Intro to Soil Mechanics: the what, why & how

José E. Andrade, Caltech
The What?
What is Soil Mechanics?

*erdbaumechanik*

The application of the laws of mechanics (physics) to soils as engineering materials

Karl von Terzaghi is credited as the father of *erdbaumechanik*
sands & gravels

clays & silts
The Why?
Sandcastles what holds them up?
Palacio de Bellas Artes
Mexico, DF

uniform settlement
The leaning tower of Pisa

differential settlement
Teton dam

dam failure

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Katrina
New Orleans
levee failure
MER: Big Opportunity

xTerramechanics

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The How?
Topics in classic Soil Mechanics

- Index & gradation
- Soil classification
- Compaction
- Permeability, seepage, and effective stresses
- Consolidation and rate of consolidation
- Strength of soils: sands and clays
**Index & gradation**

**Definition**: soil mass is a collection of particles and voids in between (voids can be filled w/ fluids or air)

- **solid particle**
- **fluid (water)**
- **gas (air)**

Each phase has volume and mass

Mechanical behavior governed by phase interaction
Index & gradation

Key volumetric ratios

\[ e = \frac{V_v}{V_s} \]  void ratio

[0.4,1] sand
[0.3,1.5] clays

\[ \eta = \frac{V_v}{V_t} \]  porosity

[0,1]

\[ S = \frac{V_w}{V_v} \]  saturation

[0,1]

Key mass ratio

\[ w = \frac{M_w}{M_s} \]  water content

<1 for most soils
>5 for marine, organic

Key link mass & volume

\[ \rho = \frac{M}{V} \]  moist, solid, water, dry, etc.

ratios used in practice to characterize soils & properties
Gradation & classification

Grain size is main classification feature

- **Sands & gravels**
  - can see grains
  - mechanics~texture
  - \( d > 0.05 \text{ mm} \)

- **Clays & silts**
  - cannot see grains
  - mechanics~water
  - \( d < 0.05 \text{ mm} \)

Soils are currently classified using USCS (Casagrande)
Fabric in coarsely-grained soils

“loose packing”, high $e$
“dense packing”, low $e$

\[ e = \frac{V_v}{V_s} \]

$e_{\text{max}}$ greatest possible, loosest packing
$e_{\text{min}}$ lowest possible, densest packing

\[ I_D = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \]

relative density

strongly affects engineering behavior of soils

(a) Loose
(b) Dense
Typical problem(s) in Soil Mechanics

- Compact sand fill
- Calculate consolidation of clay
- Calculate rate of consolidation
- Determine strength of sand
- Calculate F.S. on sand (failure?)
- Need: stresses & matl behavior
Modeling tools
Theoretical framework

- continuum mechanics
- constitutive theory
- computational inelasticity
- nonlinear finite elements
Theoretical framework

- continuum mechanics
- constitutive theory
- computational inelasticity
- nonlinear finite elements

\[
\begin{align*}
\phi \frac{\dot{p}}{K_f} + \nabla \cdot \mathbf{v} &= -\nabla \cdot \mathbf{q} \\
\nabla \cdot \mathbf{\sigma} + \gamma &= 0
\end{align*}
\]
Theoretical framework

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\[ q = k \cdot \nabla h \quad \text{darcy} \]
\[ \dot{\sigma}' = c^{\text{ep}} : \dot{\epsilon} \quad \text{hooke} \]
\[ k \quad \text{permeability tensor} \]
\[ \text{controls fluid flow} \]
\[ c^{\text{ep}} \quad \text{mechanical stiffness} \]
\[ \text{controls deformation} \]
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Displacement node
Pressure node
Finite Element Method (FEM)

- Designed to approximately solve PDE’s
- PDE’s model physical phenomena

- Three types of PDE’s:
  - Parabolic: fluid flow
  - Hyperbolic: wave eqn
  - Elliptic: elastostatics
FEM recipe

Strong from

Weak form

Galerkin form

Matrix form
Multi-D deformation with FEM

\[ \nabla \cdot \sigma + f = 0 \quad \text{in} \ \Omega \quad \text{equilibrium} \]
\[ u = g \quad \text{on} \ \Gamma_g \quad \text{e.g., clamp} \]
\[ \sigma \cdot n = h \quad \text{on} \ \Gamma_h \quad \text{e.g., confinement} \]

Constitutive relation:

given \( u \) \rightarrow get \( \sigma \)

e.g., elasticity, plasticity
1. Set geometry
2. Discretize domain
3. Set material parameters
4. Set B.C.'s
5. Solve
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Modeling Ingredients

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TIME STEP LOOP

ITERATION LOOP

ASSEMBLE FORCE VECTOR AND STIFFNESS MATRIX

ELEMENT LOOP: N=1, NUMEL

GAUSS INTEGRATION LOOP: L=1, NINT

CALL MATERIAL SUBROUTINE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

T = T + ΔT

constitutive model
Material behavior: shear strength

- Void ratio or relative density
- Particle shape & size
- Grain size distribution
- Particle surface roughness
- Water
- Intermediate principal stress
- Overconsolidation or pre-stress

Engineers have developed models to account for most of these variables

Elasto-plasticity framework of choice
A word on current characterization methods

Direct Shear

Pros: cheap, simple, fast, good for sands
Cons: drained, forced failure, non-homogeneous

Triaxial

Pros: control drainage & stress path, principal dir. cnst., more homogeneous
Cons: complex
Material models for sands should capture:

- Nonlinearity and irrecoverable deformations
- Pressure dependence
- Difference tensile and compressive strength
- Relative density dependence
- Nonassociative plastic flow
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Elasto-plasticity in one slide

Hooke’s law \( \dot{\sigma} = C^{ep} : \dot{\varepsilon} \)

Additive decomposition of strain \( \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \)

Convex elastic region \( F(\sigma, \alpha) = 0 \)

Non-associative flow \( \dot{\varepsilon}^p = \dot{\lambda} g, \quad g := \partial G / \partial \sigma \)

K-T optimality \( \dot{\lambda} F = 0 \quad \chi H = -\partial F / \partial \alpha \cdot \dot{\alpha} \)

Elastoplastic constitutive tangent

\[
C^{ep} = C^e - \frac{1}{\chi} C^e : g \otimes f : C^e, \quad \chi = H - g : C^e : f
\]
Examples
Example of elasto-plastic model

\[
F = F(\sigma', \pi_i) \\
G = G(\sigma', \bar{\pi}_i) \\
H = H(p', \pi_i, \psi)
\]
model validation: drained txc and ps

Thursday, June 23, 2011
undrained txc loose sands
true triaxial $b=\text{constant}$
Plane-strain liquefaction numerical simulation
Plane-strain liquefaction numerical simulation
(a) Pore Pressure (in kPa)

(b) Deviatoric Strain

$H - H_L$

Field scale prediction
Levee failure
(recall Katrina)
References

AN INTRODUCTION TO
Geotechnical Engineering

Robert D. Holtz
William D. Kovacs

Soil Behaviour
and Critical State
Soil Mechanics

DAVID MUIR-WOOD