Excitation of Internal Gravity Wave (IGW)

Stable Layer (buoyancy) in the atmosphere, \( N > 0 \)

- Earth \( N \approx 3 \) mHz
- Jupiter (troposphere) \( N \approx 5 \) to \( 15 \) mHz

Supports Internal Gravity Waves (IGW)

larger \( N \rightarrow \) faster wave speed, \( U \)

Model  axisymmetric \((r, z)\), long wave

Excitation  line source \( E=10^{20} \) J

Vary \( N \) (deviation of temperature profile from dry adiabat)

\( \rightarrow U? , \) Amplitude of \( \Delta T? \)
Temperature Profile in Jupiter’s Atmosphere

Voyager Data

water cloud base
Ingersoll and Kanamori (1995)
Independent variables (isobaric coordinate) \( p, r, t \)
Dependent variable  geo-potential \( \Phi(p, r, t) = gz = \phi(p) \Gamma(r,t) \)

\[
T' = -\frac{p}{R} \frac{\partial \Phi}{\partial p}
\]

Hydrostatic condition assumed. \( T \rightarrow \sigma \)
Source \( \dot{Q}(p, r, t) = \eta(p) \delta(r) \delta(t) \) \( \eta(p) \): vertical heating profile

\[
\frac{\partial}{\partial t} \left[ \left( \frac{\partial^2 \Gamma}{\partial t^2} + f^2 \Gamma \right) \frac{d}{dp} \left( \frac{1}{\sigma} \frac{d \phi}{dp} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Gamma}{\partial r} \right) \phi \right]
= \left( \frac{\partial^2 \delta(t)}{\partial t^2} + f^2 \delta(t) \right) \frac{d}{dp} \left( \frac{\kappa \eta(p)}{\sigma p} \right) \delta(r)
\]

This structure suggests vertical normal modes

\[
\frac{d}{dp} \left( \frac{1}{\sigma} \frac{d \phi}{dp} \right) = - \frac{1}{c_i^2} \phi(p) \quad \text{with} \quad c_i \quad \text{and} \quad \phi_i(p) \quad \text{eigen values and eigen functions (discretized)}
\]

\[
\Phi(p, r, t) = \sum_{i=0}^{\infty} \frac{a_i c_i}{2\pi} \phi_i(p) \frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{c_i^2 t^2 - r^2}} \right]
\]

\( f=0 \) assumed above, but the method can be extended numerically to the case \( f \neq 0 \).
Heating source, Total energy, $10^{20}$ J
Schematic figure showing IGW (Poincare) wave excited by an impulsive source

\[ \Phi(p, r, t) \equiv \sum_i \phi_i(p) E(c_i) \frac{\partial G_i}{\partial t} \]

\( c_i \): horizontal wave speed
\( E(c_i) \): Excitation given by \( \eta(p) \)
Lagrangian temperature perturbation profile at 2 hours for 3 heating profiles (at 45 mb level)

Non-dispersive wave, slower overtone than the fundamental mode

Ingersoll and Kanamori (1995)
Snap Shot of Internal Gravity Waves in Jovian Atmosphere

Outer ring: fundamental mode
Inner rings: overtones

at t=5400 sec
Crest of a jovian wave
Quick shifts of the geomagnetic field
Nitrate trapping by a bacterial mat
Molecular defect in reeler mice
Time-Distance Curve (Travel-Time Curve)

Radius (km)

Time after impact (s)

- Acoustic (v=720 m/s)
- v=450 m/s (10x solar)
- v=210 m/s (solar)
- v=130 m/s

Hammel et al., 1995
Internal Gravity Waves in Jovian Atmosphere

10000 km
Acoustic wave (at 1.5 hours, at 126 mb) \[ V=750 \text{ m/s} \]
End