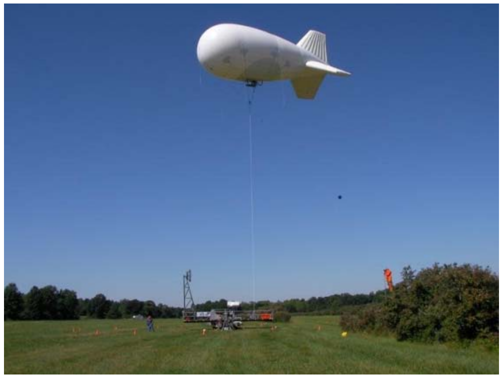
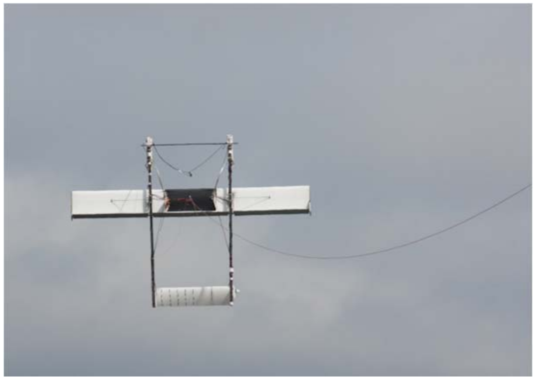
Conceptual Design and Passive Stability of Tethered Platforms

Sara Smoot

Advised by Ilan Kroo

Towed Bodies





Definition: Two or more tethered objects immersed in a moving fluid.

- Aerostats
- Underwater towed vehicles
- Kites
- Towed aircraft

Objectives

Performance Analysis: Conceptual design for tethered body

- Identify important design parameters
- Investigate sensitivities to design parameters
- Determine static stability criteria for bridled wings

Design Methodology: Quickly design a stable efficient system

Key Results:

- Reduced the static design problem to 4 design parameters
- Tether diameter is a stronger design driver than weight
- High effective lift to drag ratios are not advantageous to reach altitude
- Higher wind requires a more costly design

Performance Literature Review

Tether Static Analysis

- H. Glauert (1934)
  - Equations for heavy cable used for aerodynamic towing
  - Family of curves used to describe tether shape
- Thews and Landweber (1936)
  - Equations including tangential loading
- Pode (1955)
  - Faired sections
  - Special cable functions

Equilibrium as a special case in Dynamic Analysis

- J. DeLaurier
  - Finite differencing to calculate tether shape and height of an aerostat

Numerical Simulation

- P. Williams, B. Lansdrop, and W. Ockels.
  - Simulated the tether as rods and masses.

