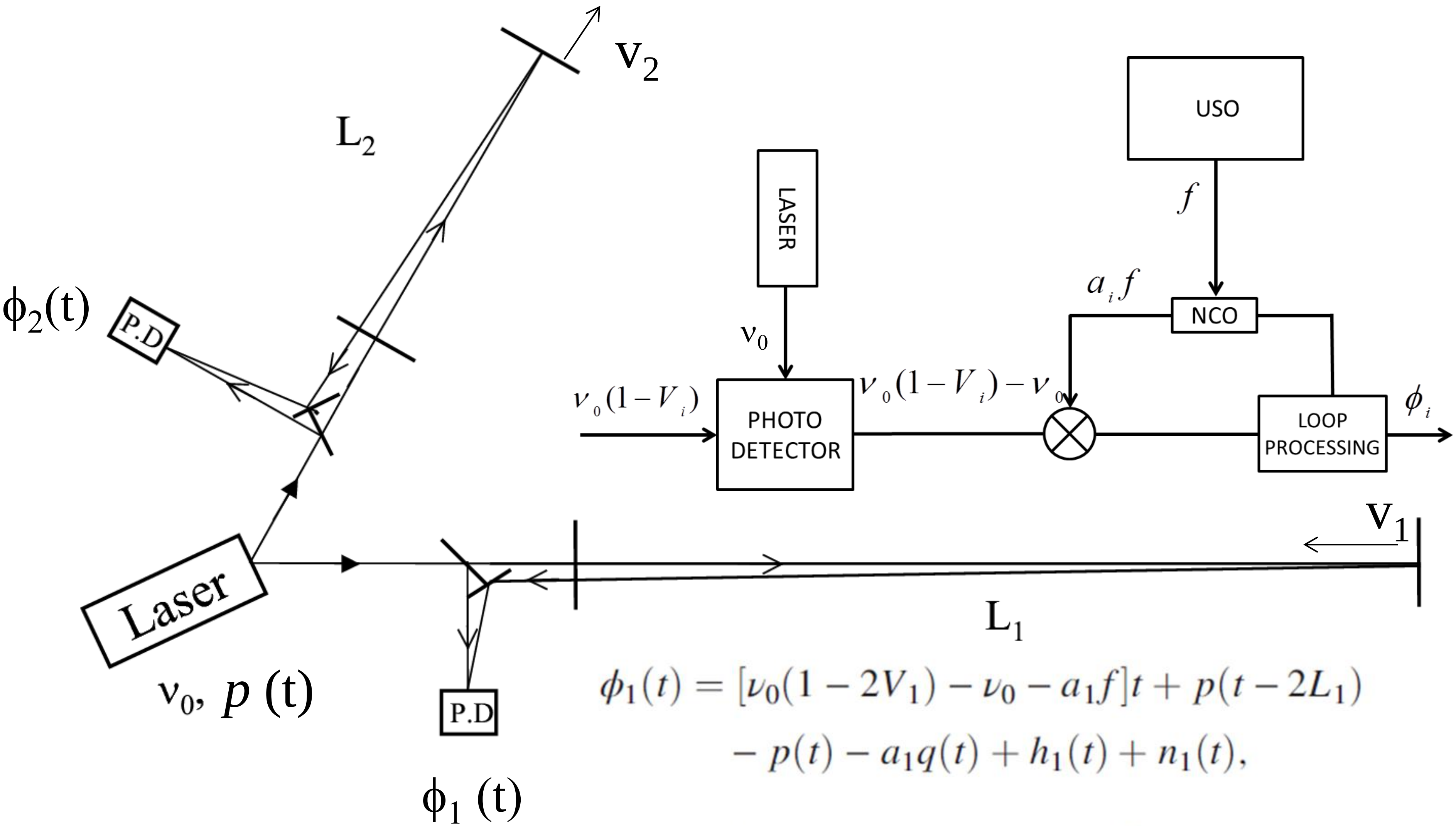
$$\phi_1(t) - \phi_2(t) = h_1(t) - h_2(t) + p(t - 2L_1) - p(t - 2L_2) + n_1(t) - n_2(t)$$

$$\phi_1(t - 2L_2) - \phi_2(t - 2L_1) = h_1(t - 2L_2) - h_2(t - 2L_1) + p(t - 2L_1) - p(t - 2L_2) + n_1(t - 2L_2) - n_2(t - 2L_1)$$

$$X(t) \equiv [\phi_1(t) + \phi_2(t - 2L_1)] - [\phi_2(t) + \phi_1(t - 2L_2)]$$

M. Tinto, & J.W. Armstrong, *Phys. Rev. D*, **59**, 102003 (1999).
D.A. Shaddock, M. Tinto, F.B. Estabrook & J.W. Armstrong, *Phys. Rev. D*, **68**, 061303 (R) (2003).
M. Tinto & S.V. Dhurandhar, *Living Reviews in Relativity*, **17**, 6, (2014).

Moving Mirrors



$$\phi_1(t) = [\nu_0(1-2V_1) - \nu_0 - a_1 f]t + p(t-2L_1) - p(t) - a_1 q(t) + h_1(t) + n_1(t),$$

$$\phi_2(t) = [\nu_0(1-2V_2) - \nu_0 - a_2 f]t + p(t-2L_2) - p(t) - a_2 q(t) + h_2(t) + n_2(t),$$

Moving...(cont.)

$$\phi_1(t) = p(t-2L_1) - p(t) - a_1 q(t) + h_1(t) + n_1(t)$$

$$a_i = -\frac{2V_i \nu_0}{f} \quad , i = 1,2$$

$$\phi_2(t) = p(t-2L_2) - p(t) - a_2 q(t) + h_2(t) + n_2(t)$$

$$b_i = -\frac{2V_i(\nu_0 + f)}{f}$$

- The above equations do not allow us to simultaneously cancel p & q and retain the GW signal!
- The LISA-solution to this problem involves generating side-bands via modulation of the laser light to produce additional side-band-to-side-band heterodyne measurements.
- One then has sufficient information for cancelling both p&q and retain the GW signal.

