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What geodesy gave us in the Earth-Moon system

KISS Next-Generation Planetary Geodesy Workshop

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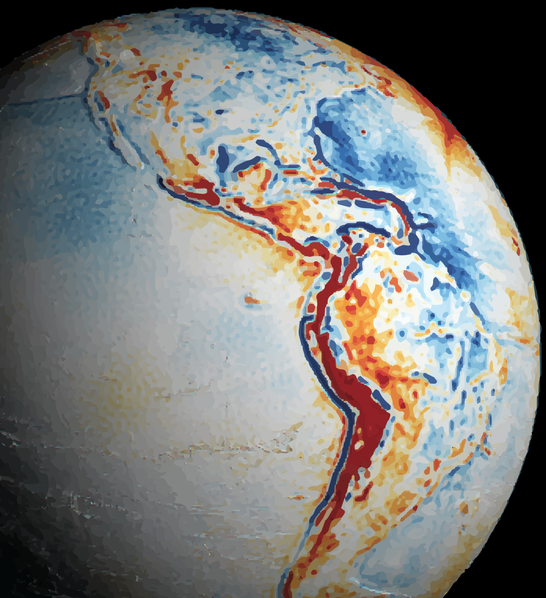
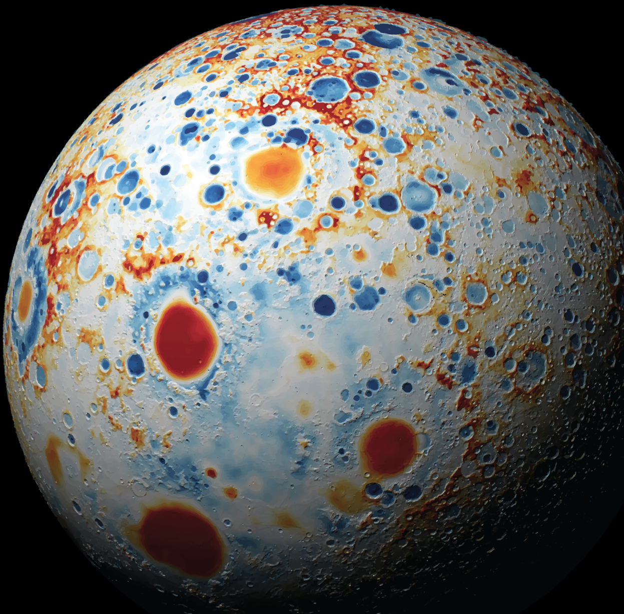
UC Berkeley

2 June 2021





Planetary Geodesy

measurement science & branch of geophysics

What: The study of a planet's shape, orientation, and gravity field.

How:

- Historically - measurements of distances and angles
- Now - Radio and laser ranging of planets, moons and spacecraft; gravimetry; gradiometry, stereo-imaging, etc.

Why: Study planetary dynamics and interior, establishment of reference frames, navigation.

Geodesy frequencies and scales

Table 3 Geophysical processes that affect geodetic observations as a function of spatial and temporal scale

Scale	*Temporal*				
Spatial	*10^{-2}–10^{3} s*	*10^{0}–10^{1} h*	*10^{0}–10^{2} day*	*10^{0}–10^{2} year*	*10^{2}–10^{6} year*
10^{0}–10^{1} km	Coseismic rupture Volcanism	Creep events Volcanism	Afterslip Poroelastic relaxation Dike injection	Viscoelastic relaxation Interseismic strain	Earthquake cycle
10^{1}–10^{2} km	M 6–7.5 seismic strain release Tropospheric moisture	Storm-surge loading Tsunami loading Tropospheric moisture	Rifting events Aquifer deformation Poroelastic relaxation Lower crustal magmatism Lake loading Snow loading	Viscoelastic relaxation Block rotation Strain partition Mountain growth Glacial loading Sedimentary loading	Fault activation and evolution Mountain range building Denudation Regional topography Sedimentary loading
10^{2}–10^{3} km	M 7.5–9 seismic strain release Traveling ionospheric disturbances Seismic waves	Coastal ocean loading	Atmospheric loading Regional hydrologic loading	Mantle–crust coupling Ice-sheet loading	Plateau rise Mountain range building Glacial cycle
10^{3}–10^{4} km	M 9+ seismic strain release Seismic waves Free oscillations	Earth tides Tidal loading	Seasonal fluid transport Ocean bottom pressure	Core–mantle coupling Climate change Solar cycle	Plate rotations Mantle flow Continental evolution