Analytic Static Equations

L_e = lift subtracted by wing weight

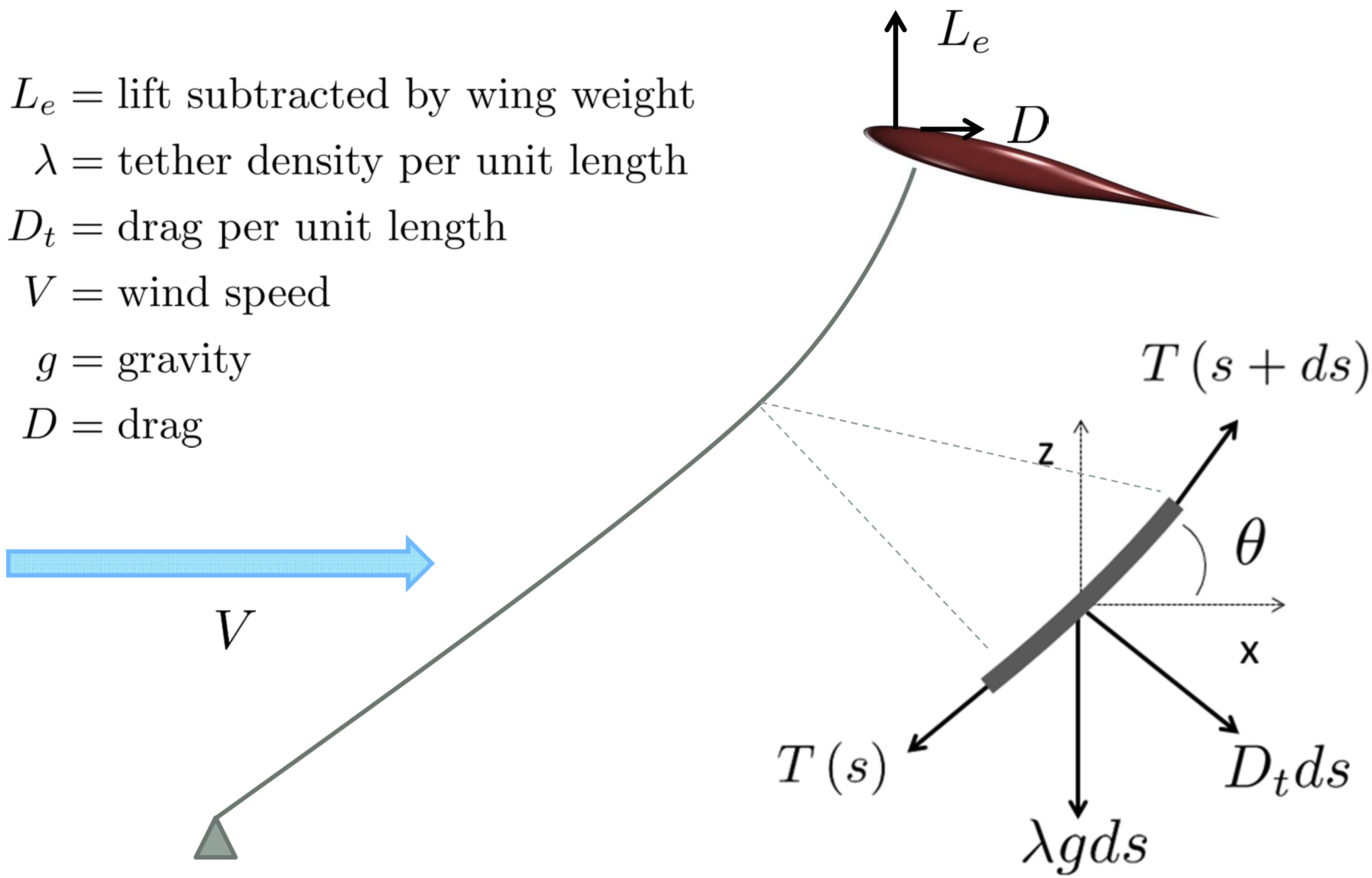
λ = tether density per unit length

D_t = drag per unit length

V = wind speed

g = gravity

D = drag



Analytic Static Equations

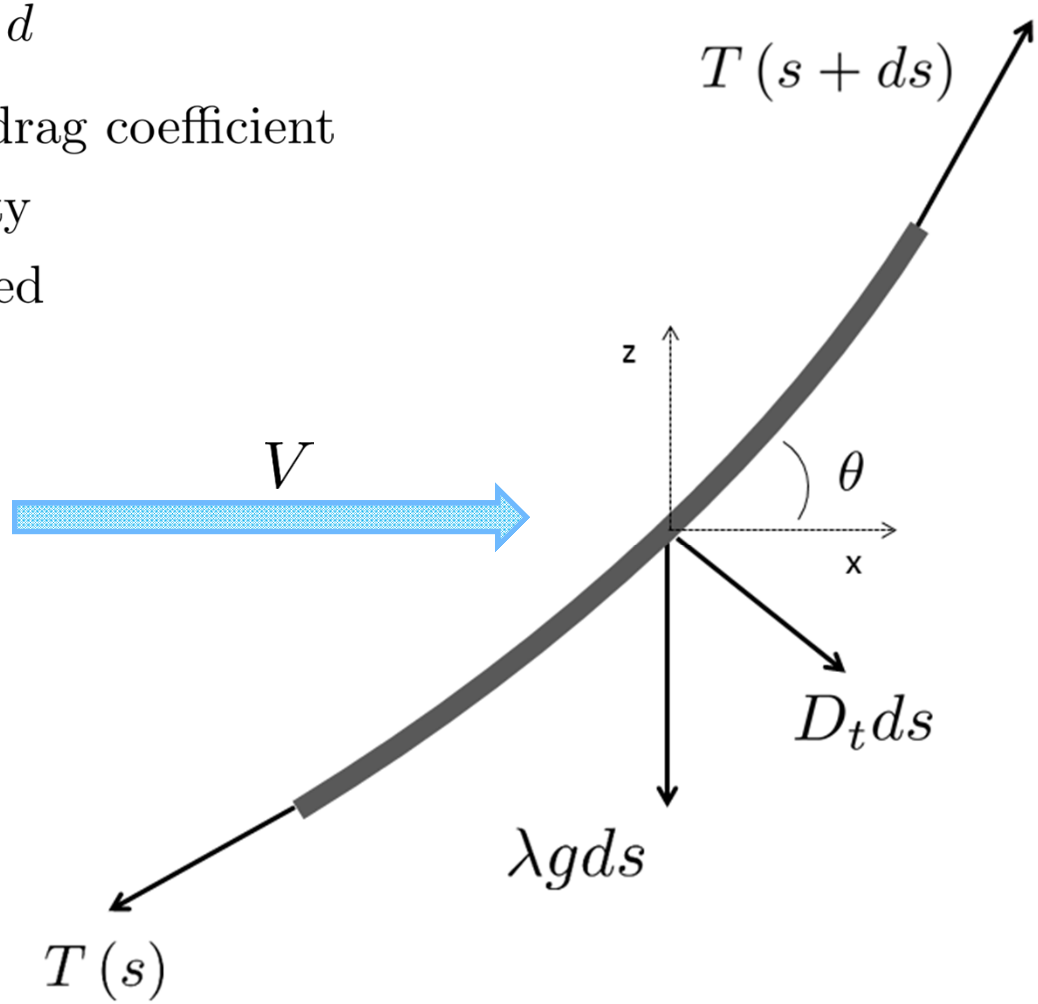
$$D_t = \frac{1}{2}\rho V^2 C_{d_x} d$$

C_{d_x} = ballistic drag coefficient

ρ = air density

V = wind speed

d = diameter



Analytic Static Tether Equations

$$\frac{d\theta}{ds} = \frac{D_t \sin^2\theta + \lambda g \cos\theta}{T}$$

$$\frac{dT}{ds} = \lambda g \sin\theta$$

$$\frac{dx}{ds} = \cos\theta$$

$$\frac{dy}{ds} = \sin\theta$$

Boundary Conditions:

$$x(0) = 0$$

$$y(0) = 0$$

One end is fixed to the ground

$$T_x(l_t) = D$$

$$T_y(l_t) = L_e$$

Effective lift and drag at the free end of the tether

There is no known analytic solution

H. Glauert. The form of a heavy cable used for towing a heavy body below an aeroplane.

Physical Dependencies: Static Tether

Buckingham Pi Theorem: The number of physical quantities subtracted from the fundamental units gives the minimum number of dimensionless quantities to describe the system.

$$[l_t] = l \qquad \text{tether length}$$

$$[h] = l \qquad \text{altitude}$$

$$[D_t] = \frac{m}{t^2} \qquad \text{tether drag per unit length}$$

$$[\lambda g] = \frac{m}{t^2} \qquad \text{tether acceleration per length}$$

$$[L_e] = \frac{ml}{t^2} \qquad \text{effective lift}$$

$$[D] = \frac{ml}{t^2} \qquad \text{drag boundary condition}$$

Dimensionless Basis

Unit exponent matrix

$$A = \begin{array}{c} \\ l \\ t \\ m \end{array} \begin{array}{c} \begin{array}{cccccc} l_t & D_t & \lambda g & L_e & D & h \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -2 & -2 & -2 & -2 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The null space gives the dimensionless basis

$$Null(A) = \begin{bmatrix} 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⇒

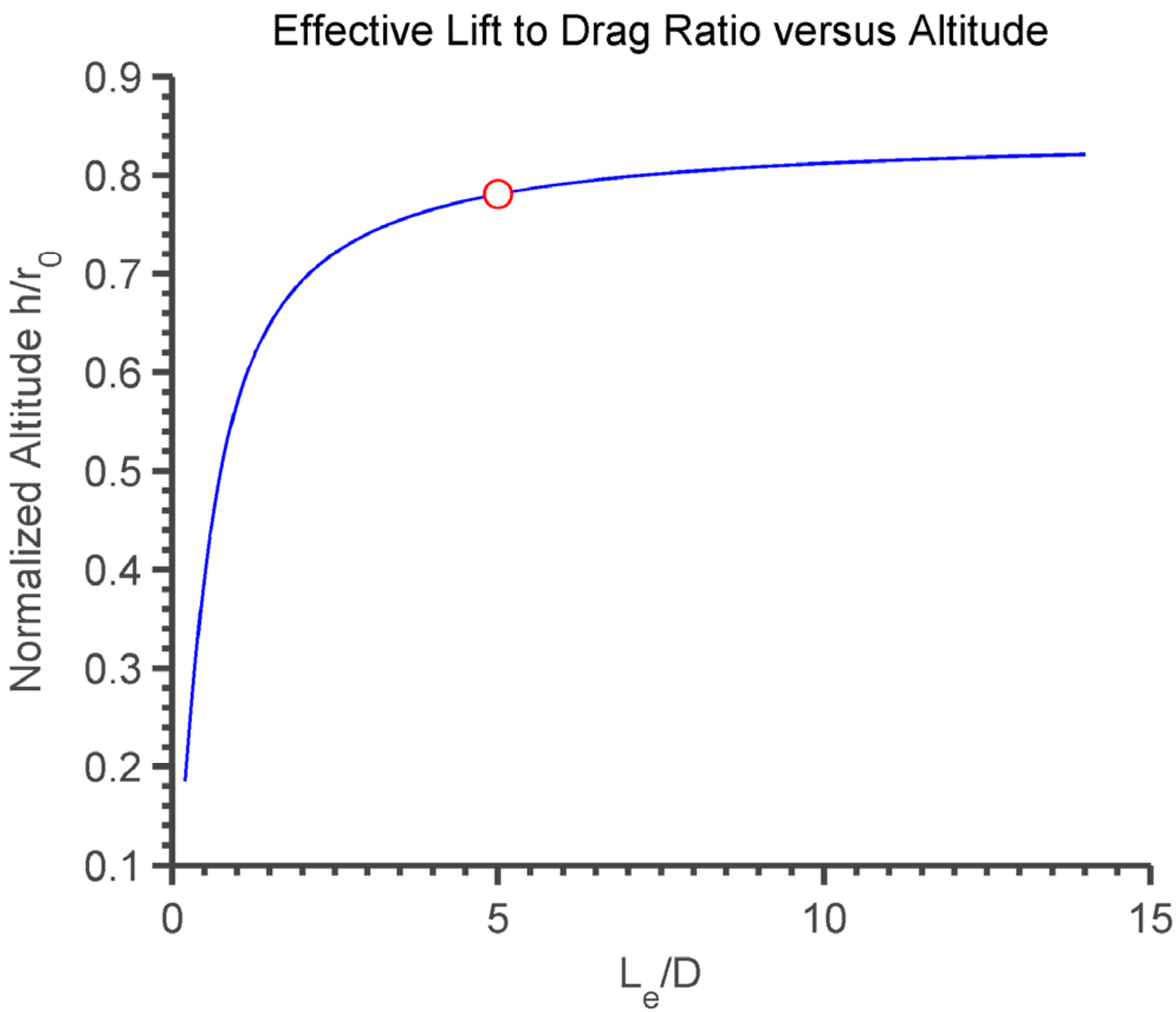
$\frac{\lambda g}{D_t}$ Gravity force to tether drag ratio

