Fundamental Sensitivity Limits for Coherent and Direct Detection

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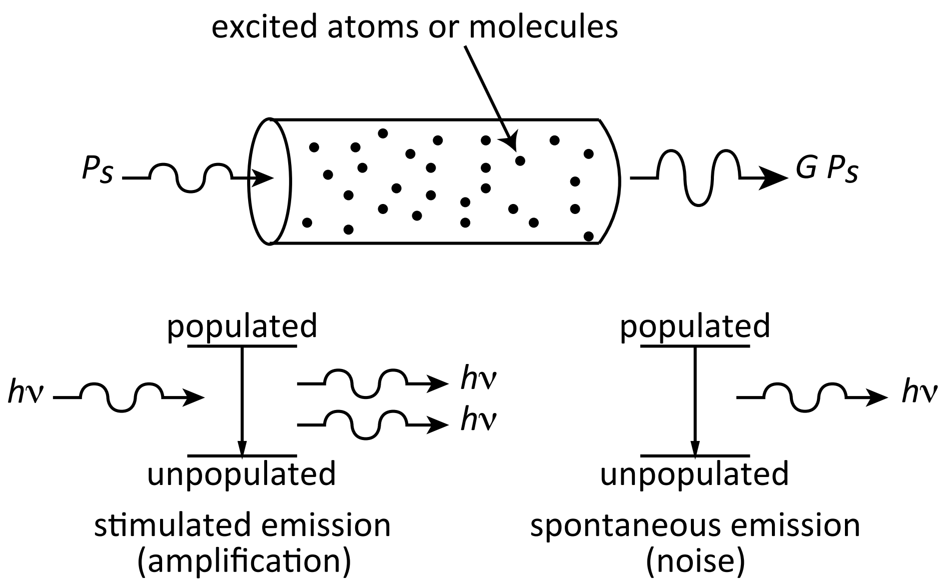
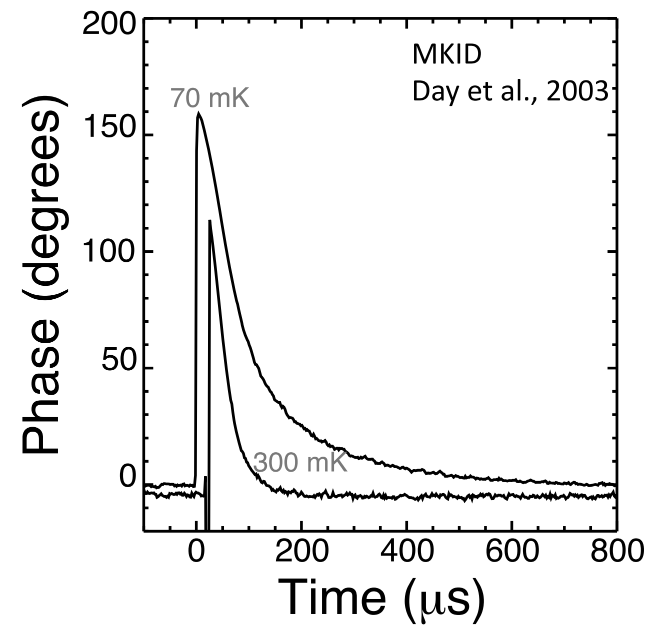
Caltech

Coherent vs. Direct Detection





- Coherent amplification using ideal maser/laser
- Gain and noise are optimized when energy level populations are perfectly inverted
- Nonzero output even for zero input
  - spontaneous emission is random
  - perfect photon counting is not possible
  - spontaneous emission = "*quantum noise*"

- Pulse represents direct detection of a single X-ray photon
- High pulse SNR means zero photon counting error
- No pulses = no photons
- Perfect photon counting *is* possible

Fundamental distinction

- Emission rate is proportional to number of photons in *final* state:

$$\Gamma_{n\rightarrow n+1}^{\text{emission}} \propto \left|\langle n+1|\, a^{\dagger}\, |n\rangle\right|^{2} = n+1$$

- Absorption rate is proportional to number of photons in *initial* state:

$$\Gamma_{n\rightarrow n-1}^{\text{absorption}} \propto \left|\langle n-1|\, a\, |n\rangle\right|^{2} = n$$

- See Feynman Lectures, vol. III, chapter 4

Quantum Noise

- Spontaneous emission = quantum noise

Quantum noise !

$$\Gamma_{n \rightarrow n+1}^{\text{emission}} \propto \left| \langle n+1 | \, a^{\dagger} \, | n \rangle \right|^2 = n + 1$$

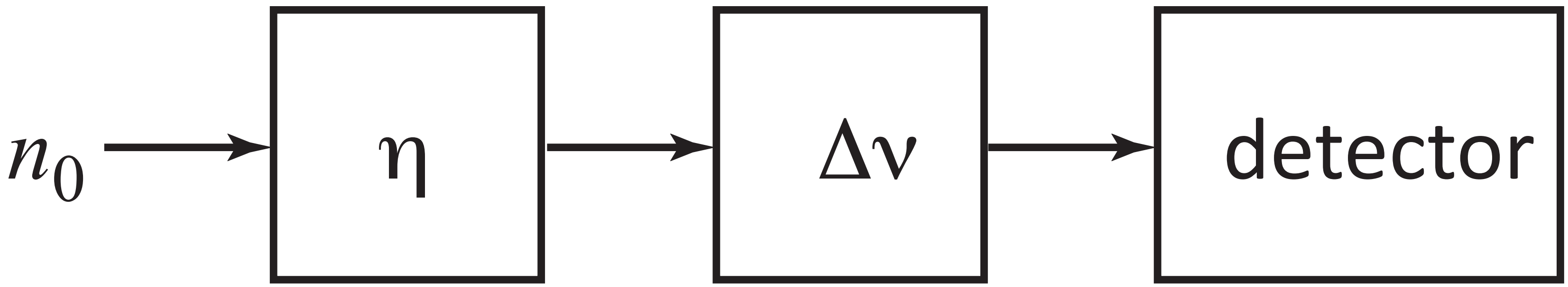
- Importance of quantum noise depends on n
  - $n << 1$ significant limiting factor
  - $n >> 1$ quantum noise is not important
- For blackbody radiation (see Feynman III.4):

$$n = \frac{1}{e^{h\nu/kT} - 1}$$

$h\nu >> kT \rightarrow n << 1$ Wien limit

$h\nu << kT \rightarrow n >> 1$ Rayleigh-Jeans limit

Photon statistics & photon bunching



- Single-mode instrument
- Cold input attenuator η
- Lossless bandpass filter $\Delta\nu$
- Ideal photon-counting detector
- n_0 is the photon occupation number at input (photons $\mathrm{Hz}^{-1}\,\mathrm{s}^{-1}$)