Blewitt, 2015

Outline: 5 case problems

1. How we figured out the shape of the Earth and what it told us about its interior
2. How we figured out that the Earth crust is in the state of isostasy
3. How geodesy helped us to accept and measure plate tectonics
4. How we constrained the loss of the Greenland and Antarctic ice sheets from the GRACE observations
5. How GRAIL helped us understand the mechanism of lunar mascon formation

Some math background

Spherical Harmonics

- **Shape**

$$r(\phi,\lambda) = R_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_{nm} Y_{nm}(\phi,\lambda)$$

- **Gravitational potential**

$$U(r,\phi,\lambda) = \frac{GM}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left(\frac{R_0}{r}\right)^n C_{nm} Y_{nm}(\phi,\lambda)$$

U **– gravitational potential**
φ **– latitude**
λ **– longitude**
r **– radial distance**
n **– degree ≈ $2\pi r$/wavelength**
m **– order**

Spherical Harmonics

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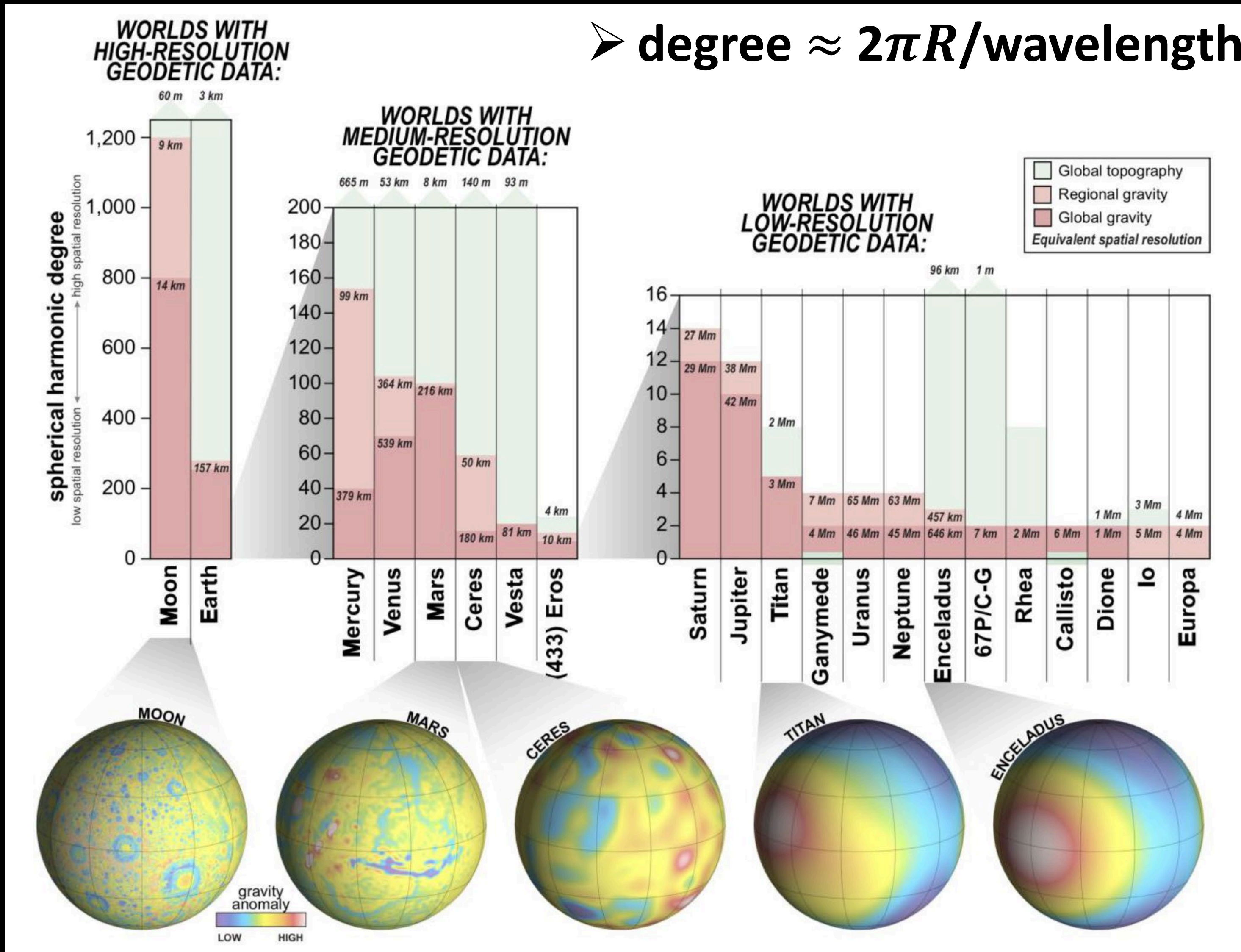
Gravity and shape data in the Solar System

