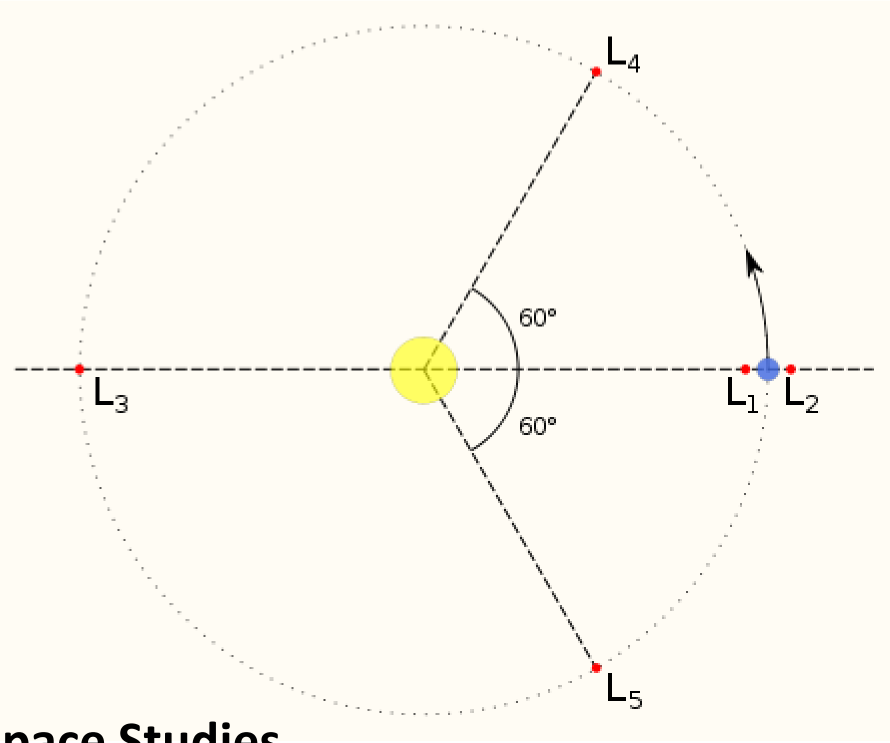
Low-deltaV Trajectories

to move a small asteroid to a Lagrange point



Keck Institute for Space Studies
Asteroid Retrieval Mission Study
Short Course
09/27/2011

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Summary

- Lagrange points: definition

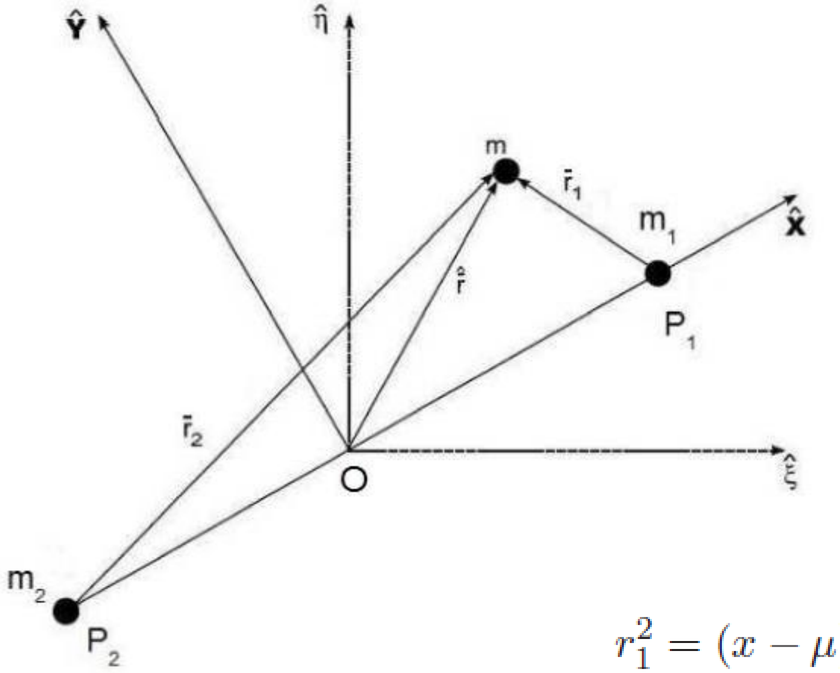
- Orbits about Lagrange points and invariant manifolds (Low DeltaV Trajectories)

- Why a NEA near EM L1/L2 or SE L2? Potential destination for human missions

- Missions to Lagrange points: overview

- Asteroid Retrieval Mission Study: matching low-thrust with invariant manifolds?

CR3BP



Equations of motion

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$
$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y}$$
$$\ddot{z} = \frac{\partial U}{\partial z}$$

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

$$r_1^2 = (x-\mu)^2 + y^2 + z^2$$

$$r_2^2 = (x-\mu+1)^2 + y^2 + z^2$$

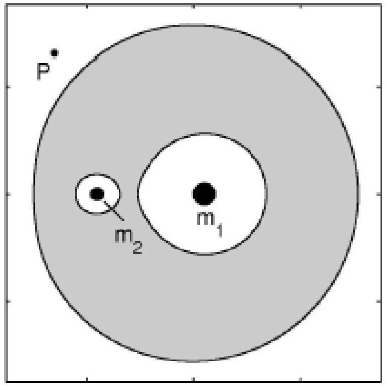
The Circular Restricted 3-Body Problem describes the motion of a massless particle under the gravitational influence of two point masses m_1 and m_2, called primaries, in circular motion around their common centre of mass.

Jacobi constant

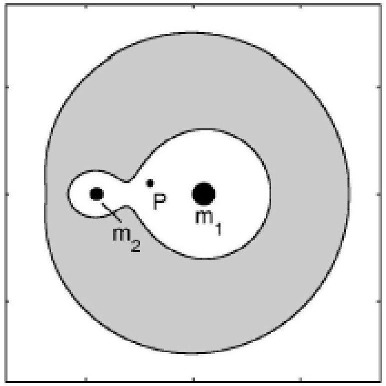
$$C = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - V^2$$