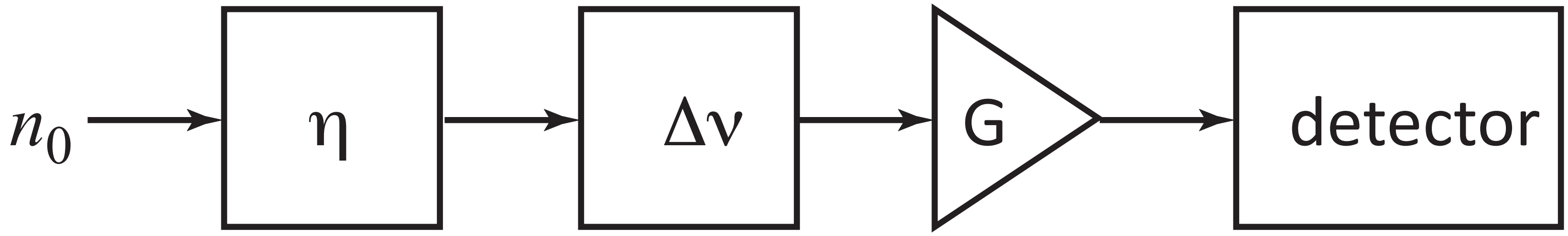
The 1 σ power sensitivity after integration time τ is:

$$\sigma_P = \frac{h\nu}{\sqrt{\Delta\nu\,\tau}}\sqrt{\frac{n_0(1+\eta n_0)}{\eta}}\,\Delta\nu \qquad \text{(second term due to bunching)}$$

$$\approx \frac{h\nu}{\eta\tau}\sqrt{\eta n_0 \Delta\nu\tau} = \frac{h\nu}{\eta\tau}\sqrt{N(\tau)} \qquad \text{(Poisson statistics; for } \eta n_0 << 1\text{)}$$

$$\approx \frac{k_B T_0 \Delta\nu}{\sqrt{\Delta\nu\,\tau}} = \frac{h\nu}{\eta\tau}\frac{N(\tau)}{\sqrt{\Delta\nu\,\tau}} \qquad \text{(Dicke formula; for } \eta n_0 >> 1\text{)}$$

Amplifiers and quantum noise



- A quantum-limited high gain (G >> 1) amplifier is now inserted before the detector
- Single-mode instrument
- Ideal photon-counting detector
- Attenuator & filter before amp

The 1 σ power sensitivity after integration time τ is:

$$\sigma_P = \sqrt{\frac{\Delta\nu}{\tau}}\,\frac{h\nu}{\eta}\,(\eta n_0 + 1)$$ (second term is quantum noise)

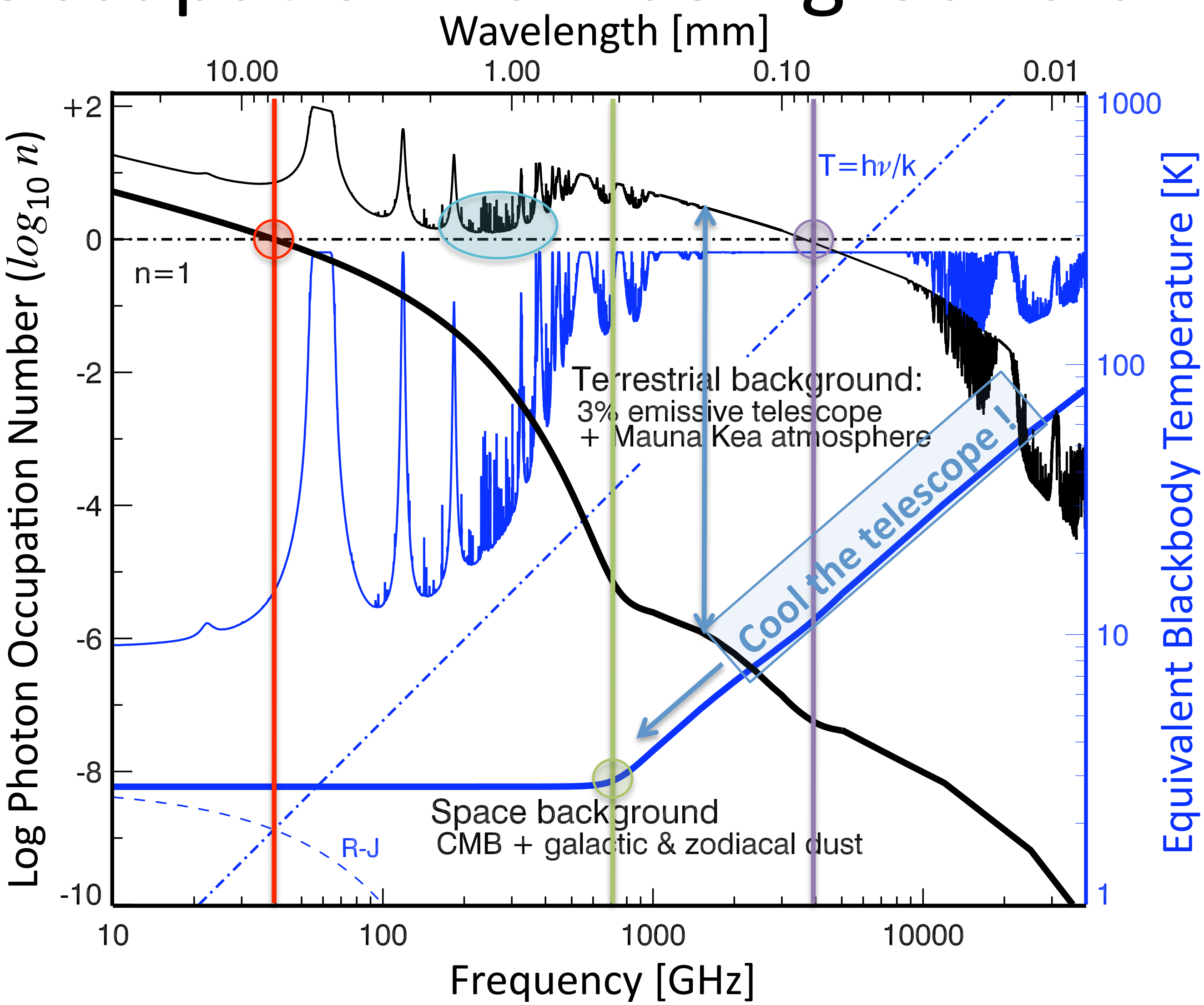
$$\approx \frac{k_B T_0 \Delta\nu}{\sqrt{\Delta\nu\,\tau}}$$ (Dicke formula; for $\eta n_0 >> 1$)