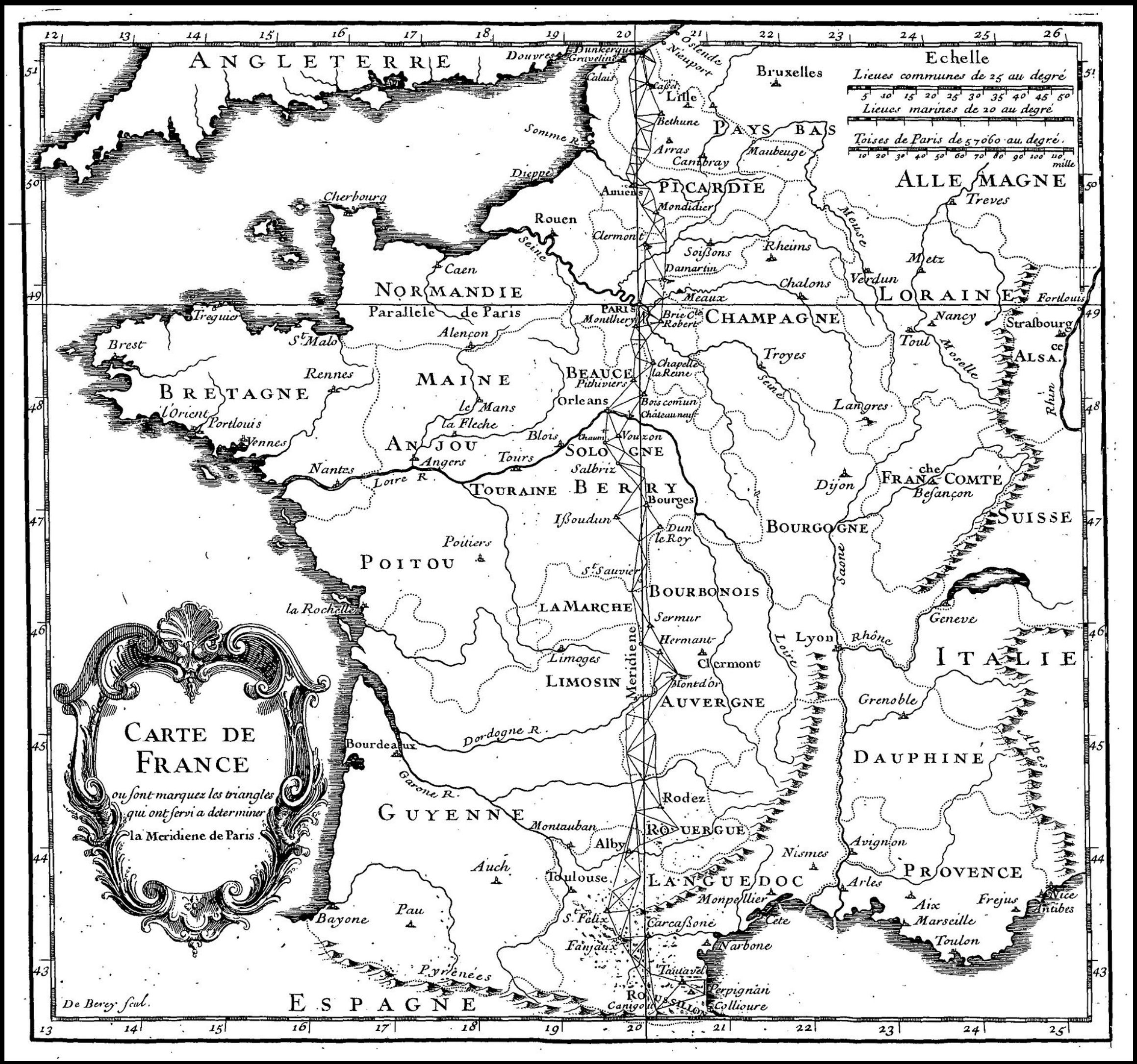
➤ **degree ≈ $2\pi R$/wavelength**



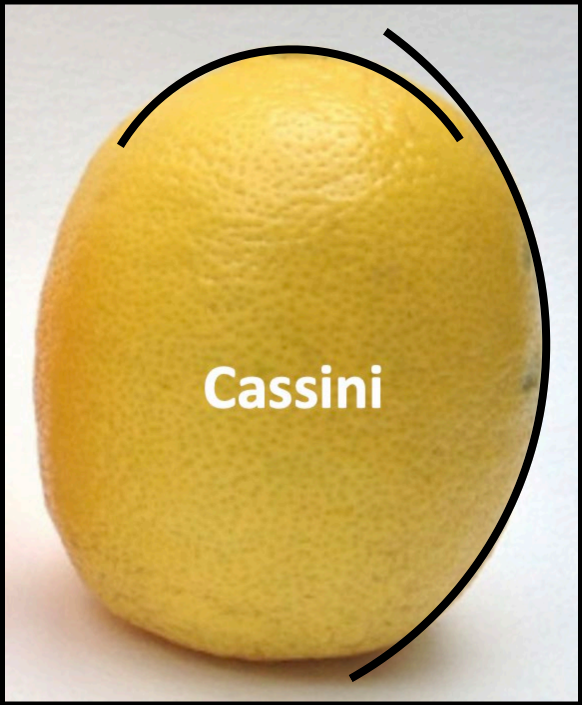
Sori et al., 2020

How we figured out the shape of the Earth and what it told us about its interior