$\frac{L_e}{l_t D_t}$ Effective lift to tether drag

$\frac{L_e}{D}$ Effective lift to tether drag

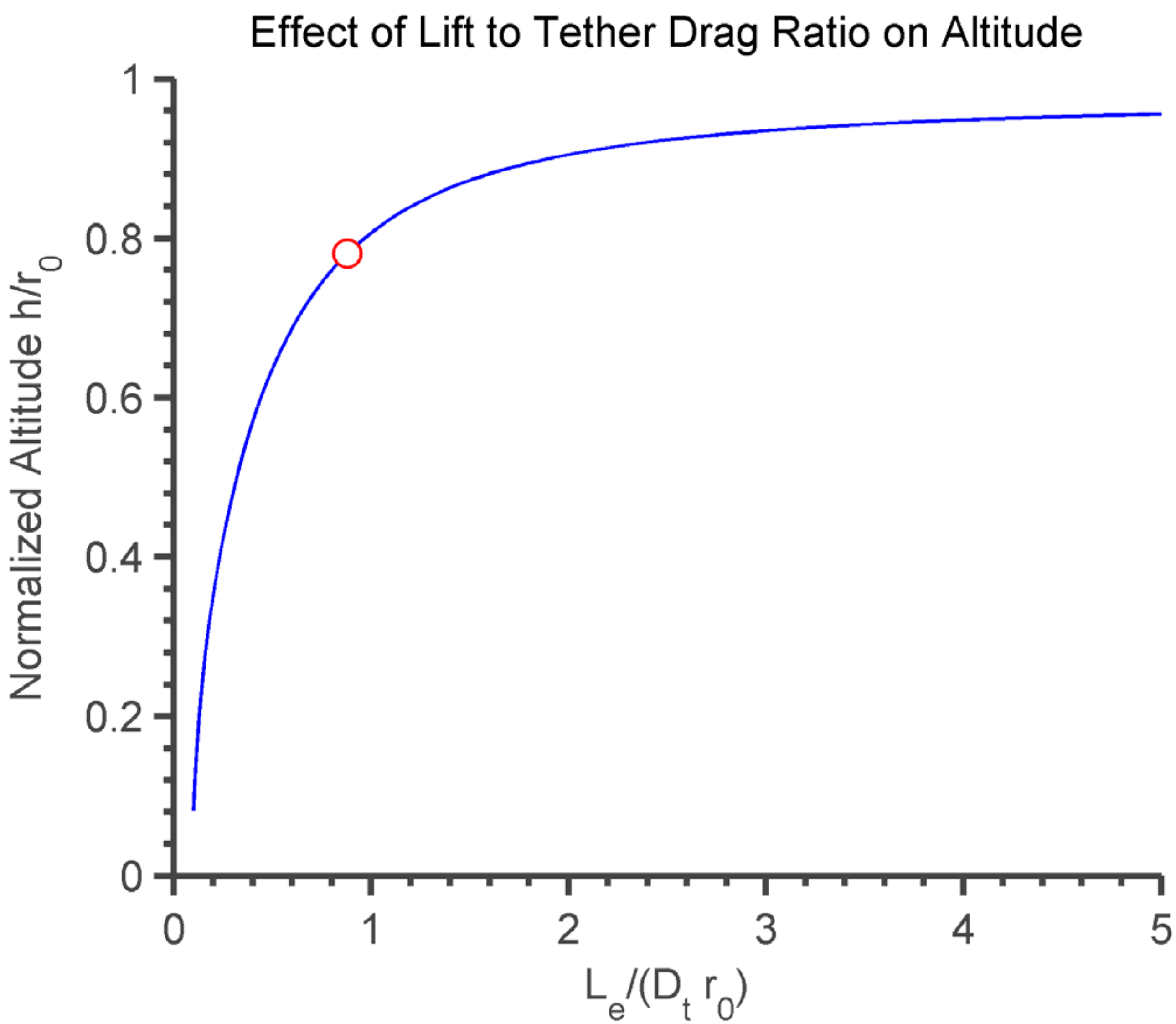
$\frac{h}{l_t}$ Altitude to tether length

Design Evaluation: End Condition



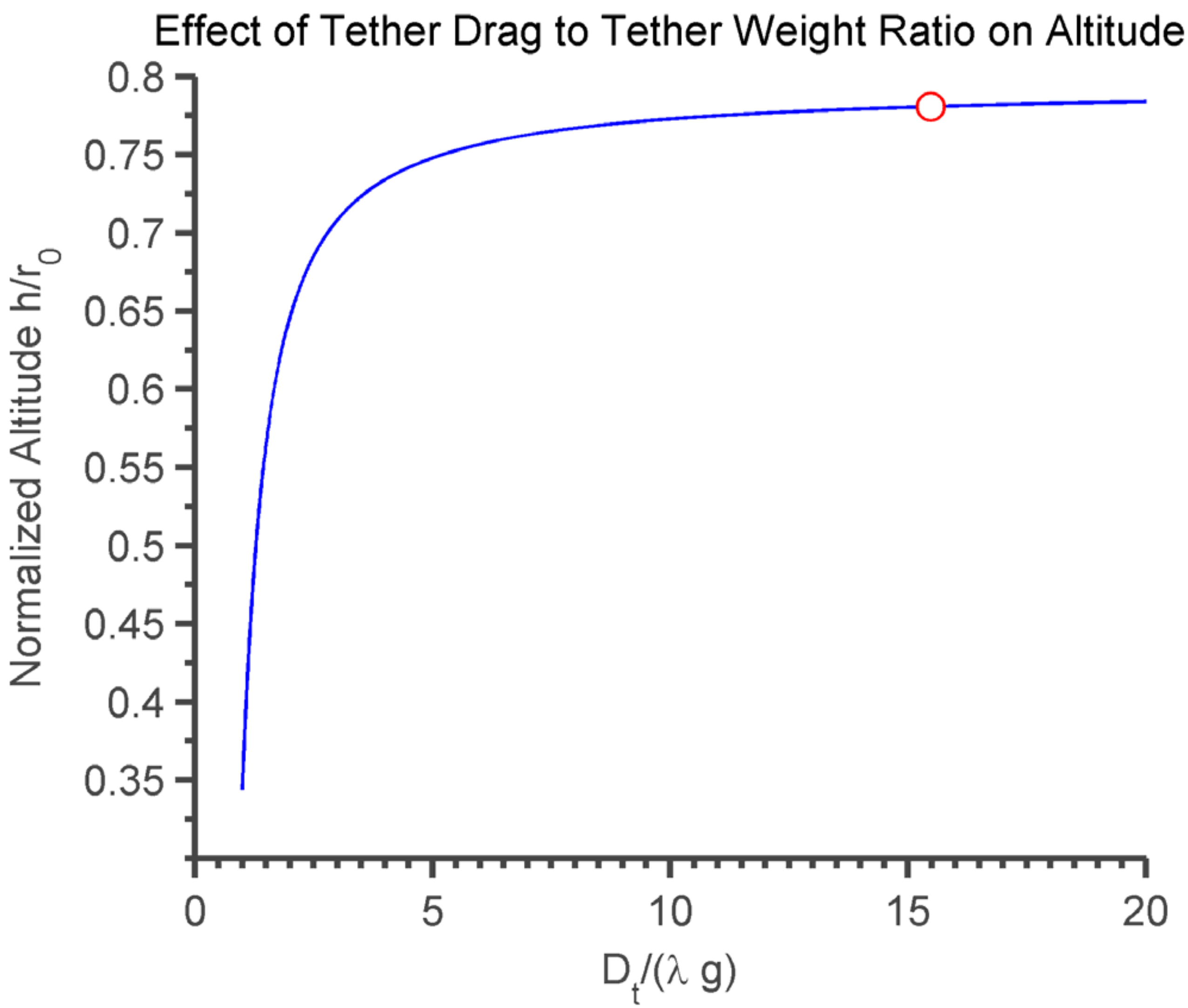
- $\frac{D_t}{\lambda g} = 15.5$
- $\frac{L_e}{D_t r_0} = 0.88$

Design Evaluation: Tether Drag



- $\frac{L_e}{D} = 5$
- $\frac{D_t}{\lambda g} = 15.5$

Design Evaluation: Tether Weight



- $\dfrac{L_e}{D} = 5$
- $\dfrac{L_e}{D_t r_0} = 0.88$

Scaling

Dimensionless parameters for static tethered system

$$\frac{\lambda g}{D_t} \qquad \frac{L_e}{l_t D_t} \qquad \frac{L_e}{D} \qquad \frac{h}{l_t}$$

Sub-scale Testing: Systems with the same dimensionless parameters will have the same normalized performance

- Reduce development cost of prototype
- Reduce development time
- Increase safety during testing phases