$$\approx \frac{h\nu\Delta\nu}{\eta\sqrt{\Delta\nu\,\tau}}$$ (quantum limit; for $\eta n_0 << 1$)

Direct detection is *more sensitive* by the factor $\sqrt{\eta n_0}$ when $\eta n_0 << 1$.

Thanks to C.M.B. !

Occupation number: ground and space



$n < 1$

CMB, ν > 40 GHz

CMB dominates background from space for ν < 700 GHz

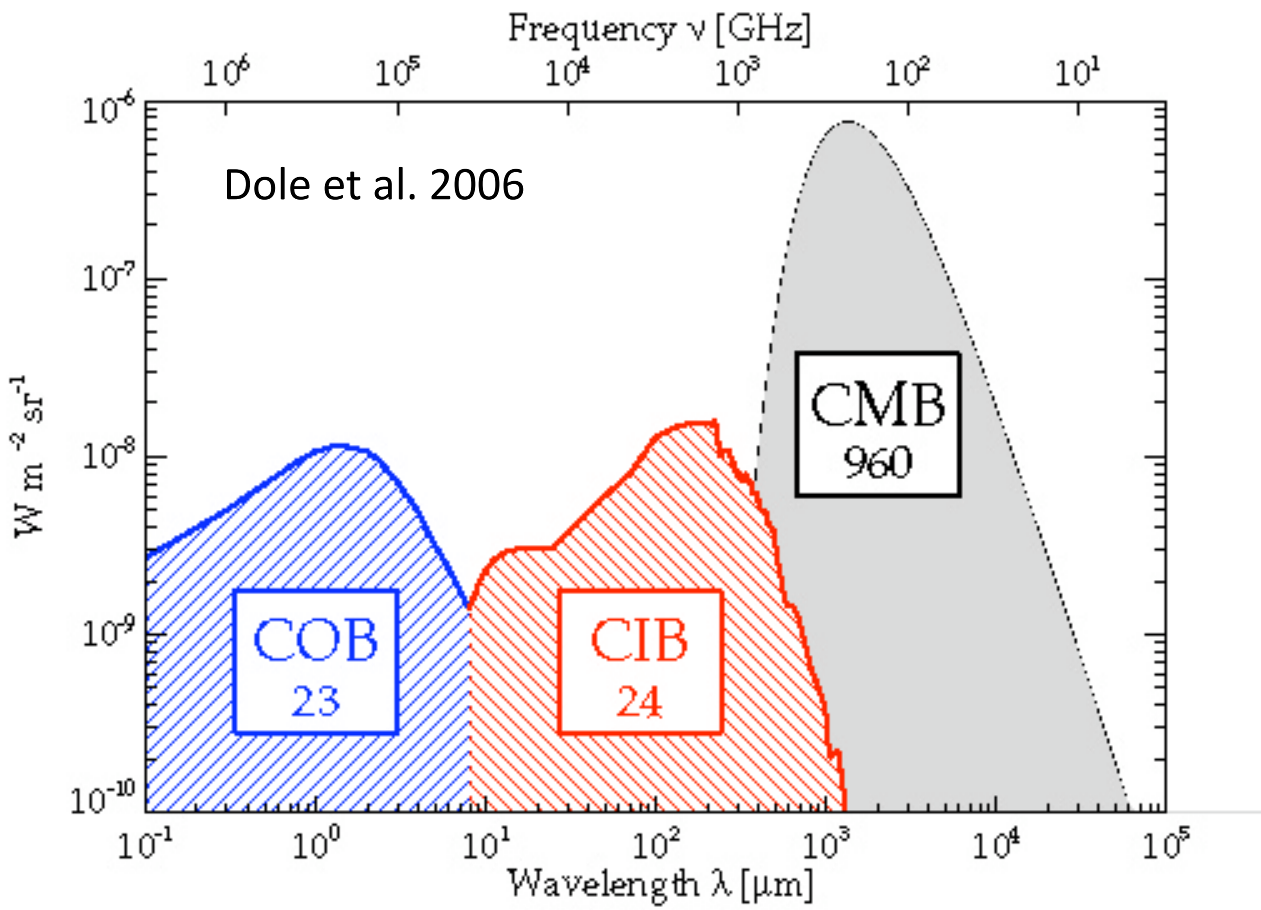
$n < 1$

from ground for ν > 4 THz

$n \sim 1$

from ground for ν ~ 200-300 GHz

Same plot, different units



