Fundamentals of Optical Interferometry for Gravitational Wave Detection

Yanbei Chen
California Institute of Technology
Gravitational Waves

accelerating matter

oscillation in space-time curvature

A perturbation of ~Minkowski space-time
Linearized Einstein’s Equations

- Near flat spacetime, metric $\eta$ is corrected by $h$ (relative correction in time$^2$ or length$^2$)

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \]

- “trace-reversed perturbation” satisfies wave eqn, sourced by energy and momentum

\[ \Box \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu} \]

$T_{00}$: energy density, $T_{01,02,03}$: momentum density, $T_{11,12,...,33}$: stress

*analogous to EM:* \[ \Box A_\mu = 4\pi J_\mu \]

- Leading multipole radiation is mass quadrupole (analogous to Electric Quadrupole)

\[ h \sim \frac{\ddot{Q}}{d} \sim \frac{ML^2\Omega^2}{d} \sim \frac{Mv^2}{d} \]

- Magnitude is very very small

1 m away from the most powerful H-bombs tested ($2 \times 10^{17}$ J): $h \sim 10^{-27}$

Total mass of $5M_\odot$ colliding at $v \sim 0.3c$, at VIRGO cluster: $h \sim 3 \times 10^{-21}$
Evidence of Gravitational Waves

• Hulse-Taylor binary pulsar discovered in 1974
• Two $1.4\ M_\odot$ neutron stars orbiting around each other with period $7.75\ h$, one emitting radio pulses
• Energy carried away by GW causing orbital period to shrink
• Current GW frequency (twice orbital freq): $71\mu Hz$
• Orbital decay will cause merger in $300M$ years (GW frequency will reach $10Hz - kHz$ during merger, the final \textit{several minutes})

estimated merger rate $20 - 1000/Myr$ in Milky Way
[\textit{e.g., Kalogera et al 2004}]
Gravitational waves are most simply thought of as ripples in the curvature of space-time, their effect being to change the separation of adjacent masses on Earth or in space; this tidal effect is the basis of all present detectors. Gravitational wave strengths are characterised by the gravitational-wave amplitude $h$, given by

$$h = \frac{2}{L^2},$$

where $L$ is the change in separation of two masses a distance $L$ apart; for the strongest-allowed component of gravitational radiation, the value of $h$ is proportional to the third time derivative of the quadrupole moment of the source of the radiation and inversely proportional to the distance to the source. The radiation field itself is quadrupole in nature and this shows up in the pattern of the interaction of the waves with matter.

The problem for the experimental physicist is that the predicted magnitudes of the amplitudes or strains in space in the vicinity of the Earth caused by gravitational waves even from the most violent astrophysical events are extremely small, of the order of $10^{-21}$ or lower. Indeed, current theoretical models on the event rate and strength of such events suggest that in order to detect a few events per year – from coalescing neutron-star binary systems, for example, an amplitude sensitivity close to $10^{-22}$ over timescales as short as a millisecond is required. If the Fourier transform of a likely signal is considered it is found that the energy of the signal is distributed over a frequency range or bandwidth, which is approximately equal to $1/t$ timescale. For timescales of a millisecond the bandwidth is approximately 1000 Hz, and in this case the spectral density of the amplitude sensitivity is obtained by dividing $10^{-22}$ by the square root of 1000. Thus, detector noise levels must have an amplitude spectral density lower than $10^{-23} \text{Hz}^{-1/2}$ over the frequency range of the signal. Signal strengths at the Earth, integrated over appropriate time intervals, for a number of sources are shown in Figure 2.

**Figure 2:** Some possible sources for ground-based and space-borne detectors.

- merging supermassive black holes
- smaller BHs falling into supermassive BHs
- binaries with larger separations
- stochastic background

- merging neutron stars/black holes
- rotating aspherical neutron stars
- collapsing stars
- stochastic background
Ground-Based Detectors

- How do we detect gravitational waves on the earth?
  - Effect of GW in a “small region” (compared with wavelength)
  - Optical Interferometry with short arms
  - Quantum enhancement on the ground
  - Limitations of GW detection on the ground
Plane Gravitational Wave

- Coordinates can be chosen such that a plane wave along \( z \) direction can be written as

\[
\begin{align*}
    g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \\
    h_{ij}^{\text{TT}}(t,x,y,z) &= \begin{bmatrix}
        h_+(t-z) & h_\times(t-z) & 0 \\
        h_\times(t-z) & -h_+(t-z) & 0 \\
        0 & 0 & 0
    \end{bmatrix}, \quad i,j = x,y,z
\end{align*}
\]

This is called the TT gauge because \( h \) is transverse, and traceless.