Testing Scaling Parameters

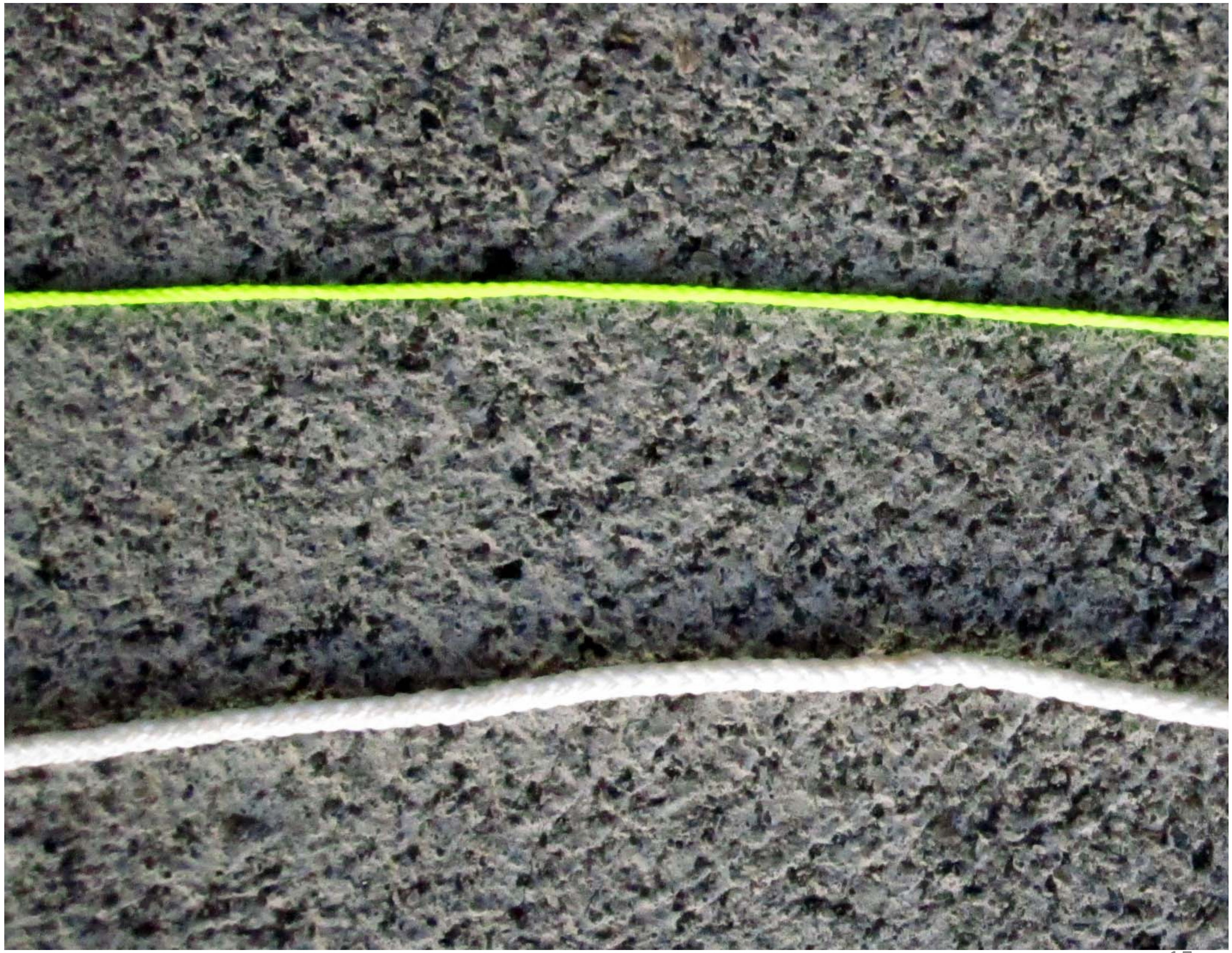
2.6 m

1.15 m

	Full Scale	Sub-Scale
Tether length (m)	30	12.7
Wind Speed (m/s)	5	3.2
Wing + Sensor mass (kg)	1.02	0.091
Wing Area (m^2)	2	0.42

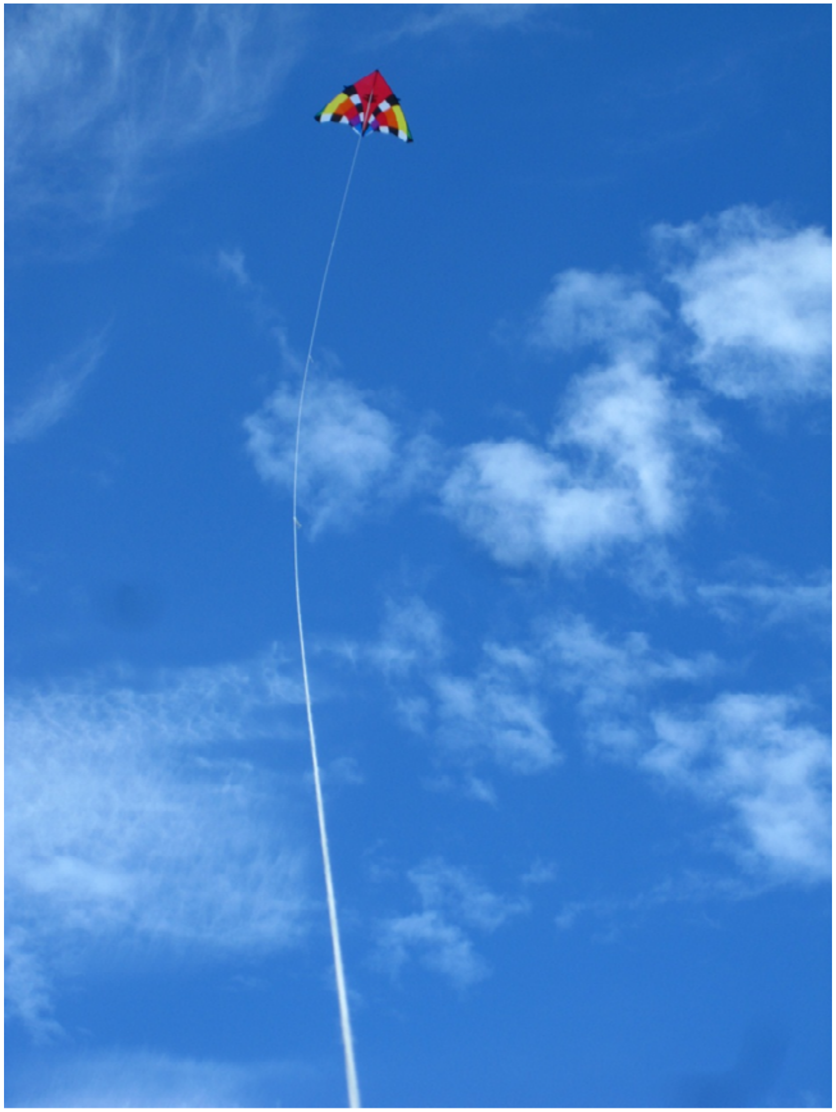
Testing Scaling Parameters

1.5 mm
1.4 g/m

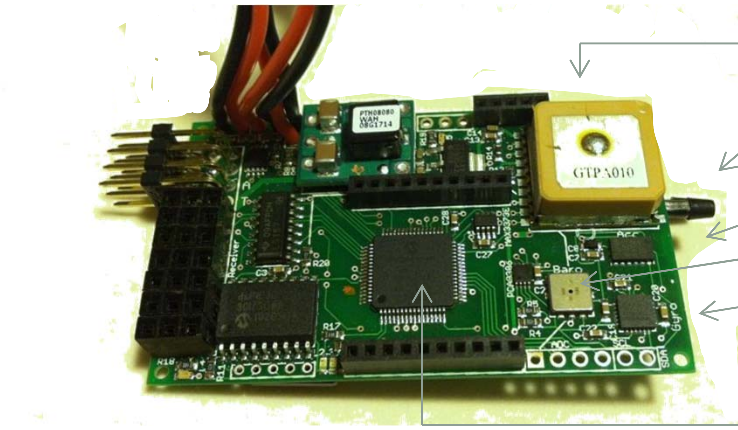


3 mm
6.6 g/m

Scaled Testing



Measurements



- GPS
- Differential Pressure
- Accelerometer
- Barometric Pressure
- Rate Gyros
- Pic Processor
- Micro SD card