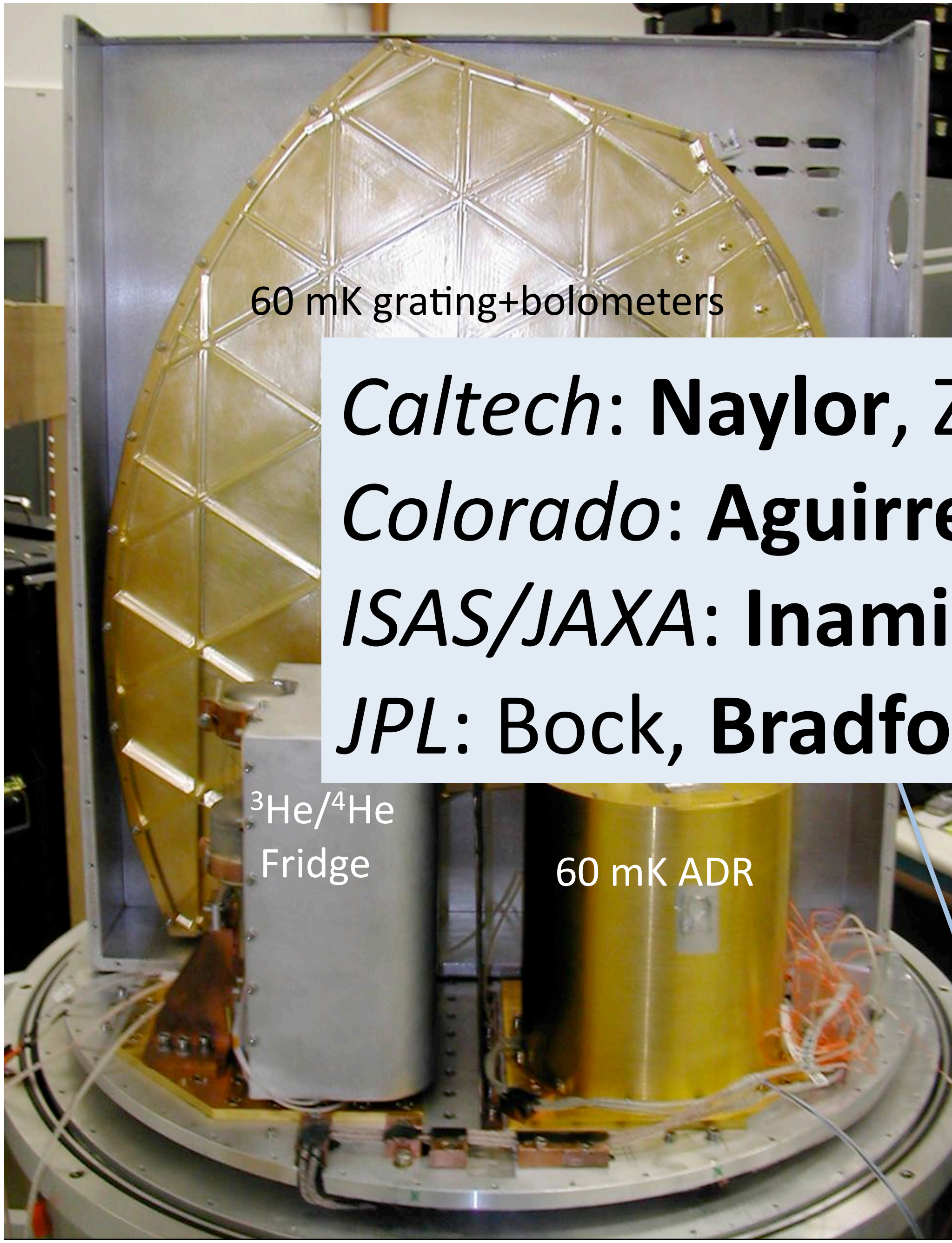
Recap

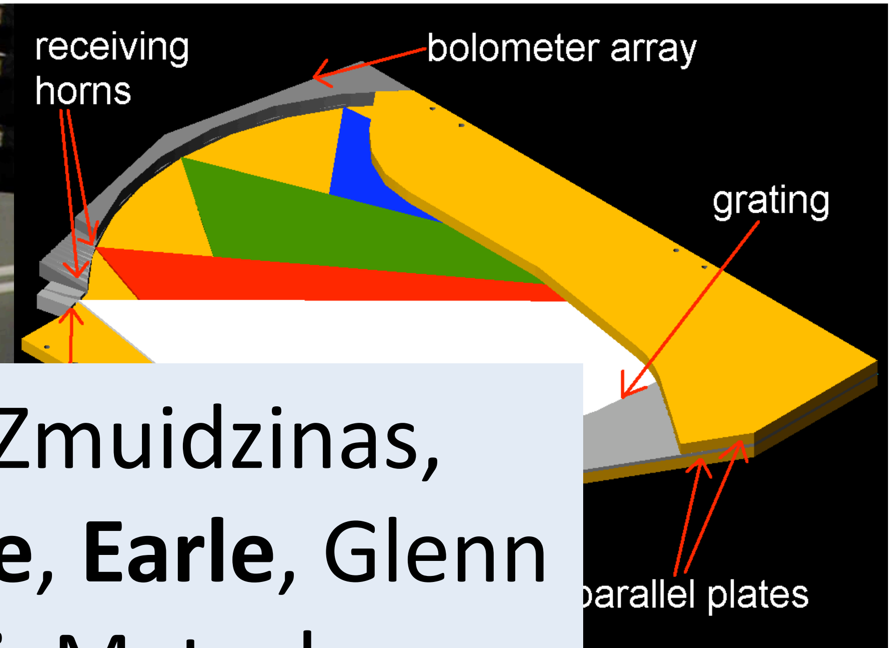
- Relative sensitivity of coherent vs. direct detection is controlled by photon occupation number
- mm/submm band represents the transition from $n >> 1$ (radio) to $n << 1$ (optical)
- Transition occurs at
  - **40 GHz** for space observatories
  - **4 THz** for ground-based observatories
  - Somewhere in between for airplanes & balloons

Spectroscopy at 1 mm: direct or coherent?