Cassini's measurements

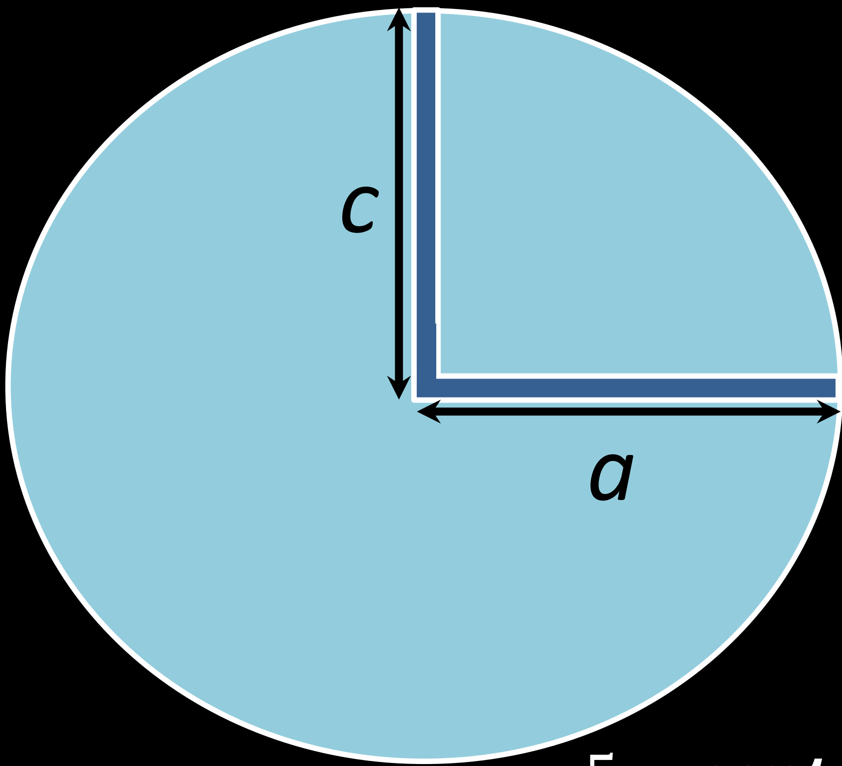


J. Cassini, 1720



- Cassinis determined that Earth's shape curvature increases toward the pole
  => Earth is a prolate ellipsoid