\( h_+ , \times \) are the two polarizations of the plane GW

This coordinate system is convenient in describing wave propagation, but not for describing relative motions of nearby objects.
Influence of GW on Light and Matter

... in a region with spatial size much less than GW wavelength we go to the Local Lorenz Frame

- low-velocity objects feel tidal gravity force:
  \[ M\ddot{x}^j = \frac{1}{2} M\dot{h}_{jk} \dot{x}^k + F^j \]

- array of free masses will be distorted with \( \text{strain} \sim h \)

\[ h_{jk}^T = \begin{bmatrix}
  h_+ & h_\times & 0 \\
  h_\times & -h_+ & 0 \\
  0 & 0 & 0
\end{bmatrix} \]

- Light propagation is unaffected by gravitational wave
- Problem reduced to the measurement of a (very weak) classical force field
A Global Network

- LIGO-H
  Hannover, Germany

- GEO 600
  Hannover, Germany

- VIRGO
  Pisa, Italy

- KAGRA
  Kamioka, Japan

- TAMA 300
  Tokyo, Japan

- LIGO-India
  ???

Monday, June 25, 12
Laser Interferometer Gravitational-wave Observatory (LIGO)

LIGO Hanford, WA site

LIGO Livingston, LA site
Ground-Based Laser Interferometer GW Detector

Schematic drawing of LIGO Detectors
Sensitivity achieved in first-generation LIGO

achieving $2 \times 10^{-23}/\text{rtHz}$ at $\sim 200$ Hz

Spectral Density: Noise Power Per Frequency Band

$$h_{\text{rms}} \sim \sqrt{f \cdot S_h}$$
Michelson Interferometer: Sensitivity Estimate

- Resolvable phase: \( \sim 1/(\text{Number of Photons})^{1/2} \)
- Photon Number: \( \text{Power} \times \text{Duration}/(\text{Energy of Photon}) \)

\[
\delta h = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \omega_0}{I_0 \tau}} \Rightarrow S_h = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \omega_0}{I_0}} = 7.5 \cdot 10^{-21} \left( \frac{4 \text{km}}{L} \right) \left( \frac{5 \text{W}}{I_0} \right)^{1/2} \text{Hz}^{-1/2}
\]

assuming \( \lambda=1\mu\text{m} \)

\( \text{rms error} \)

“shot noise” spectral density
(noise power/frequency band)

\( \text{initial LIGO } 2 \times 10^{-23} \text{ factor of 300-400 away!} \)
Resonant Enhancement of Sensitivity

\[ \frac{L_h}{2} \]

**test-mass mirror**

**arm cavity**

**beamsplitter**

**power recycling mirror**

**test-mass mirror**

Laser Light \( I_0, \omega_0 \)

**arm cavity**

LIGO I:

- Power-recycling gain \( \sim 50 \text{ -- } 60 \)
- [noise \( \sim 1/(\text{Mich. input power})^{1/2} \)]
- \# of bounces in arm cavity \( \sim 40 \)
- Total factor of improvement \( \sim 300 \)

Improvement is only below the **bandwidth** of the cavity.

\[ \Omega_{GW} > \gamma \iff \tau_{GW} < \tau_{storage} \]

Above bandwidth:

- within light storage time, GW already changes sign. Resonant enhancement deteriorates!