Surfaces of Hill

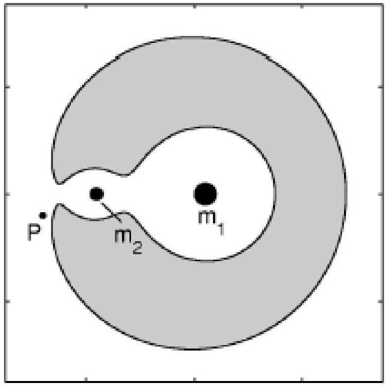
boundary for the admissable motion



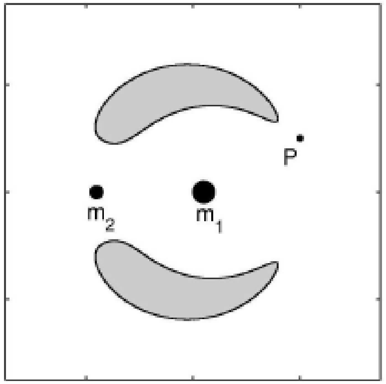
Case 1 : $C>C_1$



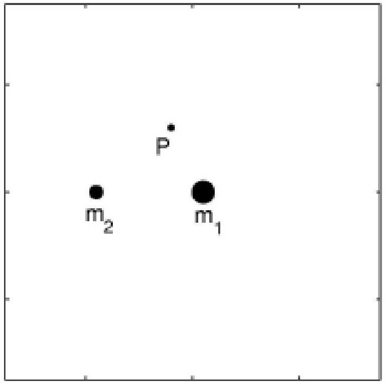
Case 2 : $C_1>C>C_2$



Case 3 : $C_2>C>C_3$

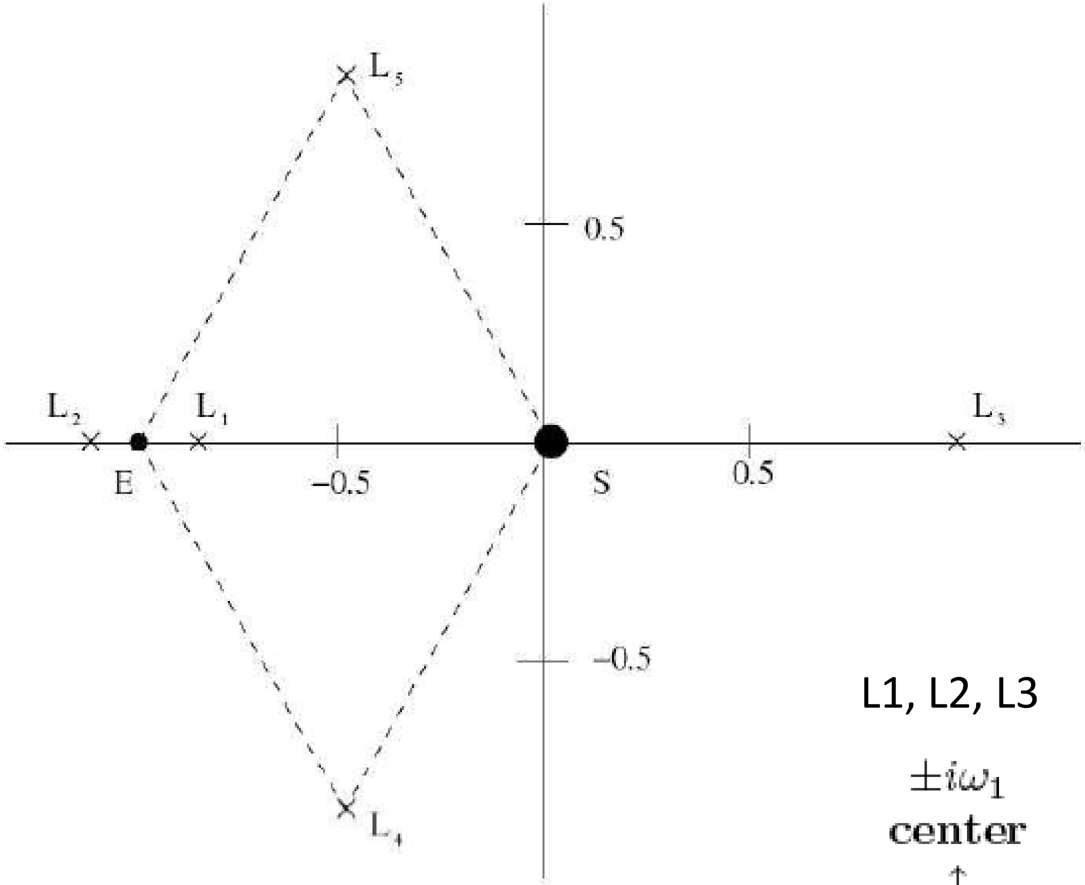


Case 4 : $C_3>C>C_4$



Case 5 : $C_4>C$

Lagrange points



SE L1/L2: about 1.5 million km from the Earth

EM L1/L2: about 60,000 km from the Moon

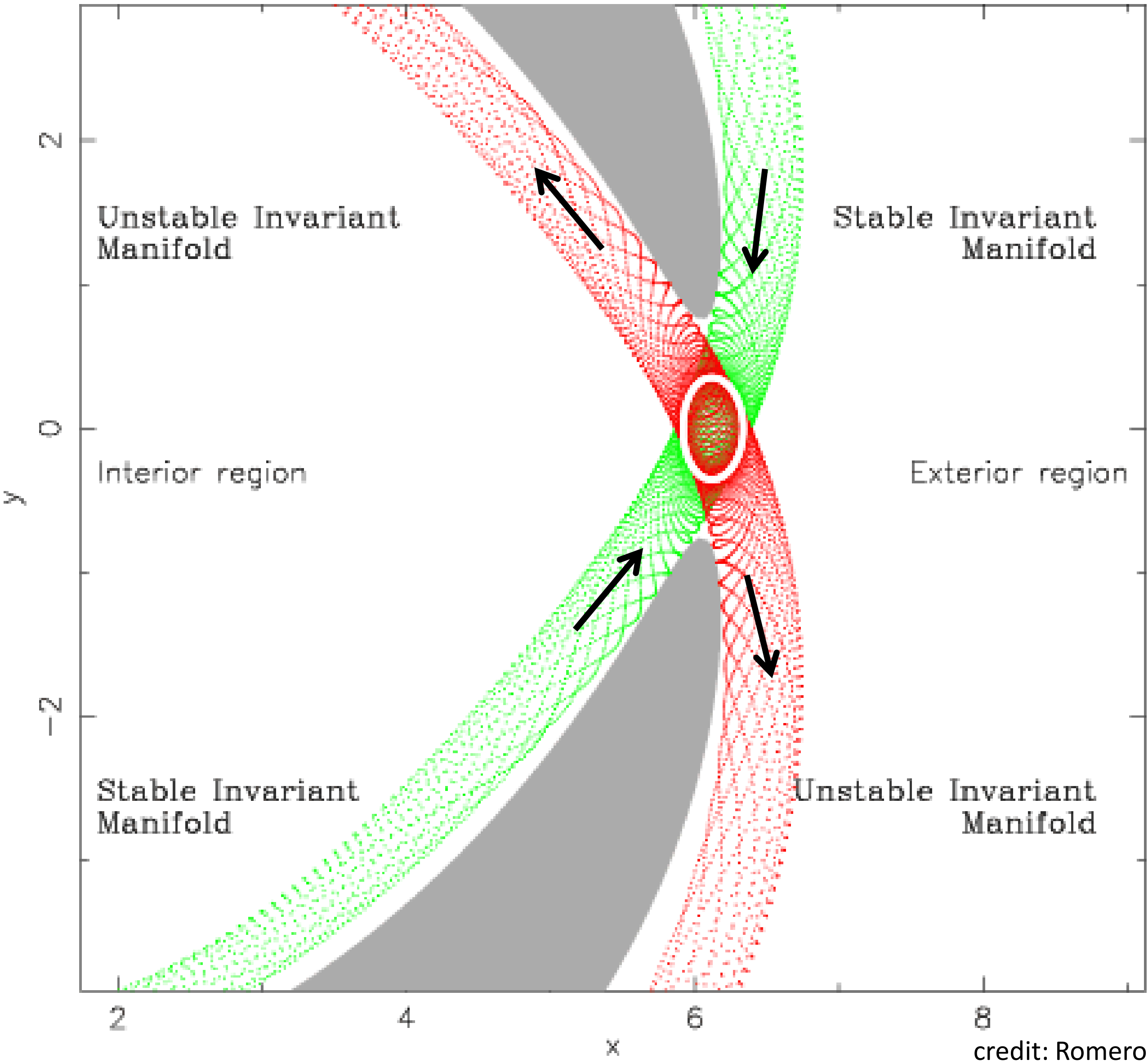
1 AU: about 150 million km

1 LD: 384,403 km

L1, L2, L3

$$\underbrace{\begin{array}{c}\pm i\omega_1\\ \textbf{center}\\ \uparrow\\ \text{planar p.o.}\end{array} \times \begin{array}{c}\pm i\omega_2\\ \textbf{center}\\ \uparrow\\ \text{vertical p.o.