- 200-300 GHz band of interest for CO redshifts
- Recall: $n \sim 1$ for $\lambda \sim 1$ mm
- Challenges for direct detection
  - Instrument size !
  - Detector sensitivity, operating temperature
- Challenges for coherent detection
  - Bandwidth (100 GHz ?)
  - Sensitivity (near quantum limit)

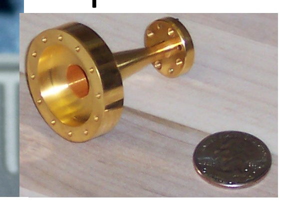
Zspec: a 2-D Waveguide Grating Spectrometer for 200-300 GHz



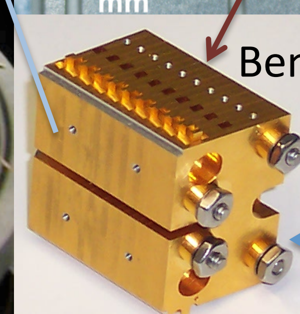


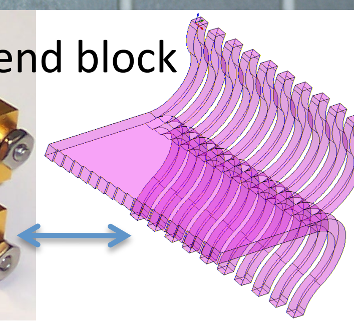
Caltech: **Naylor**, Zmuidzinas,
Colorado: **Aguirre**, **Earle**, Glenn
ISAS/JAXA: **Inami**, Matsuhara
JPL: Bock, **Bradford**, Nguyen

Input horn



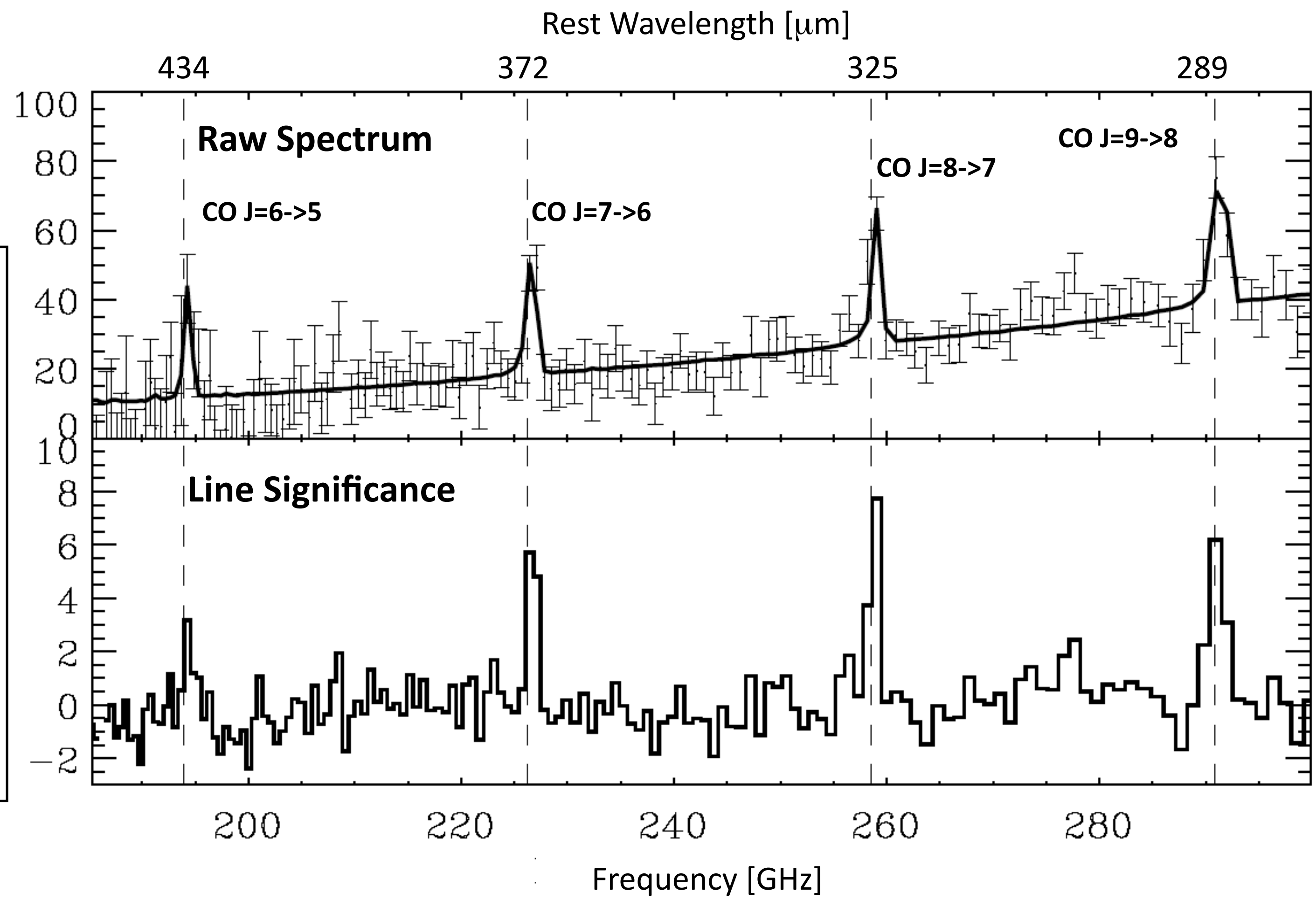
Bend block





Z-Spec high-redshift measurements

Cloverleaf QSO at z=2.55
7.9 hours with Z-Spec at CSO



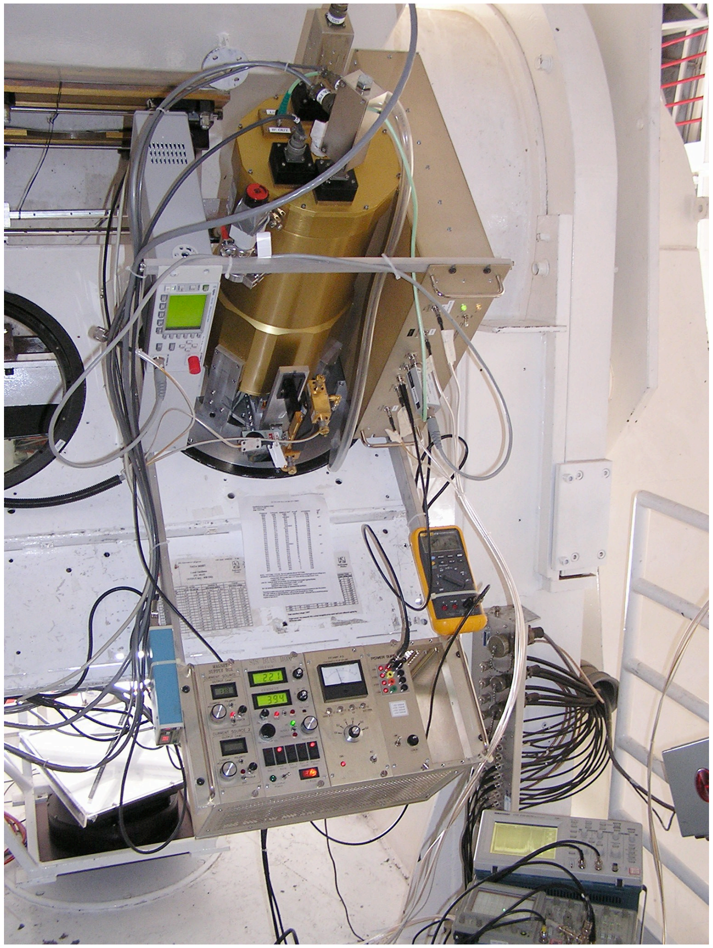
Cloverleaf host galaxy:

- A powerful lensed system, originally detected in submillimeter (redshifted dust) by Barvainis et al. (1992).

* CO 4-3 and 7-6 detected with IRAM 30m and PdB interferometer (same group in 1994).

Z-Spec at CSO:
3 new lines including 2 highest-J transitions!

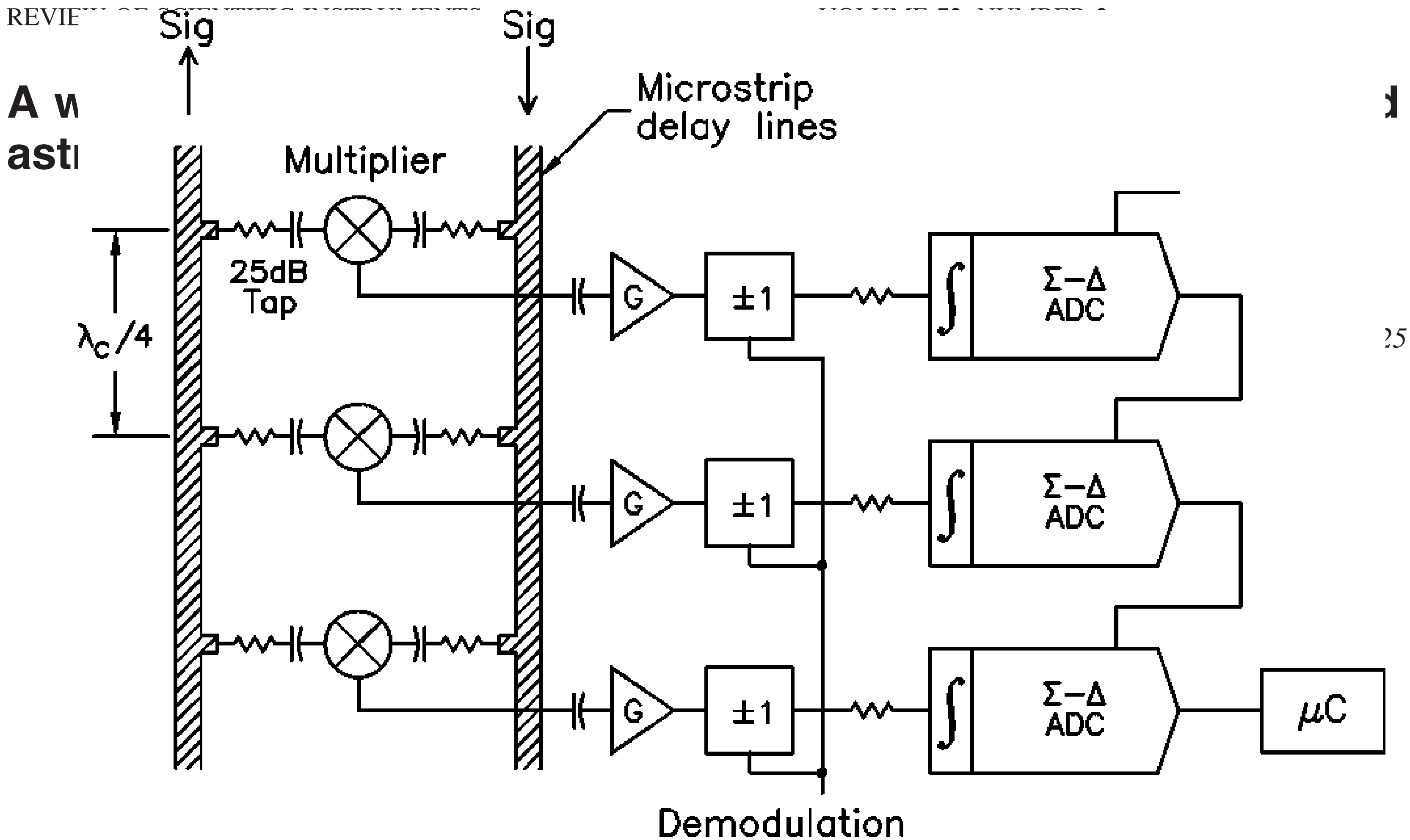
ZRx + WASP: 12 GHz IF bandwidth (DSB)



180-300 GHz SIS Receiver
fixed-tuned mixer, synthesized LO
F. Rice + C. Sumner

WASP II Backend
4 × 3.5 GHz
A. Harris, UMd

WASP: wideband analog correlator



Spectroscopy at 1mm: direct or coherent?

Date: Tue, 10 Aug 1999 11:23:48 -0400 (EDT)
From: "Andrew Harris (301)405-7531" <harris@astro.umd.edu>
To: Jonas Zmuidzinas <jonas@socrates.submm.caltech.edu>
Subject: Redshift Reinhard

Hello Jonas --

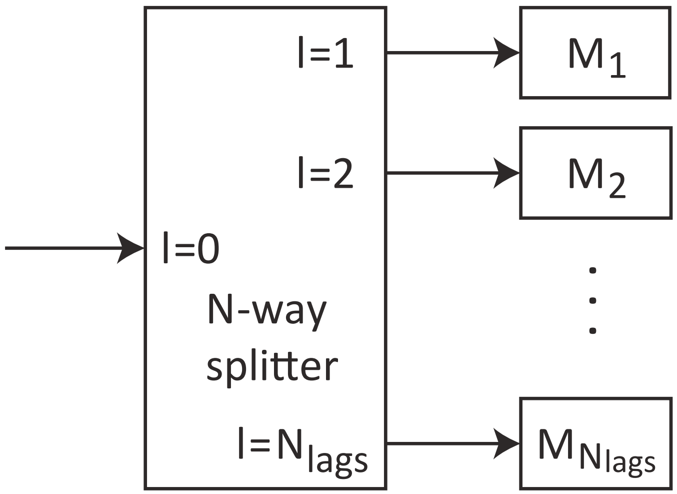
It's like deja vu all over again...

