Proof of Concept of the Quantum Capacitance Detector

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Single Cooper-Pair Box

Electrostatic gate charge

\[ n_G = \frac{C_G V_G}{2e} \]

Charging energy

\[ E_C = \frac{e^2}{2C_{\Sigma}} \]

Josephson coupling

\[ E_J = E_J^{\max} \left| \cos \left( \frac{\pi \Phi}{\Phi_0} \right) \right| \]

Hamiltonian

\[ H = 4E_C \sum_n (n-n_G)^2 |n\rangle\langle n| - \frac{E_J}{2} \sum_n (|n+1\rangle\langle n| + |n\rangle\langle n+1|) \]
Energy levels, Coulomb Staircase and Quantum Capacitance

• In the absence of Josephson coupling, Energy is given by parabolas centered at integer values of Cooper Pair Charge
  \[ E = (Q - 2ne)^2 = (C_g V_g - 2ne)^2 \]

• As the gate voltage is increased, Cooper Pairs tunnel to minimize the energy and the charge on the island changes in a stepwise fashion
• The capacitance of the island \( C_Q = 2e \frac{d\langle n \rangle}{dV_g} \) spikes up at the degeneracy points where the charge in the island is changing fast
• The Josephson Coupling introduces splittings in the energy levels
• Eigenvectors are symmetrical and anti-symmetrical combinations of the charge states
• The larger \( E_j \), the “rounder” the charge staircase and the smaller the capacitance peaks
• In the absence of tunneling, only one parabola would exist (\( n=0 \)) and the capacitance would be constant as a function of the gate voltage
• The variable capacitance is due to the quantum nature of the system and is called the quantum capacitance

\[ <\rho>(Q_n) \]
\[ C_Q(F) \]
- If there are quasiparticles present in the leads, they could tunnel in and out of the island.

- When they tunnel, they shift the effective gate voltage by e/Cg (or ng=0.5).

- Coulomb staircase and quantum capacitance curve shifts by ng=0.5 each time a quasiparticle tunnels in or out.
Measurement Technique

- Change in capacitance pulls the frequency of the oscillator
- Resonant frequency (~600 MHz) is far below qubit level spacing (~5 GHz)
- Noise and RF probe isolated from qubit
- Overall circuit capacitance is determined by qubit state
- Enables elegant multiplexing of many qubits
- Phase shift is measured with quadrature mixer
Measurement Technique

- In actual detectors we will use $\lambda/2$ resonator capacitively coupled to a feedline.
- SCB is the variable capacitor at the end of resonator.
Experimental Setup

- Two SCBs with multiplexed quantum capacitance readout
Experimental measurements
Tunnel Rate Measurements

- Phase measured at degeneracy point in real time
- One quasiparticle tunnel event shifts phase by ~140 degrees
- Can study statistics and compare with existing theory
Dependence on Quasiparticle Density

\[ \Gamma_{in} \approx Kn_{qp} \quad \Gamma_{out} \text{ independent of } n_{qp} \]

\[
K = \frac{G_N}{e^2} \frac{e^{\Delta_L/kT}}{N_L} \int_{\max(\Delta_I - \Delta_E, \Delta_L)}^{\infty} dE \frac{E(E + \delta E) - \Delta_L \Delta_I}{\sqrt{((E + \delta E)^2 - \Delta_L^2)(E^2 - \Delta_L^2)}} e^{-E/kT}
\]

- Approximations valid at low temperature
- Simple relationship between rates and QP density is ideal for detector

\[
P_{odd} = \frac{\Gamma_{in}}{\Gamma_{in} + \Gamma_{out}}
\]
The Quantum Capacitance Detector

- Radiation coupled by an antenna breaks Cooper pairs in the reservoir (absorber)
- Quasiparticles tunnel onto the island with a rate $\Gamma_{\text{in}}$ proportional to the quasiparticle density in the reservoir
- Quasiparticles tunnel out of the island with a rate $\Gamma_{\text{out}}$ independent of the number of quasiparticles in the reservoir
- At steady state the probability of a quasiparticle being present in the island is given by $P_0(N_{qp}) = \frac{\Gamma_{\text{in}}}{\Gamma_{\text{in}} + \Gamma_{\text{out}}}$
- The resulting change in the average capacitance will be $C_Q = \frac{4E_c}{E_J}(C_g^2/C_J)P_0(N_{qp})$
- This change in capacitance will produce a phase shift $\delta \Phi \approx \frac{2C_Q}{(\omega_0 Z_0 C_g^2)}$
- With the existing tank circuit parameters, phase shift per quasiparticle should be 138 degrees, in very good agreement with experiments
Noise Sources and NEP

- Phase noise – from phase measurement histogram the rms phase noise is ~ 33 degrees or 1.8x10^-3 radian/Hz^{1/2} over the 100kHz bandwidth