}\end{array}}_{\text{Cantor set of 2D tori}} \times \begin{array}{c}\pm\lambda\\ \textbf{saddle}\\ \uparrow\\ \text{inv. manif.}\end{array}$$

Invariant Manifolds



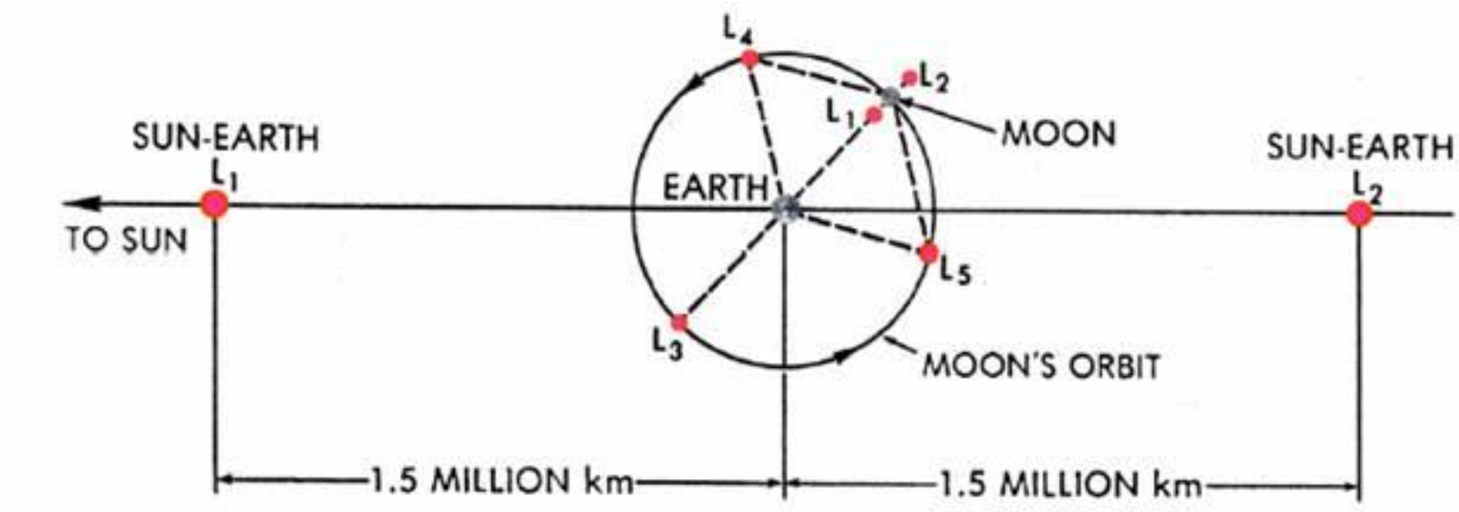
credit: Romero-Gomez

EM L1/L2 vs SE L2 as targets for human missions

EM L1	**EM L2**	**SE L2**
ΔV1 = 3.0661 km/s	ΔV1 = 3.0966 km/s	ΔV1 = 3.1537 km/s
ΔV2 = 0.8859 km/s	ΔV2 = 0.7819 km/s	ΔV2 = **0.4666 km/s**
ΔVtot = 3.9520 km/s	ΔVtot = 3.8786 km/s	ΔVtot = 3.6203 km/s
Transfer = 3.8 days	Transfer = 6.2 days	Transfer = 37.6 days

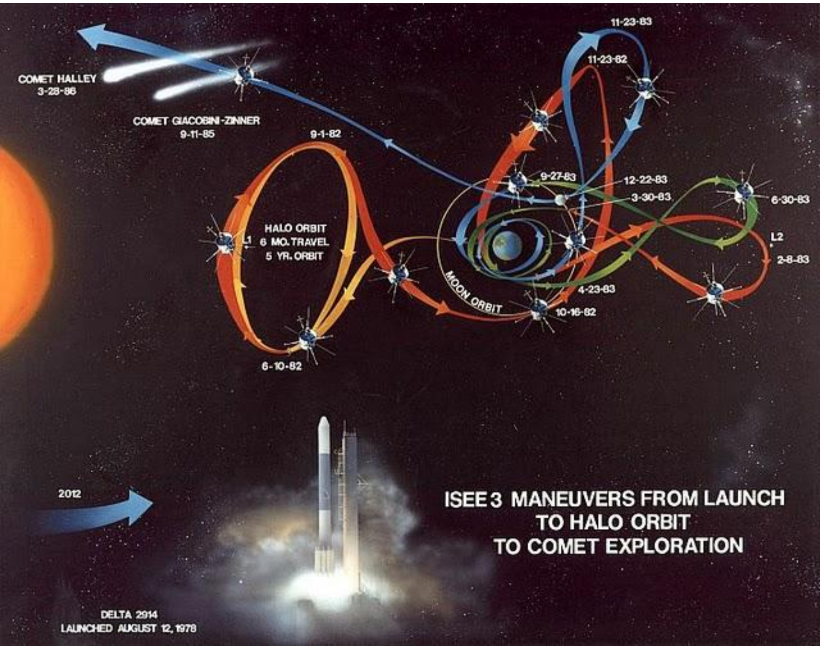
Hohmann transfer from LEO (h = 400 km)

Current NASA plans for manned missions to a NEA, total mission duration: 180 days.
Targets: 2009 OS5 (2020 and 2036), 1999 AO10 (2025), 2003 SM84 (2046)