I met up with Reinhard in Berkeley yesterday, and he's gotten very interested in the idea of wideband redshift work on distant galaxies.

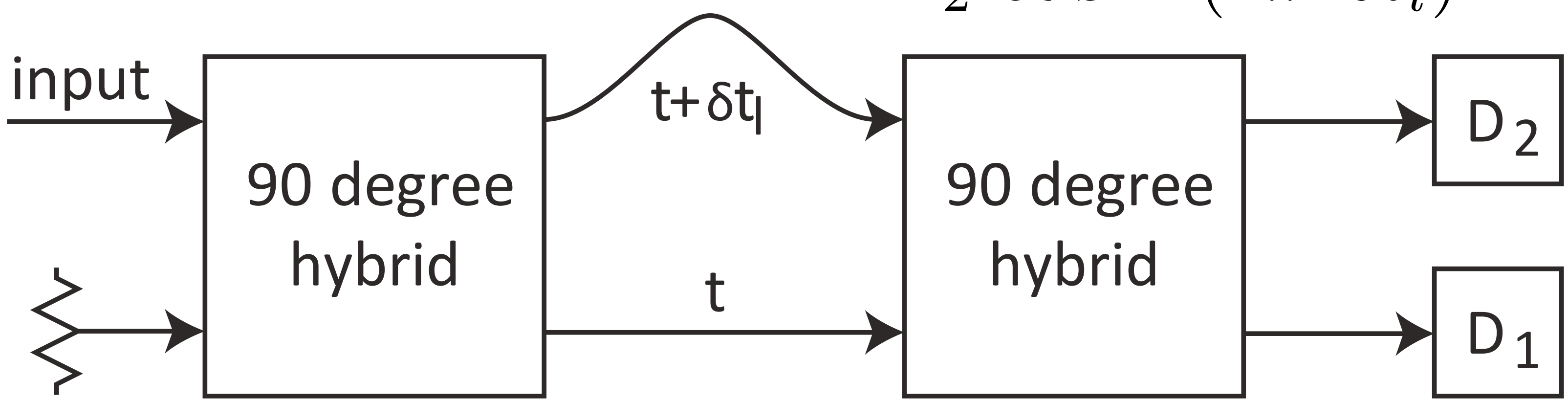
He's had Dieter Lutz and Albrecht Poglitsch looking into the astronomical and instrumental (incoherent) aspects of this, and wondered what I thought of the coherent approach. The 30m is now down to an oversubscription of 1.3 or so, and slowly headed down, so it's quite possible to think of using it for substantial integrations in the future (700 m^2).

He's been doing rather idealized coherent/incoherent comparisons. I told him that you and I have been heading in this direction for a while, that you've been working on wideband front ends and the direct spectrometer as well as the analog correlator stuff. He's very interested in exploring this further if we are. We tried to call you from his temporary office, but couldn't get you, of course. I did give him a copy of your quantum noise paper -- I hope that's ok; as far as I remember it didn't have anything he could steal away, so to speak…

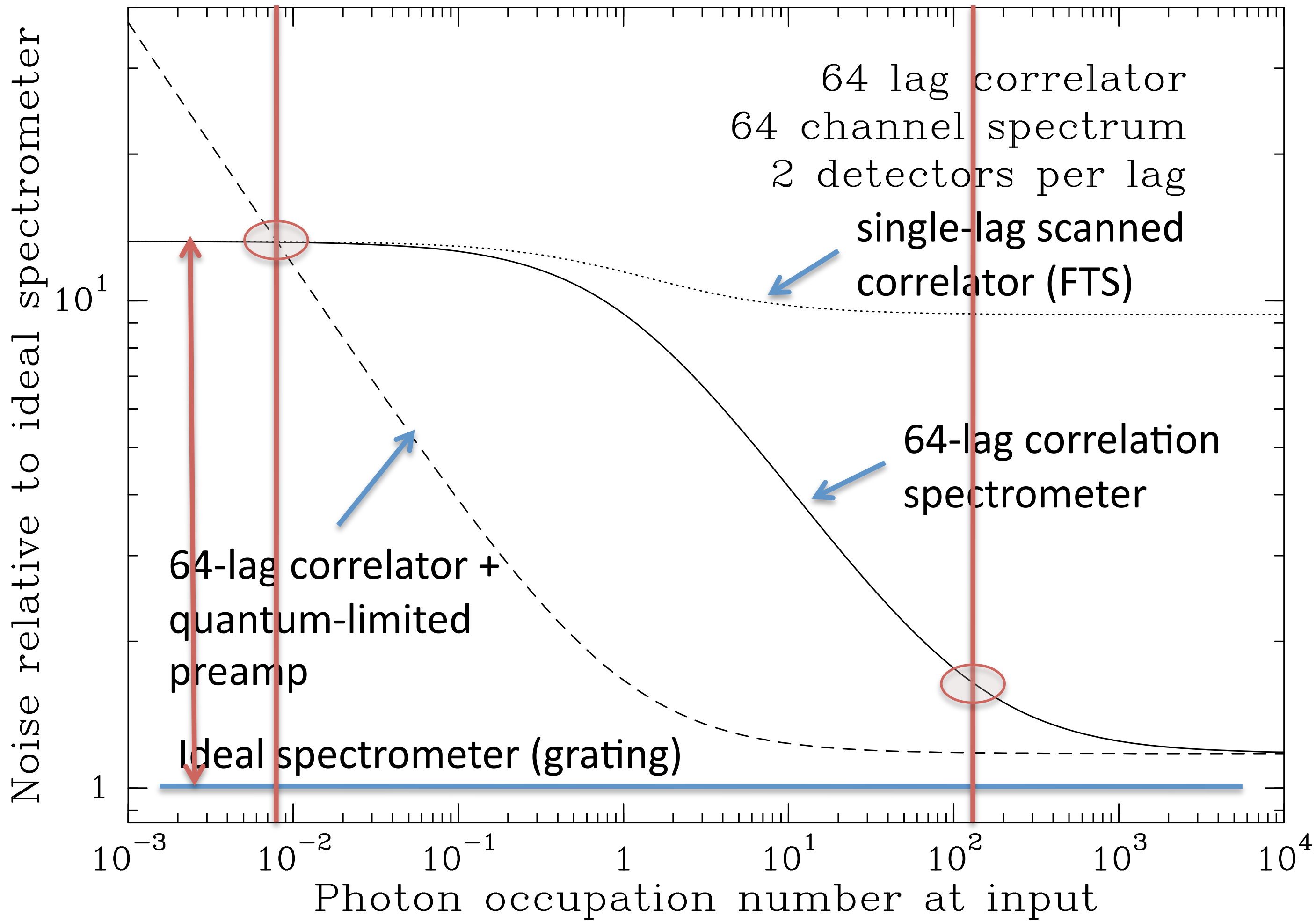
Direct-detection correlation spectrometer?