Cavity Gain \[
\frac{2 / T}{\sqrt{1 + (\Omega / \gamma)^2}}
\]

\[ \sqrt{S_h^{\text{shot}}} = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \omega_0}{I_0}} \cdot \sqrt{1 + (\Omega / \gamma)^2} \sqrt{\frac{2 / T}{2 / T}} \]
Radiation Pressure Noise

\[ \delta P = \delta N \cdot \frac{2\hbar \omega_0}{c} = \sqrt{N} \cdot \frac{2\hbar \omega_0}{c} = \sqrt{\frac{I_0 \tau}{\hbar \omega_0}} \cdot \frac{2\hbar \omega_0}{c} \]

rms momentum of mirror given by photon # fluctuation

\[ \delta F = \frac{\delta P}{\tau} = \frac{\sqrt{4\hbar \omega_0 I_0}}{c^2 \tau} \]

\[ \Rightarrow \sqrt{S_F} = \sqrt{\frac{4\hbar \omega_0 I_0}{c^2}} \]

without cavity ...

\[ \Rightarrow \sqrt{S_x} = \frac{1}{m \Omega^2} \sqrt{\frac{4\hbar \omega_0 I_0}{c^2}} \]

\[ \Rightarrow \sqrt{S_{h}^{\text{rad pres}}} = \frac{1}{m \Omega^2 L} \sqrt{\frac{4\hbar \omega_0 I_0}{c^2}} \]

with cavity gain

\[ \sqrt{S_{h}^{\text{rad pres}}} = \frac{1}{m \Omega^2 L} \sqrt{\frac{4\hbar \omega_0 I_0}{c^2}} \frac{2}{\sqrt{1 + (\Omega / \gamma)^2}} \]
If we place the two types of noise together

\[
\sqrt{S_{\text{shot}}} = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \omega_0}{I_0}} = \frac{1}{L} \sqrt{\frac{\hbar c^2}{I_0 \omega_0}} \frac{\sqrt{1 + (\Omega / \gamma)^2}}{2 / T}
\]

\[
\sqrt{S_{\text{rad pres}}} = \frac{1}{m \Omega^2 L} \sqrt{\frac{4 \hbar I_0 \omega_0}{c^2}} \frac{2 / T}{\sqrt{1 + (\Omega / \gamma)^2}} = \frac{2\hbar}{m \Omega^2 L} \sqrt{\frac{I_0 \omega_0}{\hbar c^2}} \frac{2 / T}{\sqrt{1 + (\Omega / \gamma)^2}}
\]

Their dependences on power and cavity gain are opposite.

The noise never surpasses the standard quantum limit.

\[
\sqrt{S_{h, \text{SQL}}} = \sqrt{\frac{8\hbar}{m \Omega^2 L^2}}
\]
Quantum Optical Noise in LIGO-I

INITIAL INTERFEROMETER SENSITIVITY

-19
-20
-21
-22
-23
-24
10
100
1000
10000
Frequency (Hz)

initial ligo

noise from quantum optical fluctuations

noise from thermal fluctuations

square root of noise spectral density
Generations of GW Detectors

- **initial LIGO** (2007)
  - no detections

- **Advanced LIGO**
  - Being installed
  - **2015**: first science run
  - (hopefully) first detections

- **LIGO-III** (after detections)
  - precise knowledge of waveforms

---

Monday, June 25, 12
Where does quantum noise come from?

Laser Light \( I_0, \omega_0 \)

\( \frac{Lh}{2} \)

beamsplitter

arm cavity

power recycling mirror

fluctuations entering from dark port

test-mass mirror

test-mass mirror

\( \frac{Lh}{2} \)
Quadratures, Homodyne Detection and Squeezing

- Optical field close to $\omega_0$ can be written in the **quadrature representation**
  \[ E(t) = E_1(t) \cos \omega_0 t + E_2(t) \sin \omega_0 t \]
  $E_{1,2}(t)$: slowly varying

- Act as modulations when superimposed with single-frequency carrier at $\omega_0$

**phasor diagram**
Quadratures, Homodyne Detection and Squeezing

• Optical field close to $\omega_0$ can be written in the **quadrature representation**

$$E(t) = E_1(t)\cos \omega_0 t + E_2(t)\sin \omega_0 t \quad E_{1,2}(t) \text{ : slowly varying}$$

• Act as modulations when superimposed with single-frequency carrier at $\omega_0$