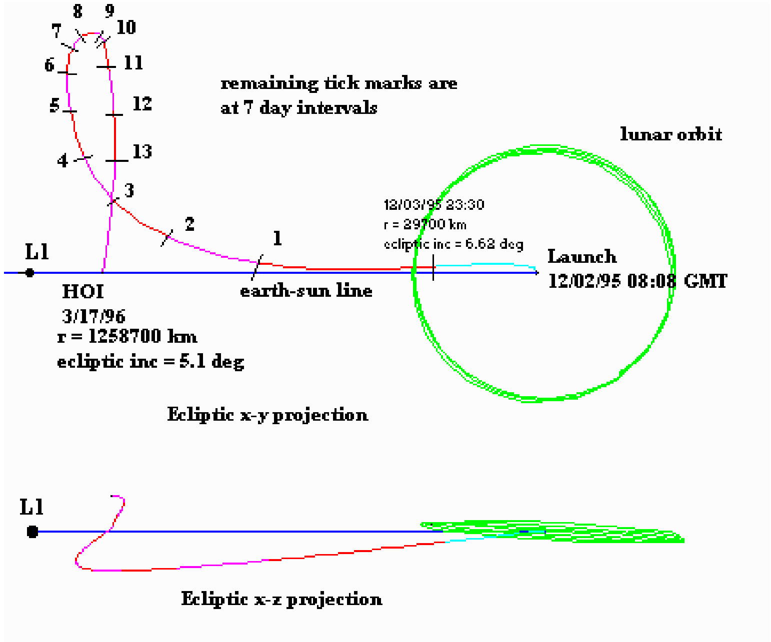


credit: Ross and Lo

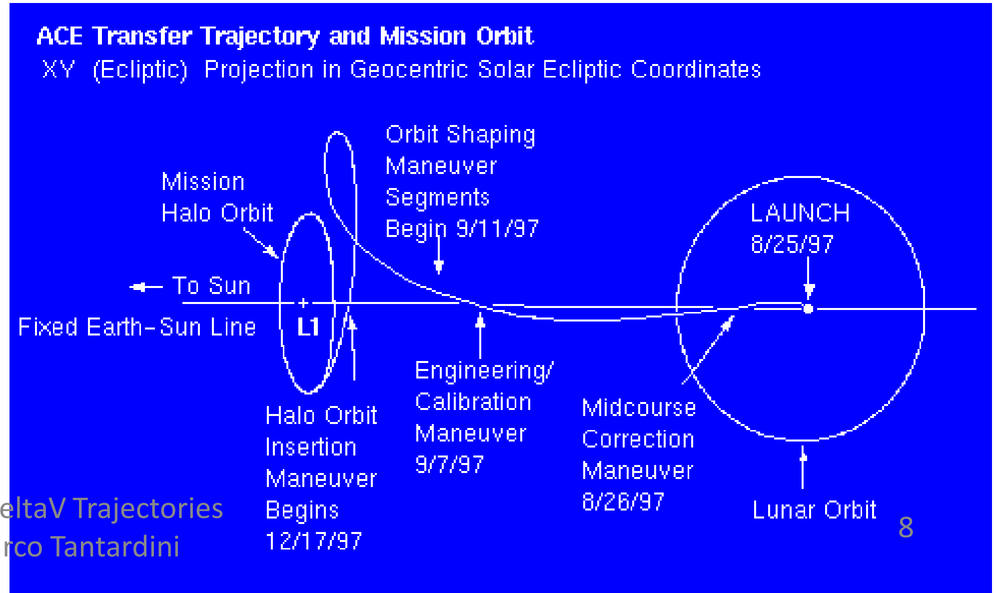
ISEE-3
International Sun-Earth Explorer-3