Board designed by Geoff Bower

Hand held scale

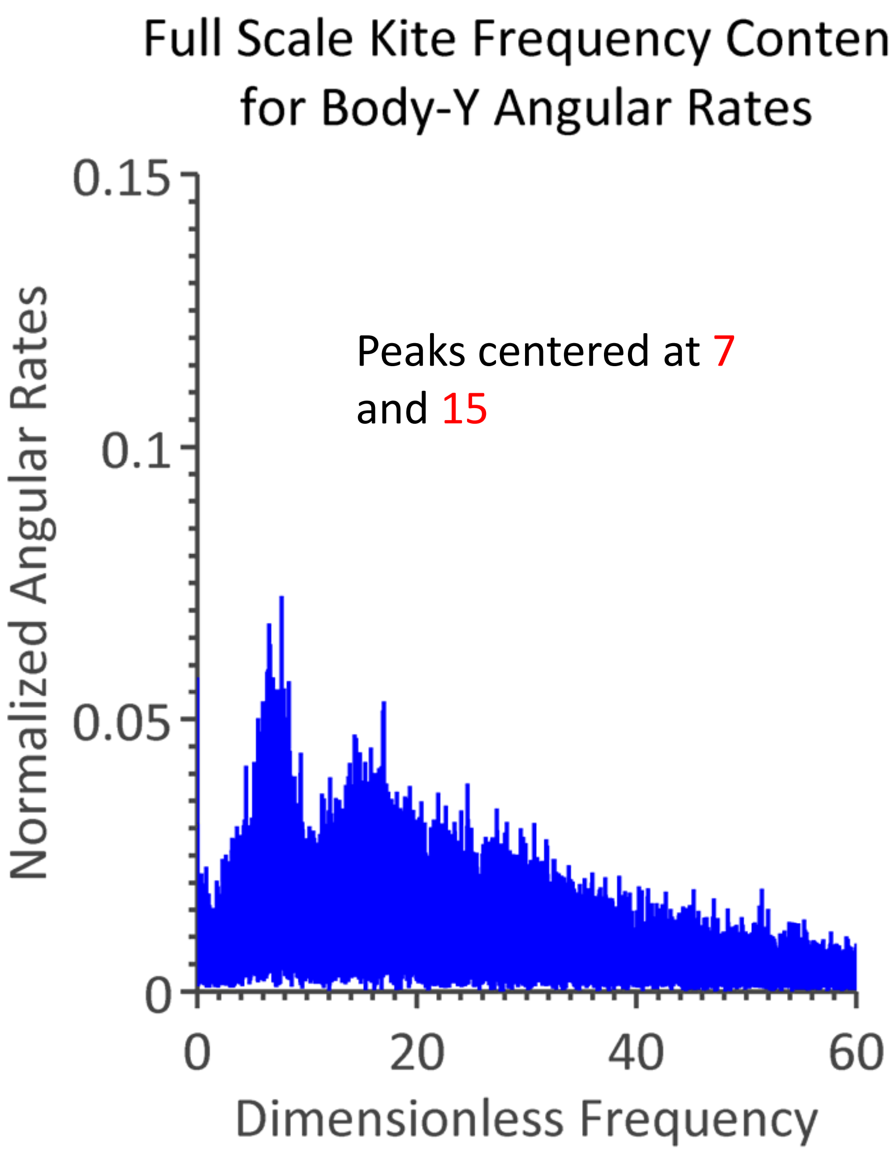
Kestrel Anemometer

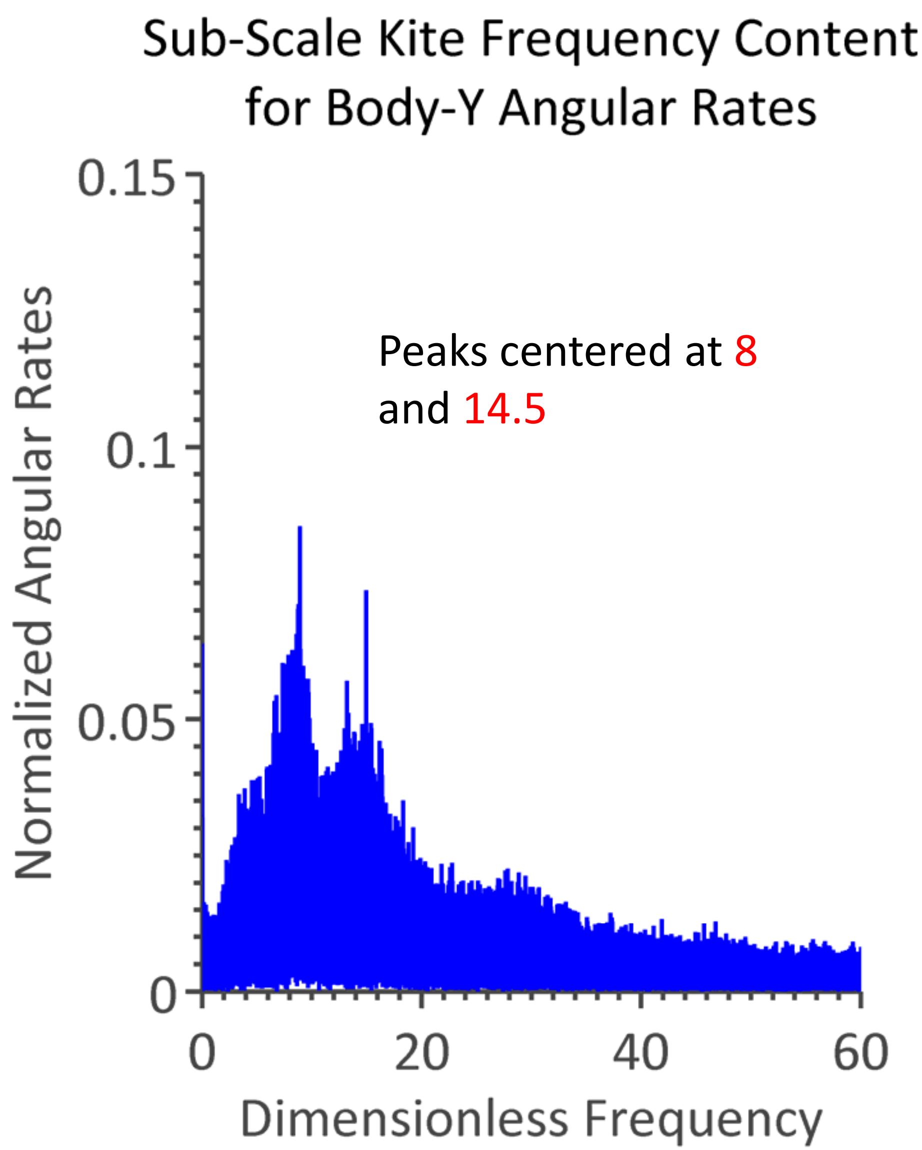
Results: Static Approximations

		Full Scale Kite	Sub-Scale Kite
Average Height	$\frac{h}{l_t}$	0.98 ± .03	0.98± .08
Estimated Tension	$\frac{T}{\lambda g l_t}$	38.9	39.5
Average Wind Speed	$\frac{V}{\overline{V}}$	1	1

Data was taken from periods of time with the necessary average wind speed

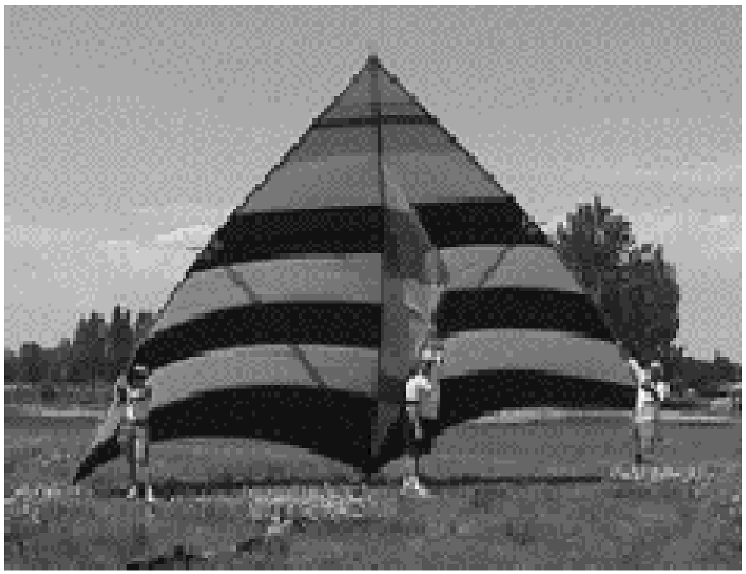
Preliminary Results: Frequency Content





Current Single Line Altitude Record

The current record holding kite built and flown by Richard Synergy.



Millibar Messenger

Area: 25 m^2
Max Wind Speed: 12 m/s
Tether: 270 pound test woven Kevlar line 3/32 inch in diameter

Current Record: 13600 ft above ground

Goal: 5000m or 16400 ft above ground

Design Method

Performance Based Design

Objective: Reach the target altitude while minimizing cost

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & f(x) & \\
\text{subject to} \quad & H(x) \leq 0 & \text{nonlinear constraints} \\
& x \leq u & \text{upper bound} \\
& x \geq l & \text{lower bound.}
\end{aligned}
$$

Variables (x)

- Tether Length
- Tether Diameter
- Tether Material
- Aspect Ratio
- Wing Area
- Lift Coefficient
- Spar Radius

Constraints H(x)

- Achievable altitude >5000 m
- Tether tension < material strength
- Bending loads < Spar bending strength

Cost Function f(x)

Estimated prices of

- Tether
- Spar cost
- Wing material

Achievable Altitude

- **Altitude is calculated by numerically integrating the static tether equations**

$$\frac{1}{\lambda g}\frac{d\theta}{ds} = \frac{\frac{D_t}{\lambda g}\sin^2\theta + \cos\theta}{T} \qquad \frac{dx}{ds} = \cos\theta \qquad x(0) = 0$$

$$y(0) = 0$$

$$\frac{1}{\lambda g}\frac{dT}{ds} = \sin\theta \qquad \frac{dy}{ds} = \sin\theta \qquad T_x(l_t) = D$$

$$T_y(l_t) = L_e$$

- **T(s) must be less than the tether material strength for all s**

- **Calculate the Boundary conditions**
  - **Weight model**
  - **Aerodynamic model**