- Feed all lags simultaneously
- All input photons absorbed
- Two detectors per lag
- $D_1 \propto \cos^2(2\pi\nu\delta t_l)$
- $D_2 \propto \sin^2(2\pi\nu\delta t_l)$



Spectrometer sensitivity

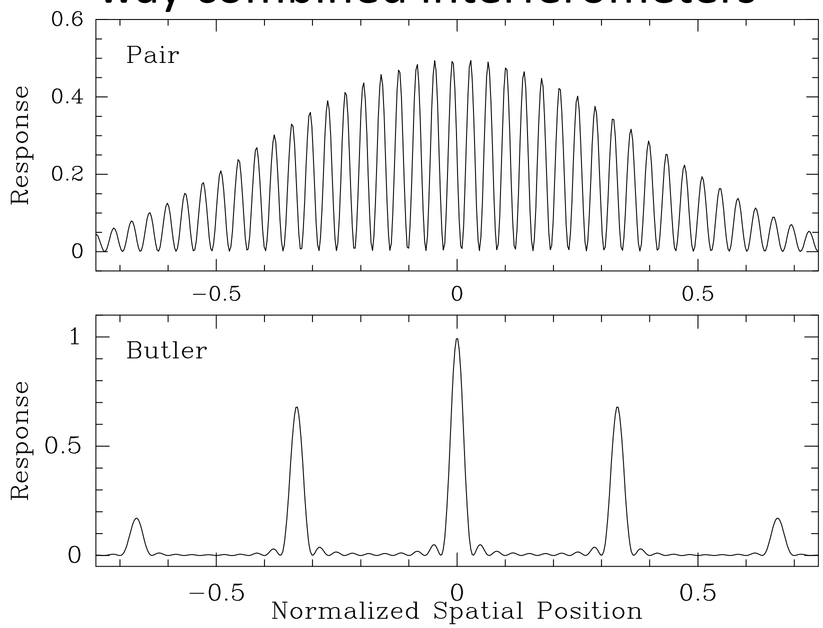


Discussion

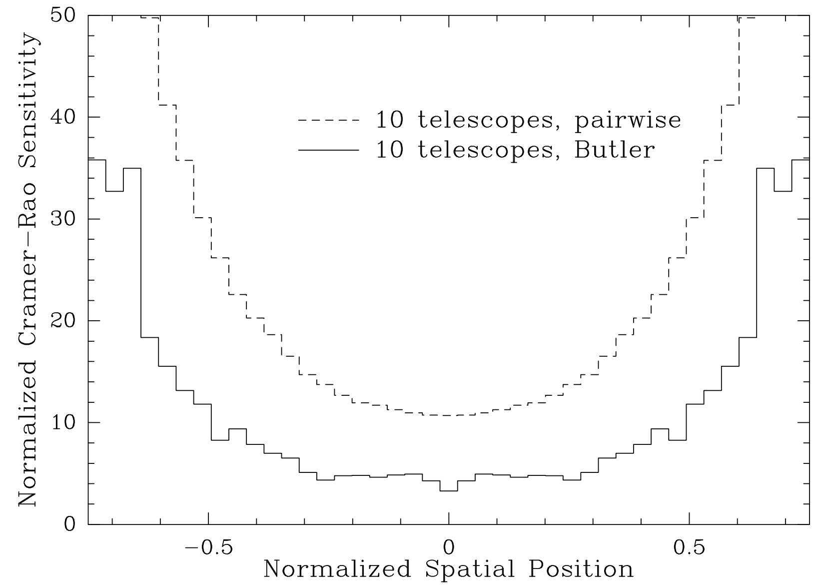
- In principle, a grating spectrometer tells you the wavelength of each detected photon
- A correlation spectrometer does not do this!
- Loss of sensitivity for correlator at low *n* arises from this wavelength ambiguity
- At high *n,* correlator receives photons in *bunches*, not individually
  - A multi-lag correlator can measure the wavelength of the bunch: take Fourier transform of photon counts
  - A single (scanned) lag correlator (FTS) cannot do this

Spatial interferometry: same story…

Instantaneous beam patterns for pairwise-combined and *N*-way combined interferometers



1-d aperture synthesis sensitivity



- *N*-way beam combination gives more compact beam patterns
  - Reduces ambiguity in photon position on sky
- Entirely analogous to correlation spectrometer vs. grating

For more information…

- A rigorous foundation for sensitivity comparisons is available
- Photon noise covariance matrix is the key:

$$\sigma_{ij}^2(N) = \langle \delta N_i \delta N_j \rangle = \tau \int_0^\infty d\nu B_{ij}(\nu)(B_{ji}(\nu) + \delta_{ij})$$

- Basically Hanbury Brown & Twiss
- See:
  - J. Zmuidzinas, J. Opt. Soc. Am. **20**, 218 (2003)
  - J. Zmuidzinas, Appl. Opt. **42**, 4989 (2003)