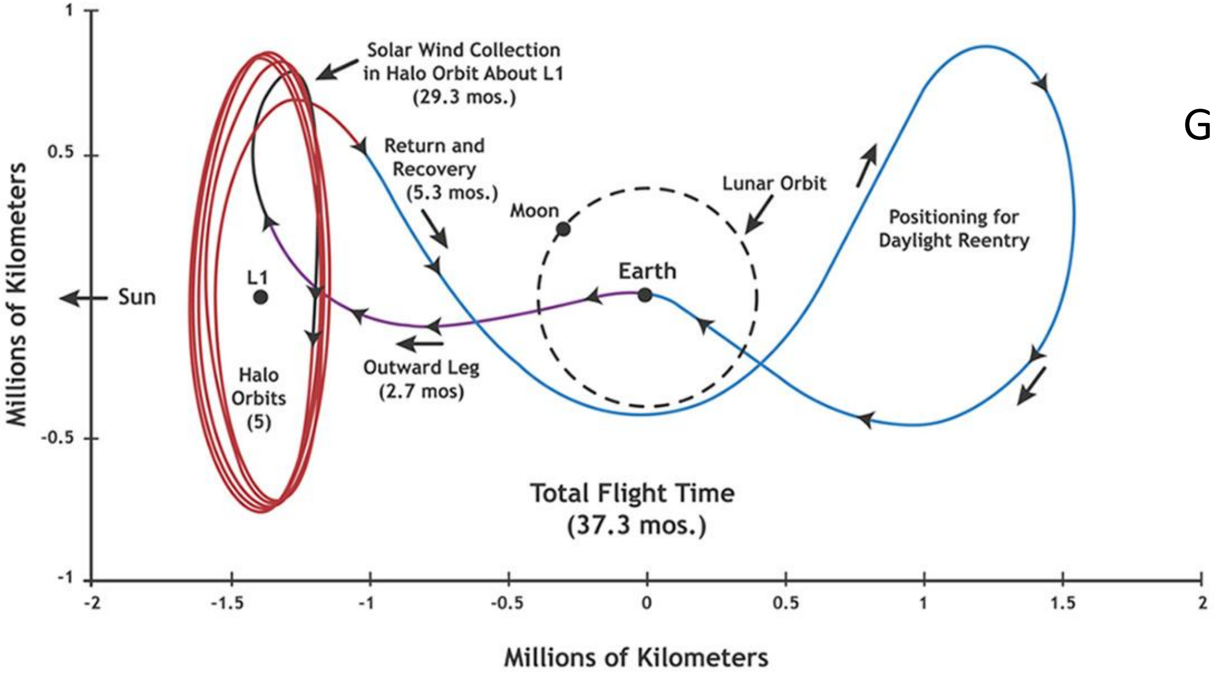


SOHO
Solar and Heliospheric Observatory

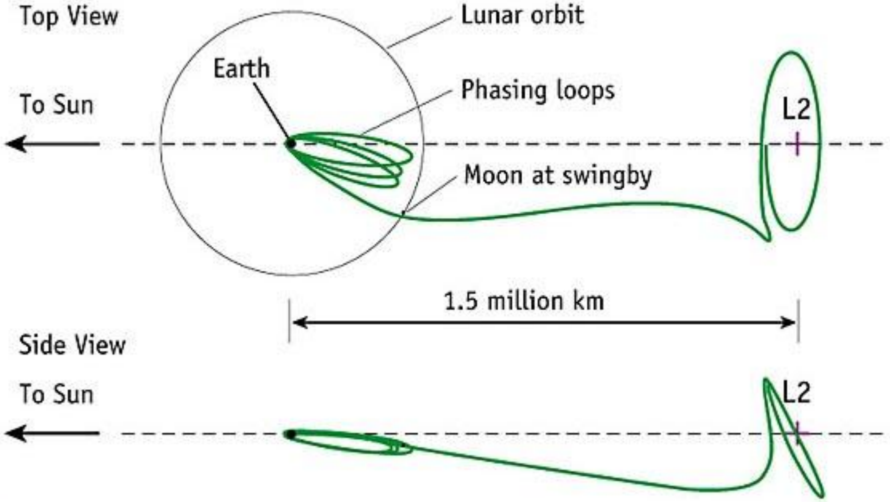


ACE
Advanced Composition Explorer





GENESIS



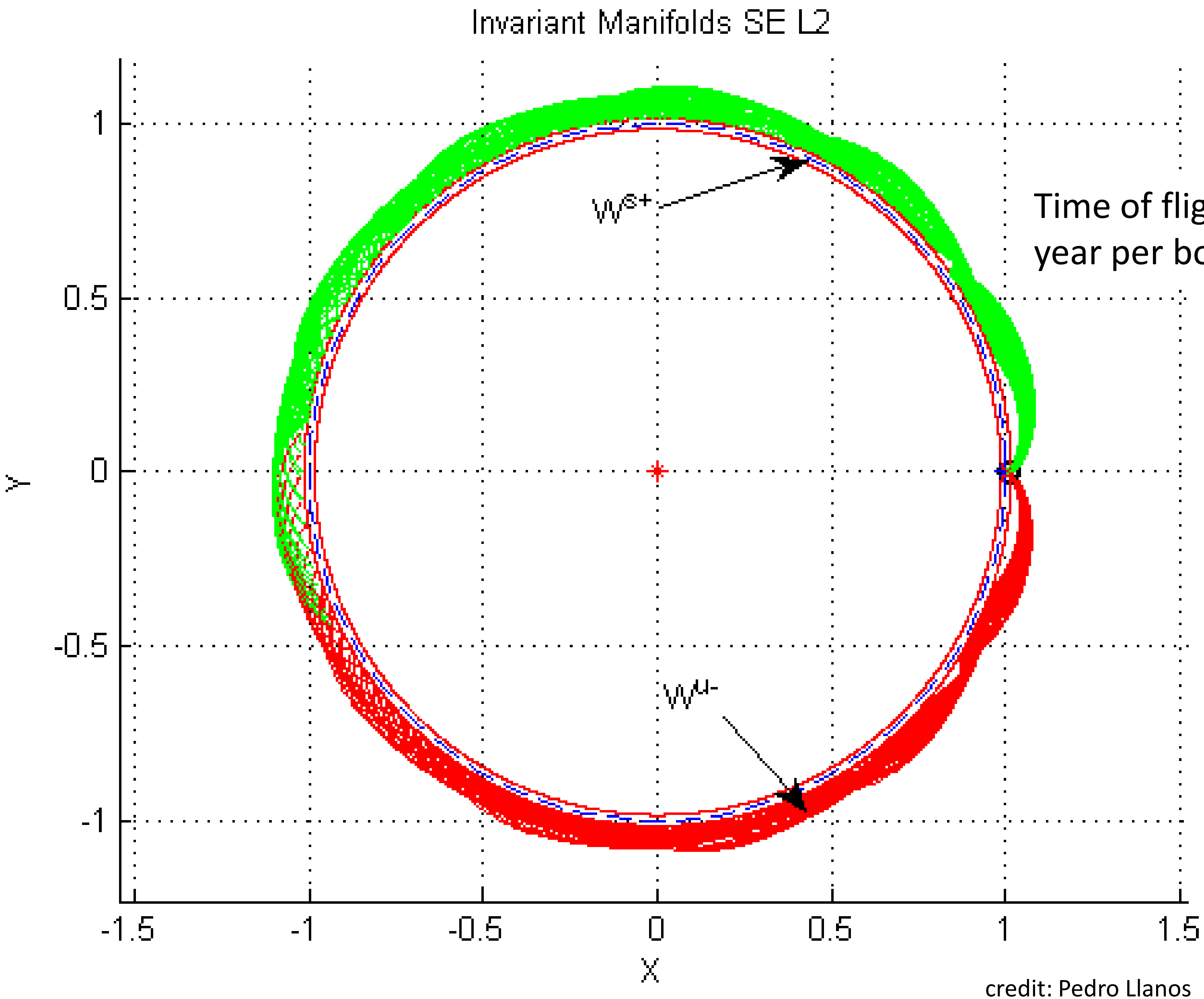
WMAP
Wilkinson Microwave Anisotropy Probe

Other missions to Lagrange points

Wind (SE L1)
Grail (SE L1)
Herschel and Planck (SE L2)
Chang’e 2 (SE L2)
Artemis (EM L1 and L2)
James Webb Space Telescope (SE L2) ?