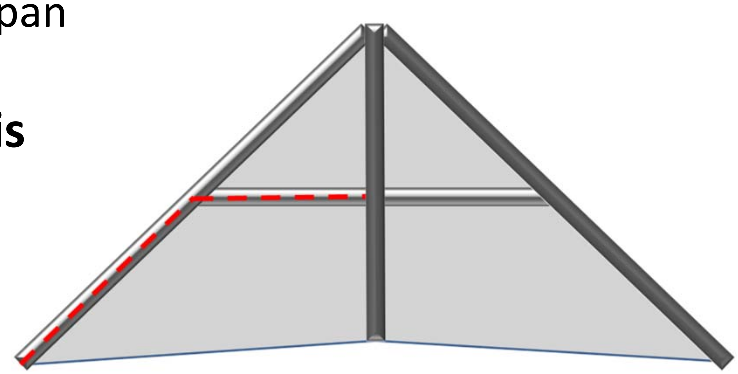
Wing Model

- **Lift determined by area and C_L**
  - Wind Speed = 12 m/s
  - Safety factor included

Elliptical Lift Distribution

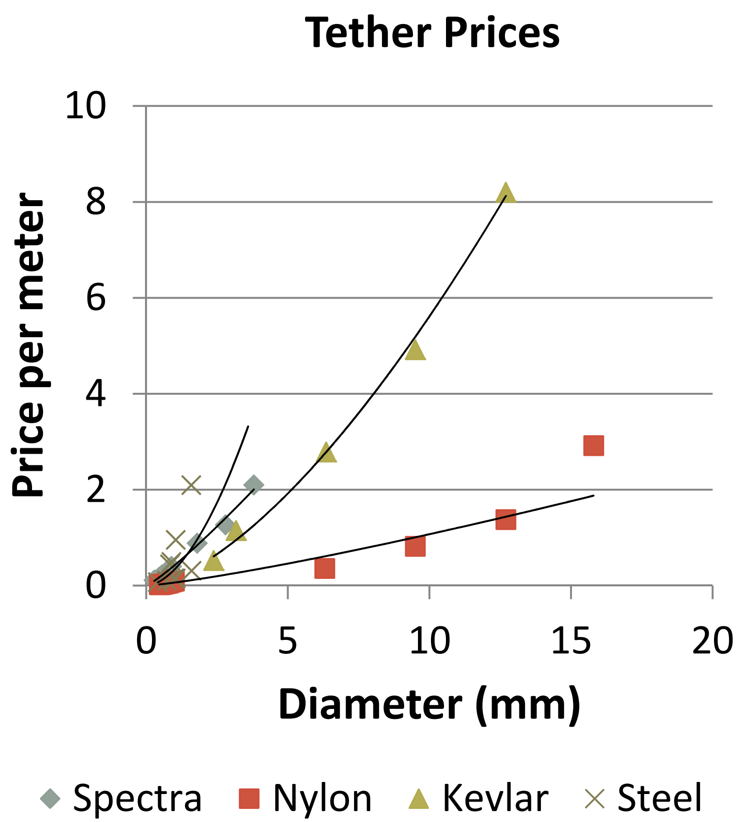
- **Fixed planform design variables**
  - Sweep = 45^0
  - Taper Ratio = 0.06
  - Crossbar intersection at half the semi-span

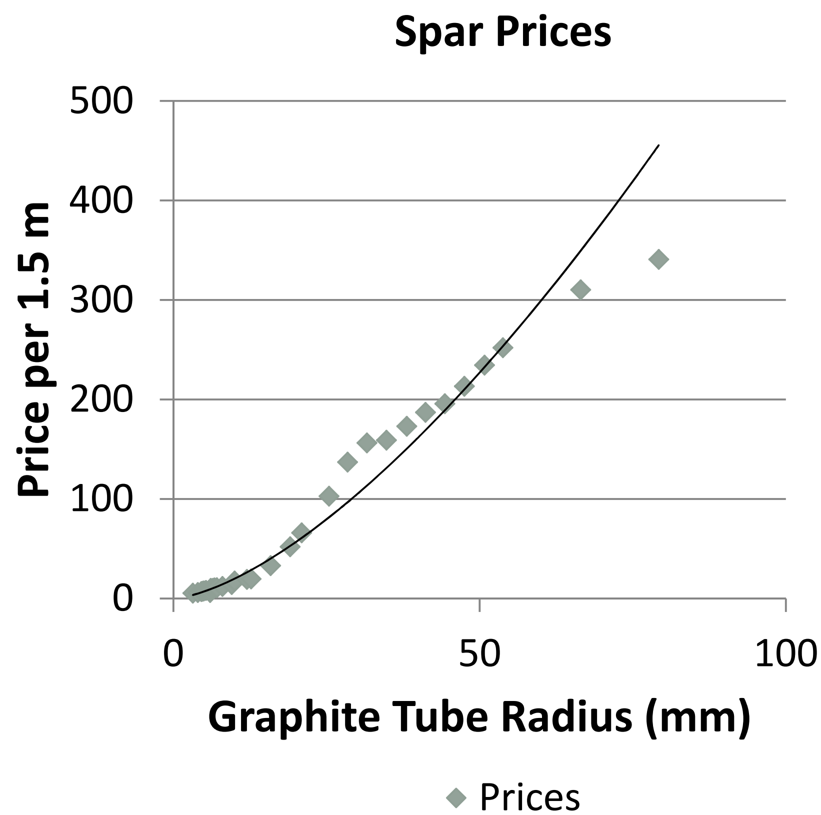
- **Moments calculated about the neutral axis**
  - Carbon composite tubes
  - Spars have uniform radius
  - Mass wing area assumed to be the mass of ripstop nylon



4 Spar Construction

Cost Function

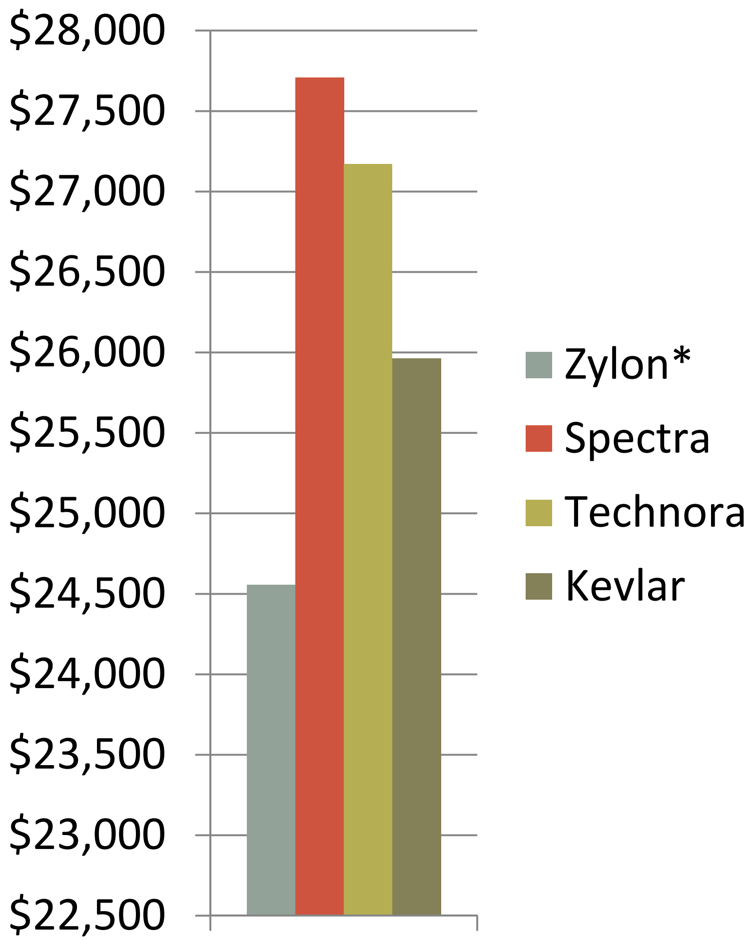




Total cost includes

- Tether cost as a function of diameter and length
- Spar cost as a function of tube radius and length
- Wing material cost as a function of area