![Phasor diagram](attachment:phasor_diagram.png)
Optical field close to $\omega_0$ can be written in the \textit{quadrature representation}

$$E(t) = E_1(t)\cos\omega_0 t + E_2(t)\sin\omega_0 t \quad E_{1,2}(t) : \text{slowly varying}$$

Act as modulations when superimposed with single-frequency carrier at $\omega_0$

\begin{align*}
E_1(t) &\quad \text{carrier} \\
E_2(t) &\quad \text{phase quadrature} \\
A \cos \omega_0 t &\quad \text{amplitude quadrature}
\end{align*}

\textit{phasor diagram}
Optical field close to $\omega_0$ can be written in the **quadrature representation**

\[ E(t) = E_1(t) \cos \omega_0 t + E_2(t) \sin \omega_0 t \quad E_{1,2}(t) : \text{slowly varying} \]

Act as modulations when superimposed with single-frequency carrier at $\omega_0$
Optical field close to $\omega_0$ can be written in the quadrature representation

$$E(t) = E_1(t) \cos \omega_0 t + E_2(t) \sin \omega_0 t$$

$E_{1,2}(t)$: slowly varying

Act as modulations when superimposed with single-frequency carrier at $\omega_0$
• Optical field close to $\omega_0$ can be written in the quadrature representation

$$E(t) = E_1(t) \cos \omega_0 t + E_2(t) \sin \omega_0 t \quad E_{1,2}(t) : \text{slowly varying}$$

• Act as modulations when superimposed with single-frequency carrier at $\omega_0$

phasor diagram
Optical field close to $\omega_0$ can be written in the quadrature representation

$$E(t) = E_1(t)\cos\omega_0 t + E_2(t)\sin\omega_0 t \quad E_{1,2}(t): \text{slowly varying}$$

Act as modulations when superimposed with single-frequency carrier at $\omega_0$.
Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1 a_1} S_{a_2 a_2} - |S_{a_1 a_2}|^2 \geq 1 \]

**Minimum Uncertainty**

Gaussian States are:

- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Quadratures, Homodyne Detection and Squeezing

- Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1a_1} S_{a_2a_2} - | S_{a_1a_2} |^2 \geq 1 \]

**Minimum Uncertainty**
Gaussian States are:
- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Quadratures, Homodyne Detection and Squeezing

- Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1 a_1} S_{a_2 a_2} - |S_{a_1 a_2}|^2 \geq 1 \]

**Minimum Uncertainty**

Gaussian States are:

- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1 a_1} S_{a_2 a_2} - |S_{a_1 a_2}|^2 \geq 1 \]

**Minimum Uncertainty**

Gaussian States are:

- vacuum state
- coherent states
- squeezed vacuua
- squeezed states

Monday, June 25, 12
Quadratures, Homodyne Detection and Squeezing

- Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1 a_1} S_{a_2 a_2} - | S_{a_1 a_2} |^2 \geq 1 \]

**Minimum Uncertainty**
Gaussian States are:

- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Quadratures, Homodyne Detection and Squeezing

- Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1 a_1} S_{a_2 a_2} - |S_{a_1 a_2}|^2 \geq 1 \]

**Minimum Uncertainty**

Gaussian States are:

- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1a_1} - S_{a_2a_2} - |S_{a_1a_2}|^2 \geq 1 \]

Minimum Uncertainty
Gaussian States are:

- vacuum state
- coherent states
- squeezed vacuua
- squeezed states

Monday, June 25, 12
Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1 a_1} S_{a_2 a_2} - |S_{a_1 a_2}|^2 \geq 1 \]

**Minimum Uncertainty**

Gaussian States are:
- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Quadratures, Homodyne Detection and Squeezing

- Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1a_1} S_{a_2a_2} - | S_{a_1a_2} |^2 \geq 1 \]

**Minimum Uncertainty**
Gaussian States are:
- vacuum state
- coherent states
- squeezed vacuua
- squeezed states
Heisenberg Uncertainty In the Frequency Domain

\[ S_{a_1a_1} S_{a_2a_2} - |S_{a_1a_2}|^2 \geq 1 \]

Minimum Uncertainty
Gaussian States are:
- vacuum state
- coherent states
- squeezed vacuua
- squeezed states

Monday, June 25, 12
• Squeezing **phase noise** will lower shot noise, but increase radiation-pressure noise (good for first-generation detectors, but doesn’t help beating the SQL)

• Squeezing a combination of input **amplitude** and **phase** will help, but only narrow band

• Squeezing a **frequency-dependent combination** will help beat the SQL broadband.