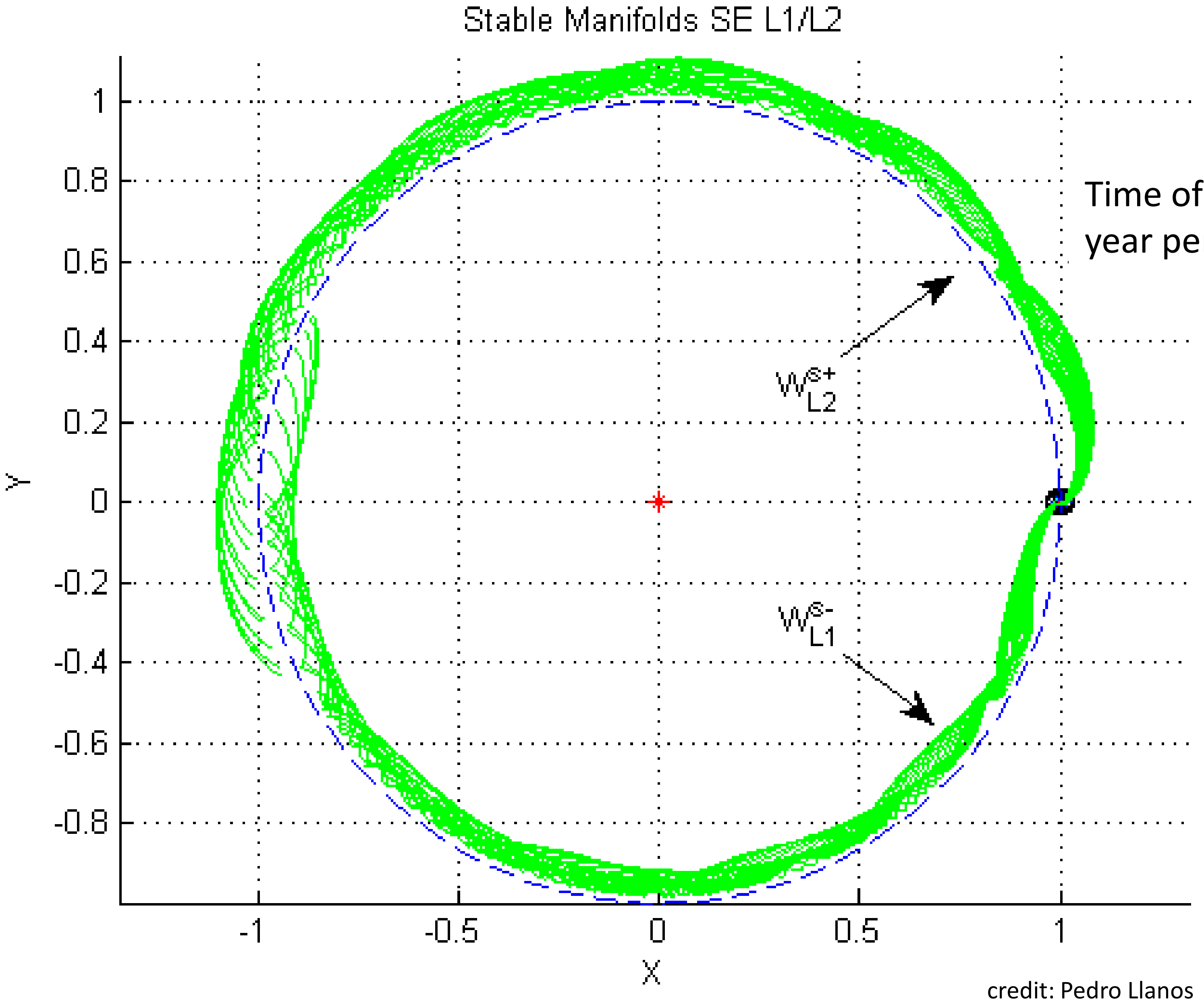
Trajectory design for Asteroid Retrieval Mission

1) Earth - NEA leg: pure low-thrust
2) NEA - Lagrange point: how?



Time of flight: one year per bounce