Performance Results



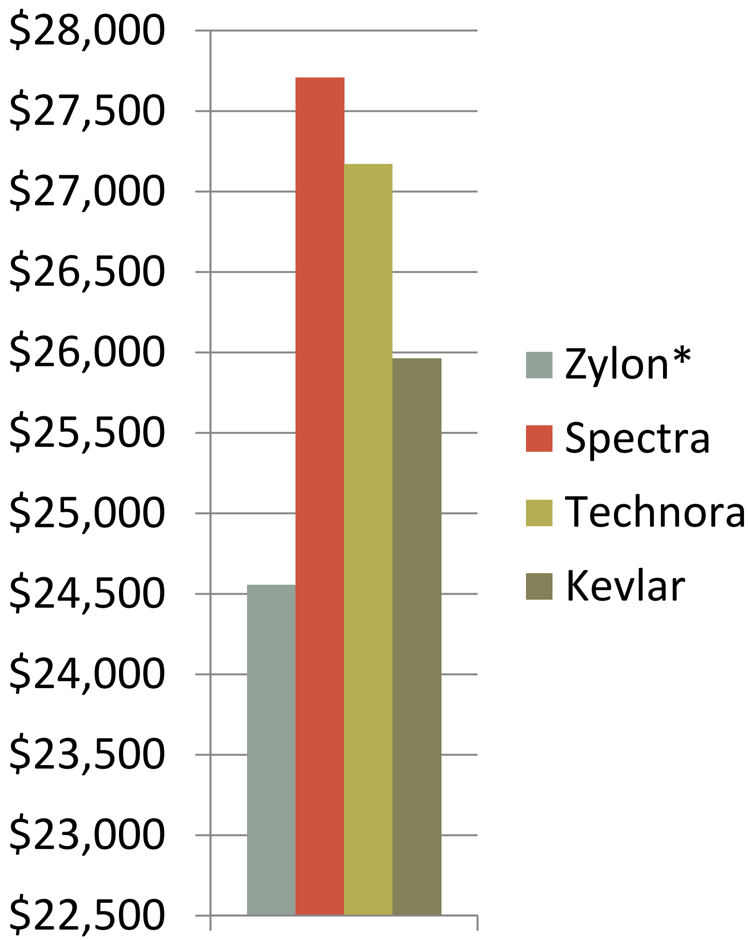
MATLAB's SQP based optimizer fmincon

Run Parameters

Wind Speed	12 m/s
Achievable Altitude	5000 m
Maximum Lift Coefficient	0.675
Spar Material Yield Stress	300 MPa
Spar Material Density	1000 Kg/m^3
Wing Material Density	0.1 Kg/m^3
Taper Ratio	0.06
Sweep	45^0
Safety Factor	10

* Price for zylon is uncertain due to limited pricing information of thin pure zylon ropes

Performance Results



	Zylon
Cost	24557
Wing Area S	24.7 m^2
Aspect Ratio AR	3.5
Tether Diameter d	1.8 mm
Tether Length l_t	8054 m
Lift Coefficient C_L	0.675
Wing Mass	36.5 Kg
Leading Edge Spar	48 mm
Cross Bar	57 mm
Lift to Drag Ratio	16
Effective Lift to Drag	9.5
Maximum Tension	1041 N
Tether Mass	18.9 Kg

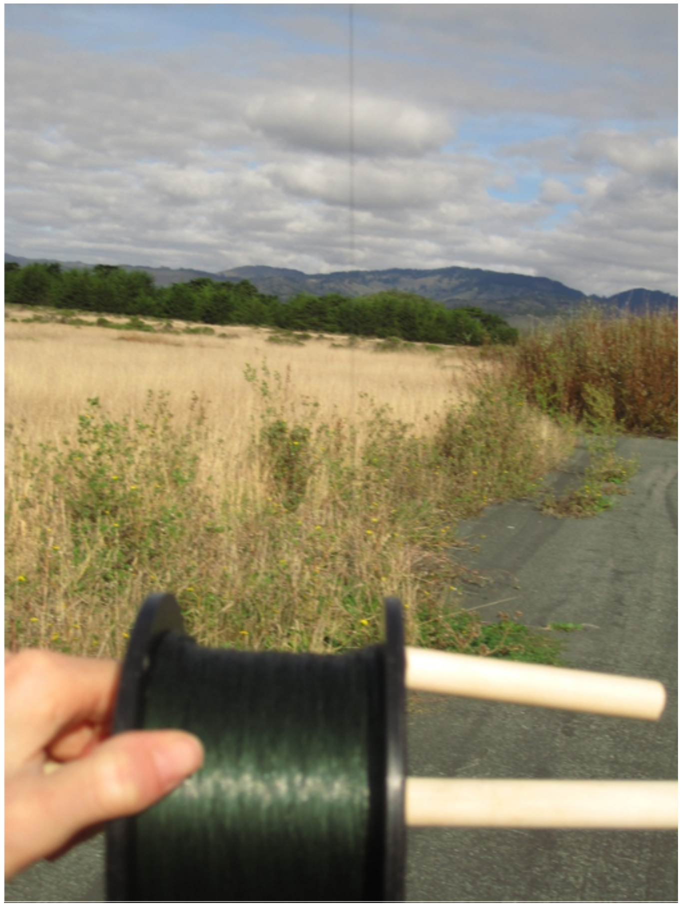
Detailed Performance Results

	Zylon	Spectra	Technora	Kevlar
Cost	24557	27708	27170	25963
Wing Area S	24.7 m^2	23.8 m^2	22.6 m^2	21.3 m^2
Aspect Ratio AR	3.5	2.9	2.9	2.8
Tether Diameter d	1.8 mm	3.0 mm	2.9 mm	3.2 mm
Tether Length l_t	8054 m	9603 m	9514 m	9794 m
Lift Coefficient C_L	0.675	0.675	0.675	0.675
Wing Mass	36.5 Kg	34.0 Kg	32.0 Kg	28.7 Kg
Leading Edge Spar	48 mm	47 mm	46 mm	46 mm
Cross Bar	57 mm	57 mm	56 mm	55 mm
Lift to Drag Ratio	16	13.7	13.6	13
Effective Lift to Drag	9.5	8.3	8.3	8
Maximum Tension	1041 N	1030 N	984 N	937 N
Tether Mass	18.9 Kg	51.4 Kg	53.0 Kg	61.2 Kg