Shape of the Earth according to Newton



Newton's idea:

Weight of polar column

=

Weight of equatorial column

$$f = \frac{a - c}{a} > 0$$

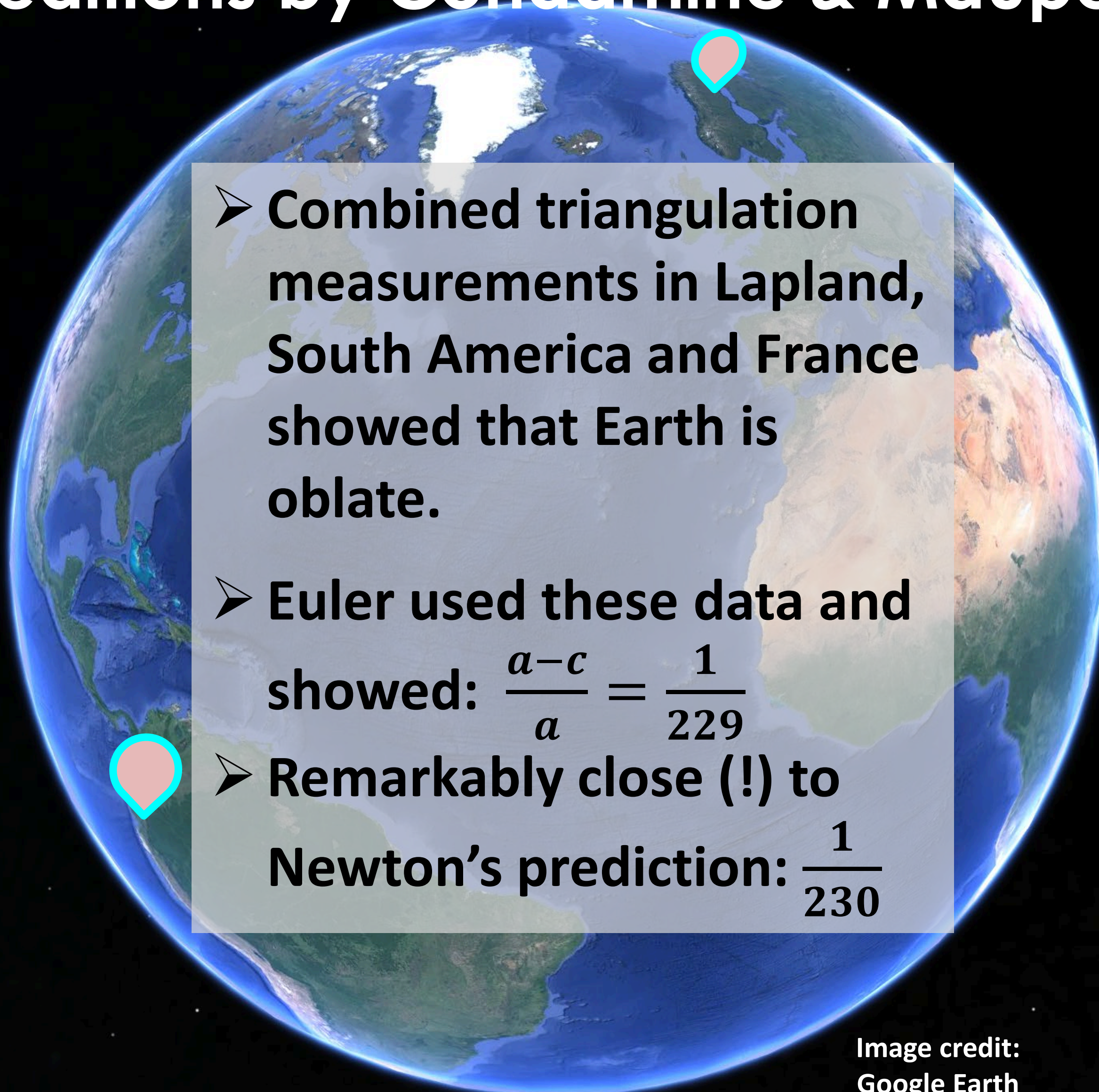
$$f = \frac{5}{4} \times \frac{\textit{centrifugal acceleration at equator}}{\textit{mean gravity over the surface}}$$