• Detecting a combination of output amplitude and phase may even completely remove back-action noise
Surpassing the SQL in a Michelson interferometer

frequency dependent input squeezed state

frequency dependent homodyne detection

[Kimble et al., 2001]
Frequency Dependent Squeezing & Detection

\[
\sqrt{S_h(f)} \ (1/\sqrt{\text{Hz}})
\]

\[
\begin{align*}
&10^{-20} \\
&10^{-21} \\
&10^{-22} \\
&10^{-23} \\
&10^{-24} \\
&10^{-25}
\end{align*}
\]

\[
\begin{align*}
&1 \\
&10 \\
&100 \\
&1000 \\
&10^4
\end{align*}
\]

\[
f (\text{Hz})
\]

Standard Quantum Limit
Frequency Dependent Squeezing & Detection

- Standard Quantum Limit
- Lossless, with filters
- 10 dB input squeezing
Frequency Dependent Squeezing & Detection

The graph depicts the relationship between frequency ($f$) and the standard deviation of the squeezed signal ($\sqrt{\langle (1/\sqrt{Hz})^2 \rangle}$). The graph shows three curves:

1. **Standard Quantum Limit**: A dashed blue line, representing the theoretical limit of squeezing.
2. **1% total loss**: A solid blue line, indicating the effect of 1% total loss on the squeezing performance.
3. **Input squeezing**: A solid red line, showing the input squeezing level.

The x-axis represents frequency in Hz, ranging from 1 Hz to 10^4 Hz, while the y-axis represents the square root of the standard deviation of the signal in $1/\sqrt{Hz}$. The graph illustrates how the squeezing performance degrades with increasing frequency, particularly under conditions of loss.
Frequency Dependent Squeezing & Detection

\[
\sqrt{S_h(f)} (1/\sqrt{\text{Hz}}) = \left(\text{loss} \cdot \text{squeeze factor}\right)^{1/4}
\]

10 dB input squeezing and 1% loss

Loss Limit = 1/5 SQL

lossless, with filters

1% total loss

Standard Quantum Limit

"Loss Limit"
Quantum Enhancement of Sensitivity Requires Low Loss!

Standard Quantum Limit

Loss Limit = 1/5 SQL

(loss · squeeze factor)\(^{1/4}\)

10dB input squeezing and 1% loss

1% total loss

lossless, with filters
Generation of Squeezed Vacuum

Nonlinear Optics

\[ H_I = \chi E^3 \]

quantize

\[ H = \ldots + a^\dagger_{2\omega_0} a_{\omega_0+\Omega} a_{\omega_0-\Omega} + a_{2\omega_0} a^\dagger_{\omega_0+\Omega} a^\dagger_{\omega_0-\Omega} \]

when non-linear medium pumped with \(2\omega_0\)
and phase-matching condition satisfied

\[ H = \ldots + \int \frac{d\Omega}{2\pi} \left[ A^*_{2\omega_0} a_{\omega_0+\Omega} a_{\omega_0-\Omega} + A_{2\omega_0} a^\dagger_{\omega_0+\Omega} a^\dagger_{\omega_0-\Omega} \right] \]

this term becomes effective and generates squeezing

T=0
cavity resonant with both \(\omega_0\) and \(2\omega_0\)

nonlinear medium

T>0
squeezed vac at \(\omega_0\)
pumped with \(2\omega_0\)
Squeezing for GW Detectors

- First demonstration of squeezing in the GW band (sub kHz) [K. McKenzie et al., 2004]
- Squeezing injection at the Caltech 40 m prototype lab [K. Goda et al., 2008]
- 3.5 dB Squeezing at GEO 600 detector [LSC, 2011; H. Vahlbruch, 2010]
- 2+dB squeezing of LIGO Hanford, achieving best-ever sensitivity to GWs at 200+Hz.
Squeezing Status & Prospects