- Telegraph noise: the tunneling on and off the island is approximated as a Poisson process with rates $\Gamma_{eo}$ and $\Gamma_{oe}$. At low frequencies the spectral density of noise associated with the process is

$$S_{\Phi}^{Tele} = \frac{\Delta \Phi^2}{\pi} \frac{\Gamma_{eo} \Gamma_{oe}}{(\Gamma_{eo} + \Gamma_{oe})^2}$$

- Fano noise: the number of quasiparticles generated by an incoming photon has an uncertainty given by $(FN_{qp})^{1/2}$ where $F \approx 0.2$ is the Fano factor for this system. The associated NEP is

$$NEP_{Fano}^{Fano} = \sqrt{\frac{FP_s \Delta}{\eta}}$$

- Generation-recombination noise: quasiparticles can be thermally excited over the superconducting gap and recombine into Cooper pairs, introducing a fluctuation in the number of quasiparticles in the reservoir. The associated NEP is

$$NEP_{GR} = 2\Delta L \sqrt{\frac{N_{eq}}{\tau_g}}$$

- The noise equivalent power at low frequencies will be given by

$$NEP = \sqrt{\left(\frac{dP_s}{dN_{qp}}\right)^2 \left(\frac{d\Phi}{dN_{qp}}\right)^2 (S_{\Phi}^{Phase} + S_{\Phi}^{Tele}) + NEP_{GR}^2 + NEP_{FANO}^2}$$

$N_{eq}$ is the number of equilibrium quasiparticles.

$$N_{eq} = \Omega_{f} D(E_F) \sqrt{\frac{\pi k_B T \Delta l}{2}} \exp\left(-\frac{\Delta}{k_B T}\right)$$
Left: NEPs from various noise sources calculated for devices optimized for $\lambda = 100\mu m$, optical loading $10^{-19}$ W and $R=1000$ as a function of temperature. Right: NEPs of various noise sources as a function of wavelength as compared to the requirements for a spectrometer with $R=1000$ and the expected optical loading at L2 for a cold (4.2K) telescope. The operating temperature was chosen to be 0.1K at which the GR noise contribution is negligible.
• *Detector is background limited over a wide range of operation*
Theoretical Sensitivity vs. Absorber Volume

- Absorber volume is a key parameter
- Can be used to trade sensitivity for saturation power
Single Photon Detection

Photon arrival rate for a cold (4K) telescope with an R=1000 spectrometer at L2 as a function of wavelength.

- From the NEP, the energy resolution will be:
  \[ \delta U = \text{NEP}(0) \sqrt{\tau_R} \]

From Fano limit:
\[ \frac{U}{\delta U} = \frac{1}{2.36} \sqrt{\frac{\eta hc}{F \lambda \Delta}} \]
Experimental Confirmation
Quasiparticle Injection with SIS junctions
• Ran a current through SIS junction to inject quasiparticles on reservoir
• AC component of current simulates signal and DC optical loading
• $\tau_D$ is the time for quasiparticles to diffuse through constriction
• Graph shows lock-in response as a function of number of quasiparticles present in the reservoir.
• The measured noise in number of quasiparticles in the reservoir was $\delta N_{qp}\sim 11$ qp/Hz$^{1/2}$, which would yield an NEP $\sim 3 \times 10^{-18}$ W/Hz

$$N_{qp} = \frac{I}{e} \left( \frac{1}{\tau_R} + \frac{1}{\tau_D} \right)^{-1} \sim \frac{I}{e} \tau_D$$

$$NEP = \eta \frac{\Delta}{\tau_R} \delta N_{qp}$$
Experimental demonstration
Response x loading
Multiplexing scheme

- Gates are swept with a low frequency $f$ signal of amplitude $e/C_g$
- A mixer demodulates the reflected RF signal to the modulation frequency $f$ and a down converter translates the results to DC.
- In the multi-pixel readout, A low frequency comb function (0-200MHz) containing several frequency components is produced digitally using a D/A converter and then block up-converted, resulting in a comb of RF carrier frequencies with each frequency corresponding to a particular detector.
- All of the SCB gates are tied together through a common bias line and modulated at the same frequency.
- The reflected RF comb, containing the phase shift information for the entire array, is demodulated at the bias modulation frequency, down-converted to the 0-200MHz band, then digitized and digitally demultiplexed.
Applications in FIR-Submillimeter Astronomy

- Cold (4.2K) telescope at L2 with R=1000 spectrometer
Detector Advantages

- SCB has extreme sensitivity to the presence of quasiparticles
- Sensitivity of QCD rivals MKID and TES
- Frequency-domain multiplexing allows scaling to large arrays
- Applicable to submillimeter wavelengths for far-infrared astrophysics
- Can be easily incorporated with existing technology for MKID arrays
- Detector (SCB) is separate from resonator – flexibility of design
- NEP and saturation power easily tailorable