credit: Pedro Llanos



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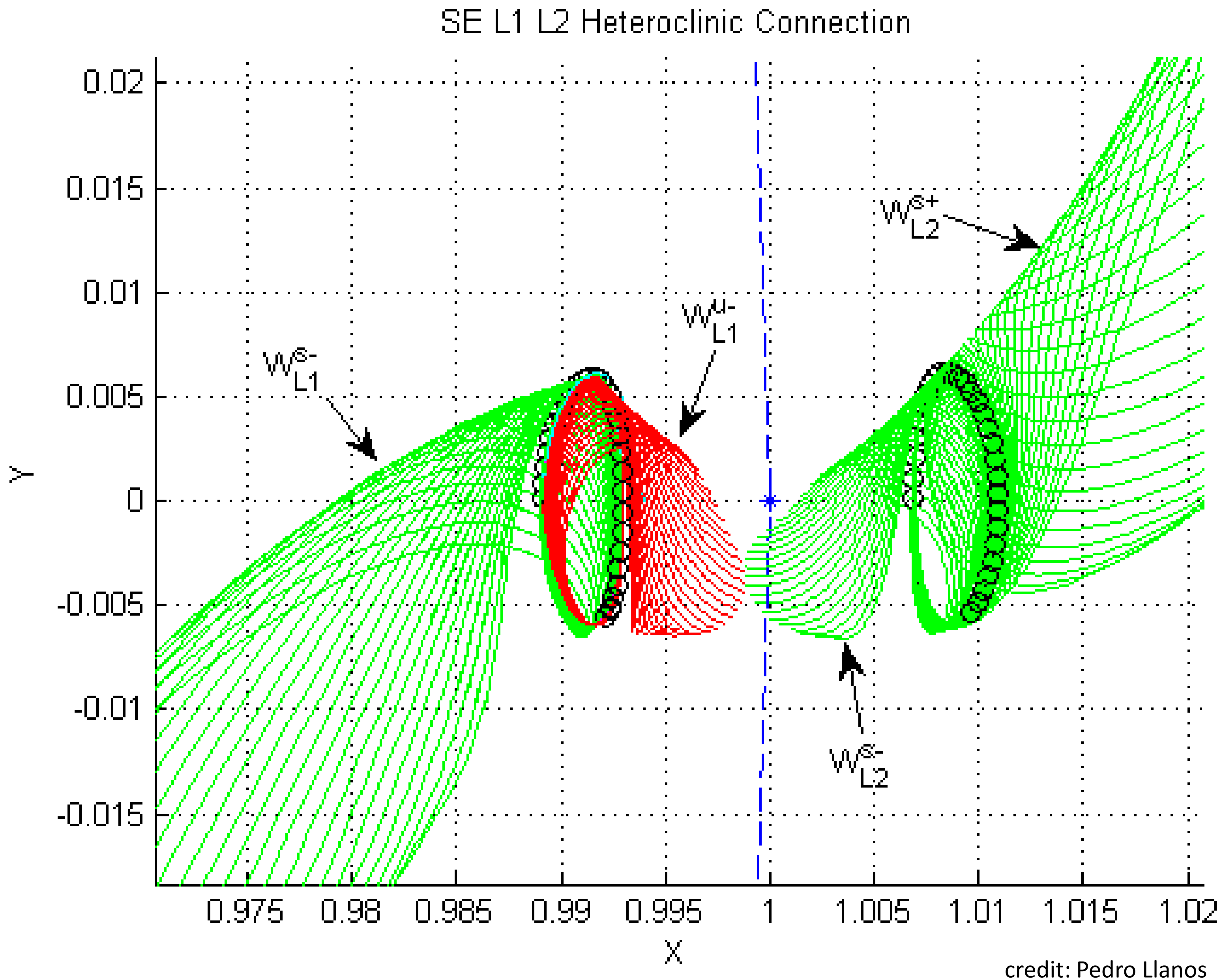
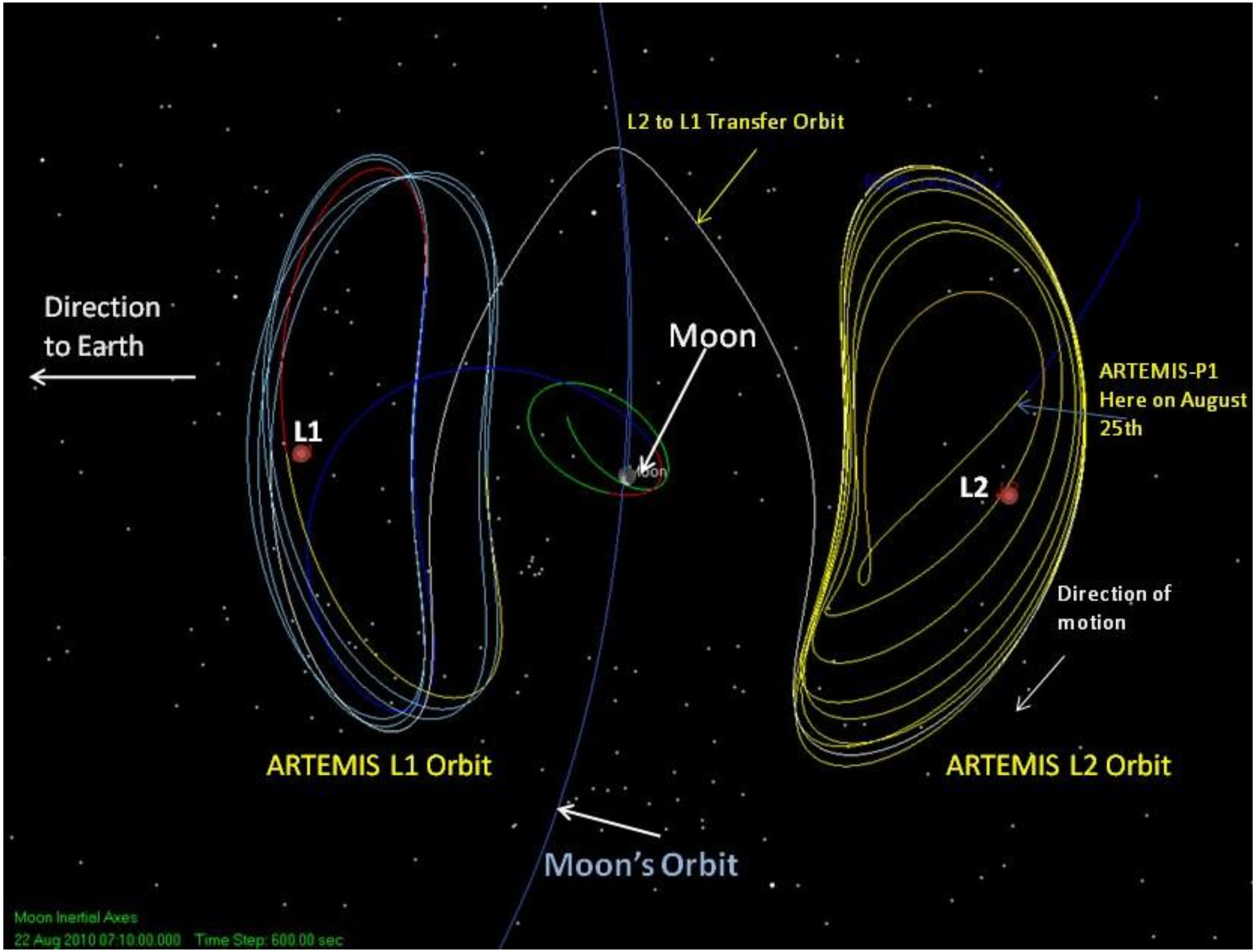