<table>
<thead>
<tr>
<th></th>
<th>initial LIGO</th>
<th>Advanced LIGO</th>
<th>aim of future LIGO</th>
</tr>
</thead>
<tbody>
<tr>
<td>total loss</td>
<td>55-60%</td>
<td>20%</td>
<td>&lt;2%</td>
</tr>
<tr>
<td>detected</td>
<td>2+dB</td>
<td>6dB</td>
<td>10-15dB</td>
</tr>
</tbody>
</table>

[slide & numbers from Sheila Dwyer, GW Adv Detector Workshop, 2012]
Low-frequency barrier on the earth?

- Suspension Thermal Noise
  - a pendulum’s thermal noise seems a strong limitation
  - other methods are being considered
    - magnetic levitation
    - juggling mirrors?
    - atom interferometers
    - TOBA?

[Dimopoulos, Graham, et al.]

[Ando et al., 2010]
Gravity Gradient Noise

- Seismic motion driving fluctuations in newtonian gravity field

- Can be suppressed by monitoring ground motion and subtracting the predicted effect.
- For LIGO (between 10 Hz and 20 Hz)
  - 5x suppression required to not affect Advanced LIGO
  - 30x suppression required to not affect 3rd generation designs [J. Driggers, 2012]

- Moving detector underground may suppress level and allow better subtraction.
Space-Based GW Detection

- Space-based GW
  - interferometers with long arms (compared with GW wavelength)
  - Laser Interferometer Space Antenna (LISA)
    - *quantum enhancements of a LISA-like mission?*
  - Other space missions
Plane Gravitational Wave

- Coordinates can be chosen such that a plane wave along $z$ direction can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$h_{ij}^{TT}(t, x, y, z) = \begin{bmatrix}
     h_+(t - z) & h_\times(t - z) & 0 \\
     h_\times(t - z) & -h_+(t - z) & 0 \\
     0 & 0 & 0 \\
\end{bmatrix}, \quad i, j = x, y, z$$

This is called the TT gauge because $h$ is transverse, and traceless.

$h_+, \times$ are the two polarizations of the plane GW
Influence of GW on Light and Matter

- Propagation of Light in the “Transverse Traceless” gauge

The scalar wave equation

\[ g^{ab} \nabla_a \nabla_b \Phi = 0 \Leftrightarrow \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \Phi) = 0 \]

\[ \Phi = A \exp(i k_\mu x^\mu + i \delta \phi) = A \exp\left(-i \omega t + i k \cdot x + i \delta \phi\right) \]

Flat-space solution plus additional phase due to GW

\[ k^\mu = (\omega, k) = \omega(1, \hat{k}) \]

4-wavevector \quad ang freq \quad 3-wavevector \quad propagation direction

\[ \delta \phi \text{ slowly varying} \quad k^\mu \partial_\mu \delta \phi = \frac{h^{TT}_{\mu \nu} k^\mu k^\nu}{2} \]

Additional phase accumulates along rays as wave propagates

\[ \delta \phi(t_0 + L, x_0 + \hat{k}L) = \frac{\omega}{2c} \int_0^L \hat{k}^i \hat{k}^j h_{ij}(t_0 + \xi, x_0 + \hat{k} \xi) d\xi \]

Light propagation is modified by GW
Response of Laser Interferometers: TT Gauge

- For larger separation (∼ reduced wavelength): oscillatory nature matters

\[ h_p = H_p e^{-i\Omega(t-z)}, \quad p = +, \times \quad \text{... plane wave with propagation direction } \mathbf{N} \]

\[ \delta \phi(t_0 + L, x_0 + \hat{k}L) = \frac{\omega_0}{c} \frac{LH_p e^{p \hat{k}_i \hat{k}_j}}{2} e^{-i\Omega(t-z)} \]

GW along z

\[ \begin{align*}
\text{same as before} & \quad \text{this favors } \hat{k} \text{ orthogonal to } \mathbf{N} \\
\text{(transverse wave)} & \quad \text{proportional to } L \\
\text{additional phase factor} & \quad \text{due to propagation effect} \\
\text{this favors } \hat{k} \text{ along } \mathbf{N} \\
\end{align*} \]