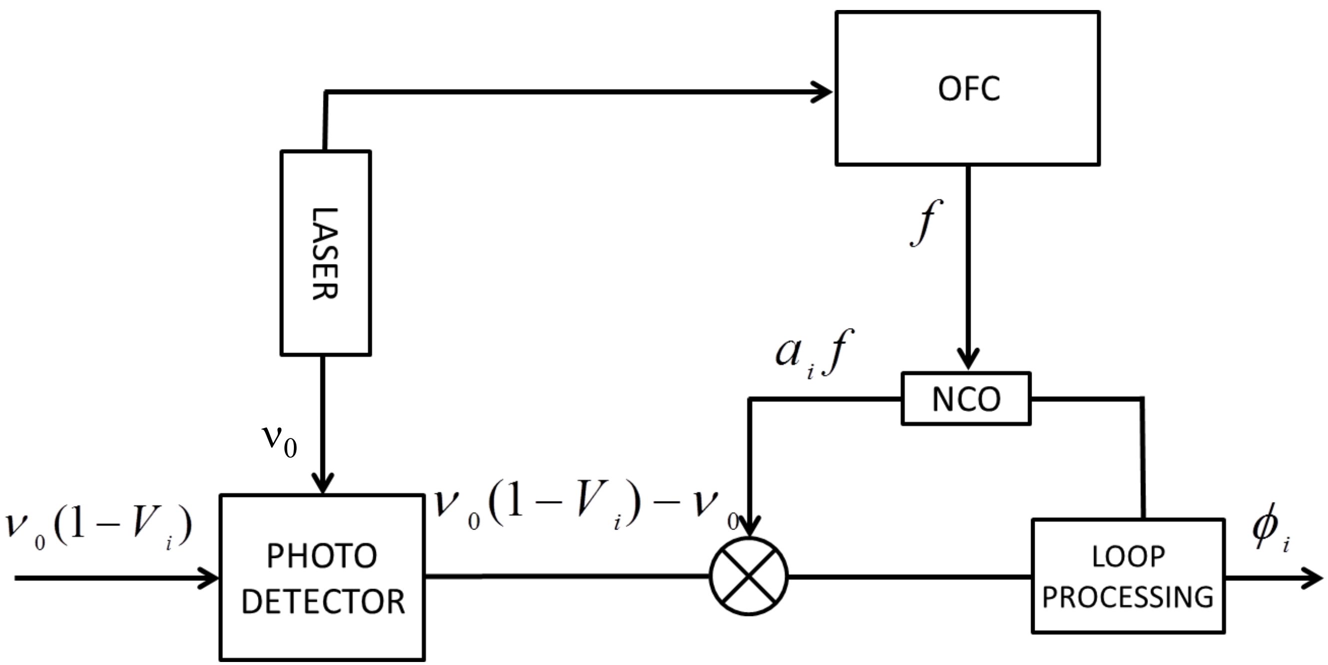
$$\phi_{1'}(t) = [(\nu_0 + f)(1-2V_1) - (\nu_0 + f) - b_1 f]t + q(t-2L_1) - (1+b_1)q(t) + p(t-2L_1) - p(t) + h_1(t) + n_{1'}(t)$$

$$\phi_{2'}(t) = [(\nu_0 + f)(1-2V_2) - (\nu_0 + f) - b_2 f]t + q(t-2L_2) - (1+b_2)q(t) + p(t-2L_2) - p(t) + h_2(t) + n_{2'}(t)$$

M. Tinto, F. B. Estabrook, and J.W. Armstrong, *Phys. Rev. D* **65**, 082003 (2002).
R.W. Hellings, G. Giampieri, L. Maleki, M. Tinto, K. Danzmann, J. Homes, and D. Robertson, *Opt. Commun.* **124**, 313 (1996).
R.W. Hellings, *Phys. Rev. D* **64**, 022002 (2001).

TDI with OFC

- An elegant alternative to the "modulation scheme" is to rely on the OFC technique!



TDI…(cont.)

$$q(t) = \frac{f}{\nu_0} p(t) + \Delta q(t)$$

$$\phi_1(t) = p(t - 2L_1) - p(t) + 2V_1 p(t) + h_1(t) + n_1(t),$$

$$\phi_2(t) = p(t - 2L_2) - p(t) + 2V_2 p(t) + h_2(t) + n_2(t).$$

$$X^{OFC}(t) = [\phi_1(t - 2L_2) + (1 - 2V_1)\phi_2(t)] \\ - [\phi_2(t - 2L_1) + (1 - 2V_2)\phi_1(t)]$$

L. S. Ma, Z. Bi, A. Bartels, L. Robertsson, M. Zucco, R. S. Windeler, G. Wilpers, C. Oates, L. Hollberg, and S. A. Diddams, *Science* **303**, 1843 (2004).

Conclusions

- With the advent of self-referenced optical frequency combs, it is possible to generate a heterodyne microwave signal that is coherently referenced to the onboard laser.
- In this case the microwave noise can be canceled directly by applying modified time-delay interferometric combinations to the heterodyne phase measurements.
- This approach avoids the use of modulated laser beams as well as the need for additional ultra-stable oscillator clocks.
- Nan Yu and I have been awarded a NASA APRA grant to test TDI with OFC. Stay Tuned!