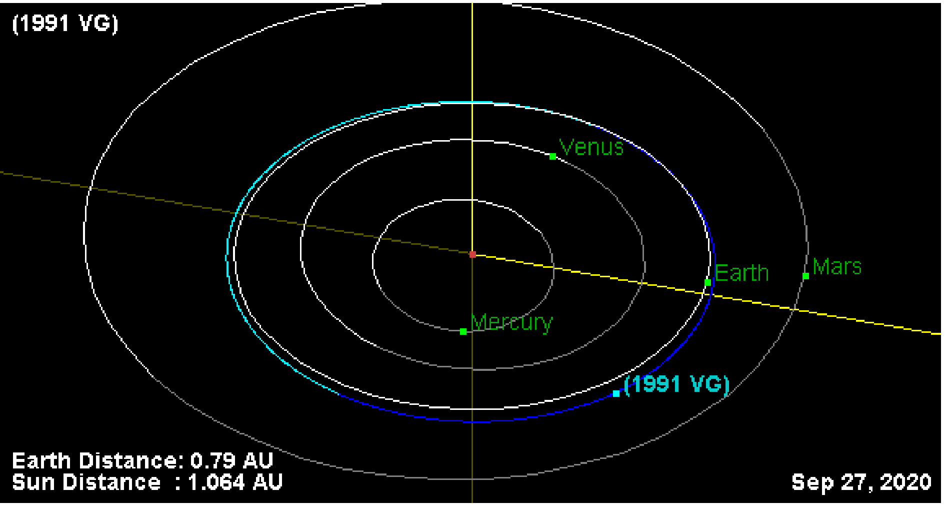
credit: Pedro Llanos

Connection between L1 and L2



credit: NASA GFSC

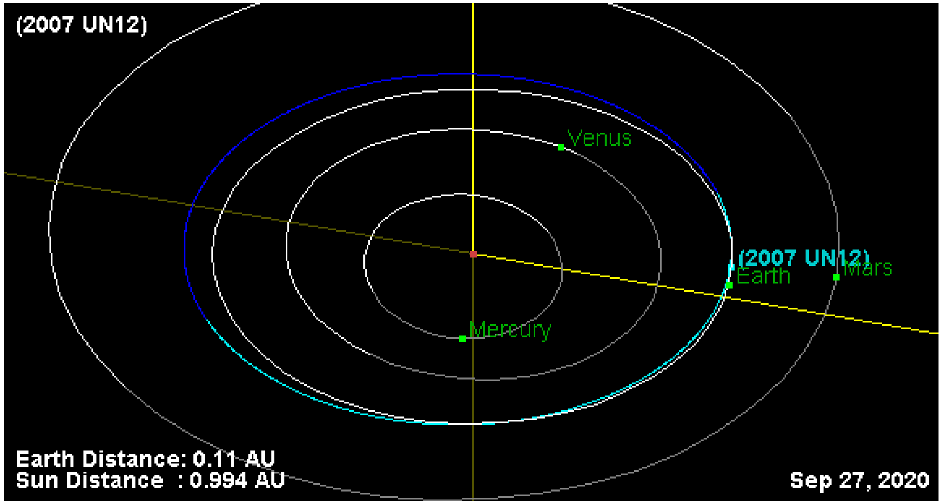
1991 VG



Earth Distance: 0.79 AU
Sun Distance : 1.064 AU

Sep 27, 2020

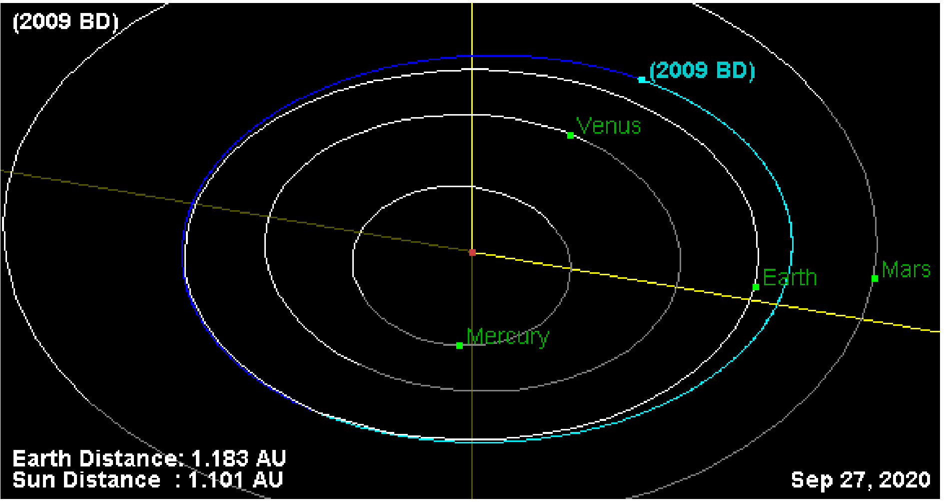
2007 UN12



Earth Distance: 0.11 AU
Sun Distance : 0.994 AU

Sep 27, 2020

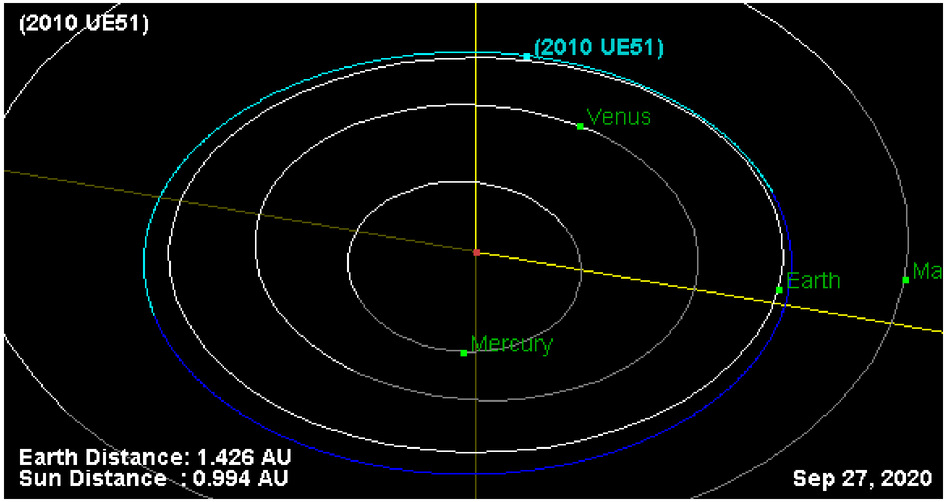
2009 BD



Earth Distance: 1.183 AU
Sun Distance : 1.101 AU

Sep 27, 2020

2010 UE51



Earth Distance: 1.426 AU
Sun Distance : 0.994 AU

Sep 27, 2020

ssd.jpl.nasa.gov

Combining low-thrust + invariant manifolds has been studied

G. Mingotti, F. Topputo, F. Bernelli-Zazzera (2006)

GTO -> thrust arc -> stable manifold -> halo orbit

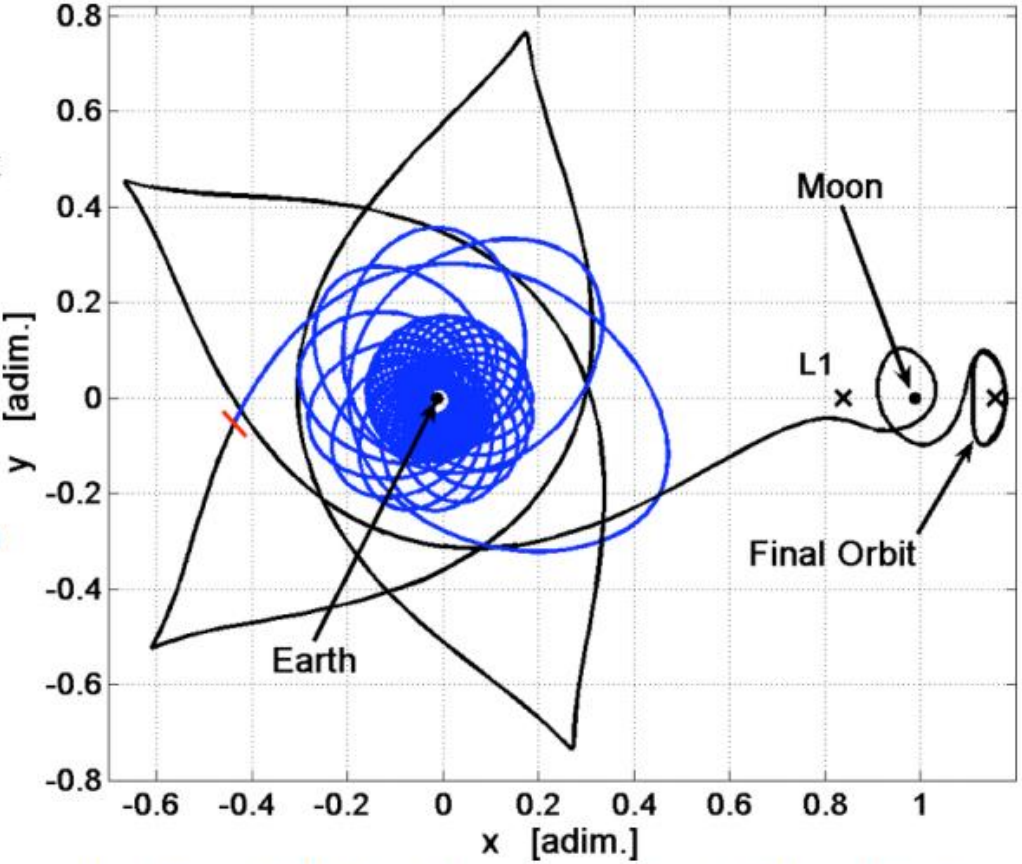
- GTO:

  chosen as the starting orbit
- thrust arc:

  solution of an optimal control problem
- stable manifold:

  associated to the final halo
- halo orbit:

  computed through the Richardson method [1980]



G. Mingotti, F. Topputo, and F. Bernelli-Zazzera - Low Thrust, Stable Manifold Trajectories to the Earth-Moon Halo Orbits

IDEA for Asteroid Retrieval

NEA -> thrust arc -> stable manifold -> target Lagrange point (SE L1/L2)

Why study SE L1/L2 first? Because if the NEA is in SE L1/L2 then it can be moved to EM L1/L2 through invariant manifolds (EM CR3BP)

If the final destination is a Lagrange point, low-thrust + invariant manifolds might be more energy efficient than pure low-thrust. Thus, for a selected NEA, we might need smaller SEP.