+ polarized

\[ \pi/8 \]

\[ \pi/4 \]

\[ \pi/2 \]

\[ \delta \phi/(LH_+) \]

\[ \Omega L/(2\pi) \]

suppression of phase shift from simple \( Lh \)
Response of Masses and Building an Interferometer

- **In TT gauge**: low-speed motion of test masses not affected by GW!
- But test masses won’t stay at fixed locations; they will be moving under noisy forces!
- Simplest interferometer
  - A, B, and C freely fall + noisy motion
  - A sends light to B and C
  - B and C reflect light back to A
  - A compares phase between light from B and light from C.
- This gives signal of \((\delta\phi_1 + \delta\phi_2) - (\delta\phi_3 + \delta\phi_4)\)
- Plus local displacement noises (driven by force noise) & shot noise
Arm Length?

- What if frequency is $f = 10\text{mHz}$.
- Reduced wavelength is $\lambda/(2\pi) \sim 5 \times 10^9 \text{m} \sim 5 \times 10^6 \text{km}$
- This is the most optimal arm length to reduce effect of local force noise

Very small amount of light ($\sim 0.2\%$) is received by B

to collect most of the light, the mirror diameter has to be $> (\lambda L)^{1/2} \sim 71 \text{ m}$
or, reduce to $L < D^2/\lambda \sim 250 \text{ km}$
L = 5 \times 10^6 \text{km} = 5 \times 10^9 \text{m}

Equilateral Triangle, tilted at 60 degrees
LISA’s Time-Delay Interferometry (TDI)

- LIGO-like interferometry does not work for LISA, because
  - light is too weak
  - arm lengths are not equal enough
- Armstrong, Estabrook & Tinto’s **Time-Delay Interferometry**
  - light not bounced back by mirrors, but detected
  - interferometry signal synthesized, with length difference accounted for

**naive view: test masses compare each other's clock by sending & receiving light pulses**

6 links between the 3 spacecraft, each with 1 clock
6 channels - 3 clock noises = 3 noise-free channels

- \( t_{21}(L_{12}) + t_{12}(2L_{12}) \): cancels noise of 1
- \( t_{23}(L_{23}) + t_{32}(2L_{23}) \): cancels noise of 3

subtracting the two doesn’t cancel clock noise of 2!!
but we can complete the loop!!
The Real Time-Delay Interferometry

- Two Lasers & Two Test masses on board each spacecraft
  - 6 additional links
  - 3 additional channels of laser noise
  - 3 additional test-mass degrees of freedom
  - these are arranged to also cancel

Tinto, Estabrook & Armstrong (2002)
Figure 3-1: LISA Sensitivity Curve. The strain amplitude spectral density of the Instrument Sensitivity Model is plotted. The measurement bandwidth extends from 0.03 mHz to 100 mHz.

3.1.1 Instrument Noise Model

The single link equivalent position uncertainty is expressed as an amplitude spectral density whose power spectral density is the sum of two terms – the displacement noise of the Interferometry Measurement System (IMS), and the acceleration noise of the Disturbance Reduction System (DRS), which is responsible for minimizing the residual acceleration of the proof masses:

\[ S_{\text{single link}}(f) = S_{\text{IMS}}(f) + S_{\text{DRS}}(f) \]

The displacement noise amplitude spectral density \( S_{\text{IMS}}(f) \) for the uncertainty in the interferometry measurement system is given by:

\[ S_{\text{IMS}}(f) = X_0 \left( \frac{10}{1 + f_0 f} \right) \]

with \( X_0 = 18 \), \( f_0 = 0.002 \) Hz.
Squeezing?

- Signal mode: very wide Gaussian cut by B’s aperture, **flat-top mode**
- Local oscillator at B must match this mode (mixing in any other mode will only lose)
- Can we squeeze this mode (or approximately this mode)?
  - let’s propagate it backwards ...
  - it’s not possible to squeeze this mode, unless we have larger apertures!!