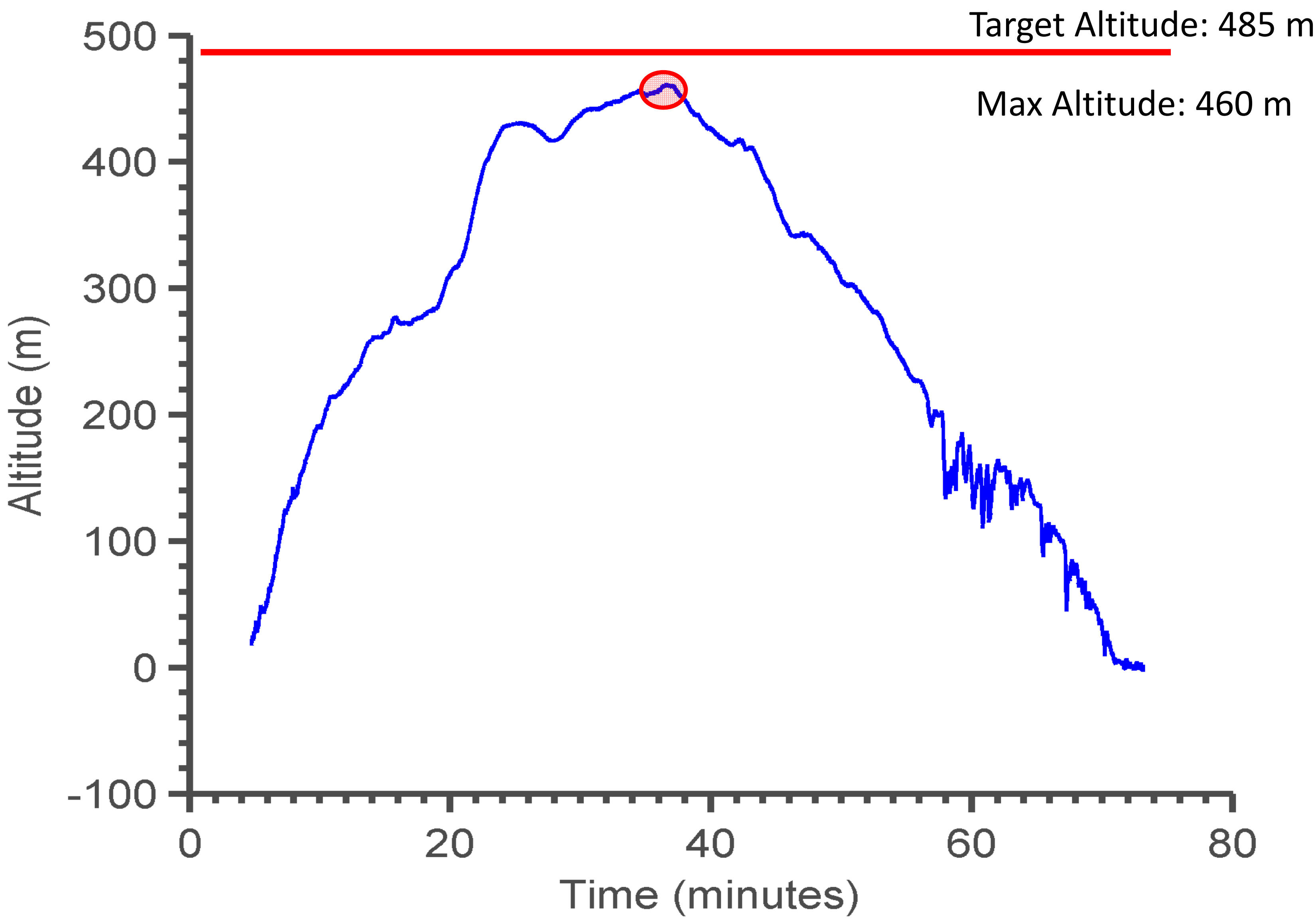
Scaled Design

Subscale Flight

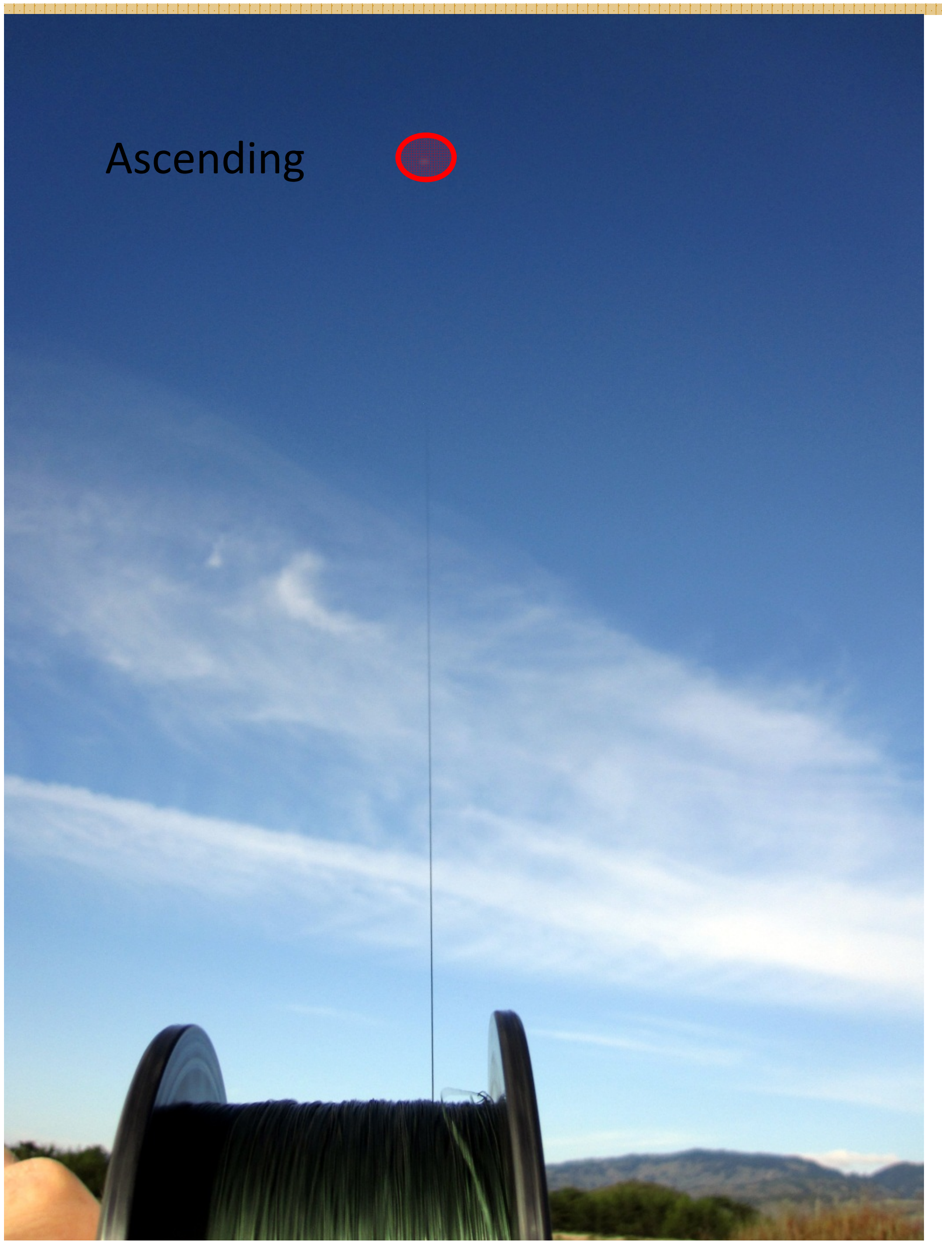
- IMU, GPS and air pressure sensors (50g)
- Average wind speed 4 m/s
- 20 lb test spectra



Experimental Results

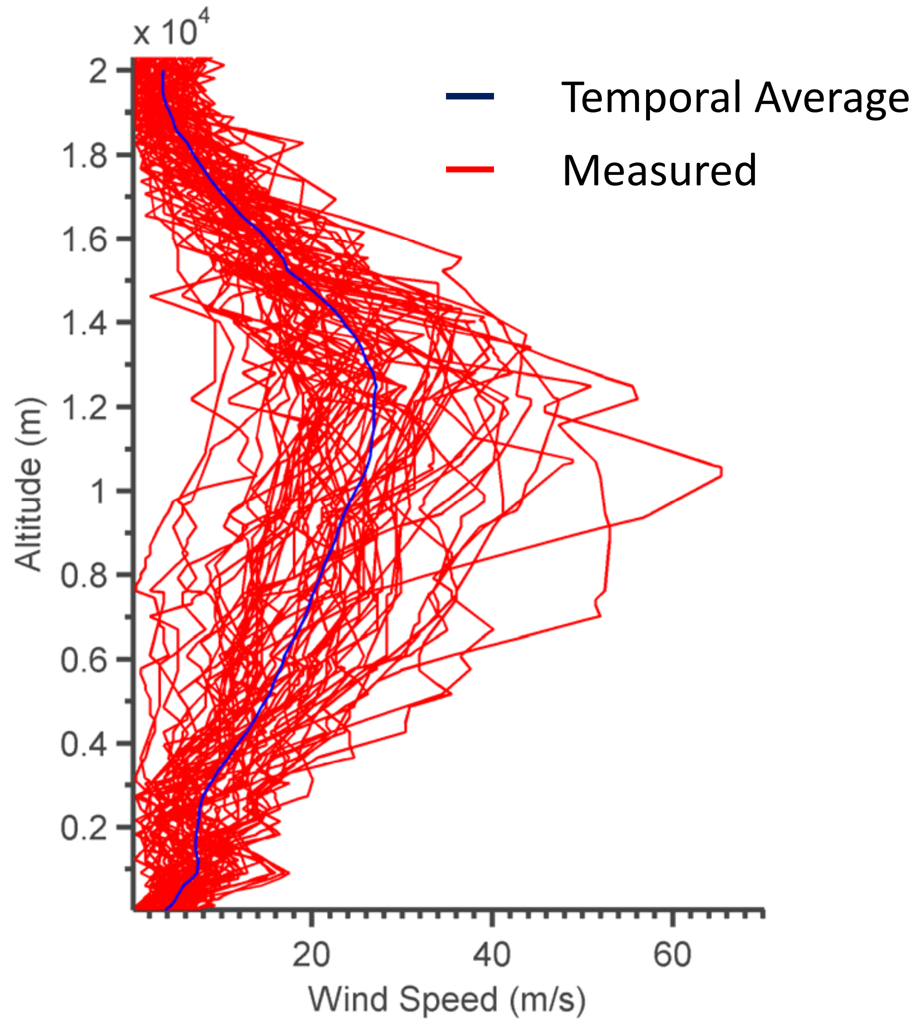


Experimental Results



Wind Profiles

Sounding Data for Oakland in June



Wind Speed Profile

1. Assume a constant wind profile
2. Choose the reference wind speed

Data: Oakland sounding records for January 2010

- Collect every 12 hours
- 64 samples total