*Being limited by Aperture Size & Acceleration Noise, LISA cannot be improved quantum mechanically ...*
attempts have been made in recent decades to collect such data (Ulysses, Mars Observer, Galileo, Mars Global Surveyor, Cassini) with broadband frequency sensitivities reaching $10^{-16}$ (see [85] for a thorough review of gravitational-wave searches using Doppler tracking). There are currently no plans for dedicated experiments using this technique; however, incorporating Doppler tracking into another planetary mission would provide a complimentary precursor mission before dedicated experiments such as LISA are launched.

The technique of Doppler tracking to search for gravitational-wave signals can also be performed using pulsar-timing experiments. Millisecond pulsars [219] are known to be very precise clocks, which allows the effects of a passing gravitational wave to be observed through the modulation in the time of arrival of pulses from the pulsar. Many noise sources exist and, for this reason, it is necessary to monitor a large array of pulsars over a long observation time. Further details on the techniques used and upper limits that have been set with pulsar timing experiments can be found from groups such as the European Pulsar Timing Array [187], the North American Nanohertz Observatory for Gravitational Waves [190, 191], and the Parkes Pulsar Timing Array [179].

All the above detection methods cover over 13 orders of magnitude in frequency (see Figure 1). This broadband coverage allows us to probe a wide range of potential sources.

Figure 1: The sensitivity of various gravitational-wave detection techniques across 13 orders of magnitude in frequency. At the low frequency end the sensitivity curves for pulsar timing arrays (based on current observations and future observations with the Square Kilometre Array [108]) are extrapolated from Figure 4 in [325]. In the mid-range LISA, DECIGO and BBO are described in more detail in Section 7, with the DECIGO and BBO sensitivity curves taken from models given in [323]. At the high frequency the sensitivities are represented by three generations of laser interferometers: LIGO, Advanced LIGO and the Einstein Telescope (see Sections 6, 6.3.1 and 6.3.2). Also included is a representative sensitivity for the AURIGA [88], Allegro [226] and NAUTILUS [239] bar detectors.
Mirror

FP cavity

Laser

Photo detector

Beam splitter

Drag-free spacecraft

It should be emphasized that the frequency band of DECIGO, 0.1–10 Hz, is appropriate to reach a very high sensitivity, since the confusion limiting noise caused by irresolvable gravitational wave signals from many compact binaries in our galaxy is expected to be very low above 0.1 Hz. Note also that this frequency band is between that of LISA and ground-based detectors. Thus DECIGO will be able to play a follow-up role for LISA by observing inspiral sources that have moved above the LISA band, as well as a predictor for ground-based detectors by observing inspiral sources that have not yet moved into the ground-based detector band.

3. Pre-conceptual design

The pre-conceptual design of DECIGO is the following. DECIGO consists of four clusters of spacecraft; each cluster employs three drag-free spacecraft containing freely-falling mirrors as shown in figure 1. A change in the distance between the mirrors caused by gravitational waves is measured by three pairs of differential Fabry–Perot (FP) Michelson interferometers. The distance between the spacecraft is 1000 km, the diameter of each mirror is 1 m and the wavelength of the laser is 0.5 µm. This ensures a finesse of 10 in the FP cavities, which is determined by the diffraction loss of the laser power in the cavity. The mass of each mirror is 100 kg and the laser power is 10 W. DECIGO will be delivered into heliocentric orbits with two clusters nearly at the same position and the other two at separate positions.

We chose the FP configuration rather than the light transponder configuration because the FP configuration could provide a better shot-noise-limited sensitivity than the transponder configuration, since gravitational wave signals can be enhanced by the FP cavity. Note that the FP configuration requires a relatively short arm length to avoid the optical loss of the diverging laser light; this makes the requirement of the acceleration noise considerably stringent.

The implementation of the FP cavity using the drag-free spacecraft is feasible. Each spacecraft follows the motion of the mirror inside each spacecraft as a result of the function of the drag-free system. Each mirror is, on the other hand, controlled in position in such a way...
Summary

• Laser Interferometry can be used to detect gravitational waves.

• Squeezing already improves sensitivity of ground-based interferometry.

• Space-based GW detection goes after low-frequency sources