Newton's result: $f = \frac{5}{4} \times \frac{1}{290} = \frac{1}{230}$

Shape of the Earth according to Newton

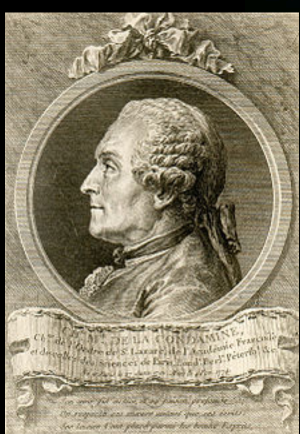
- Planetary materials behave as liquids over geologic time scale
- Hydrostatic equilibrium:
  gravity + *pressure gradient* + *centrifugal force* = *ZERO*
- Surfaces of constant density, pressure and potential coincide

Expeditions by Condamine & Maupertuis



- Combined triangulation measurements in Lapland, South America and France showed that Earth is oblate.
- Euler used these data and showed: $\frac{a-c}{a} = \frac{1}{229}$
- Remarkably close (!) to Newton’s prediction: $\frac{1}{230}$

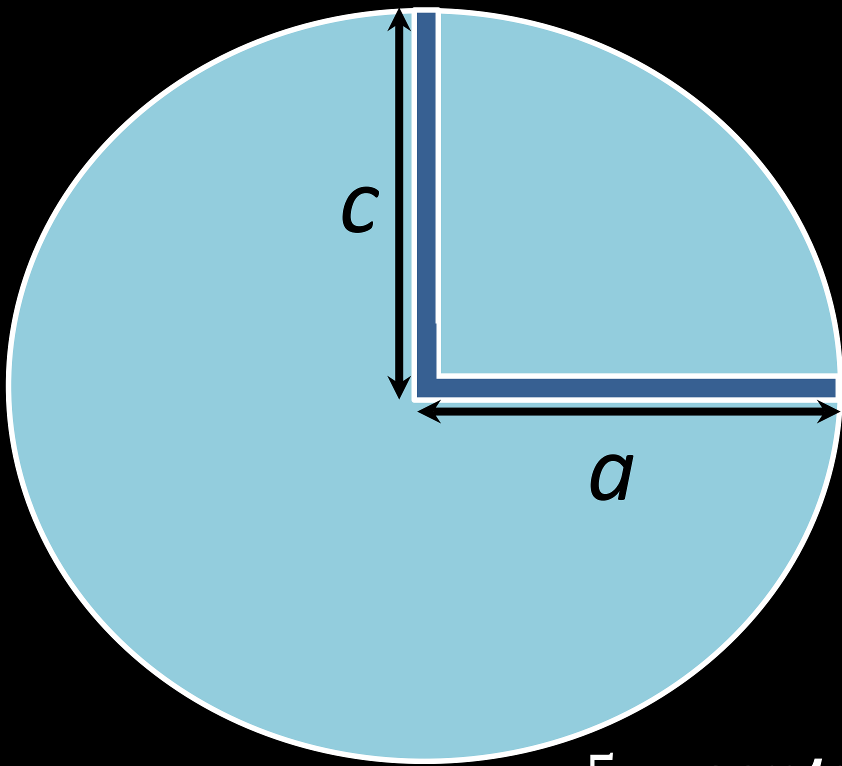
Pierre Louis Maupertuis



Charles Marie de La Condamine

Image credit: Google Earth

Shape of the Earth according to Newton



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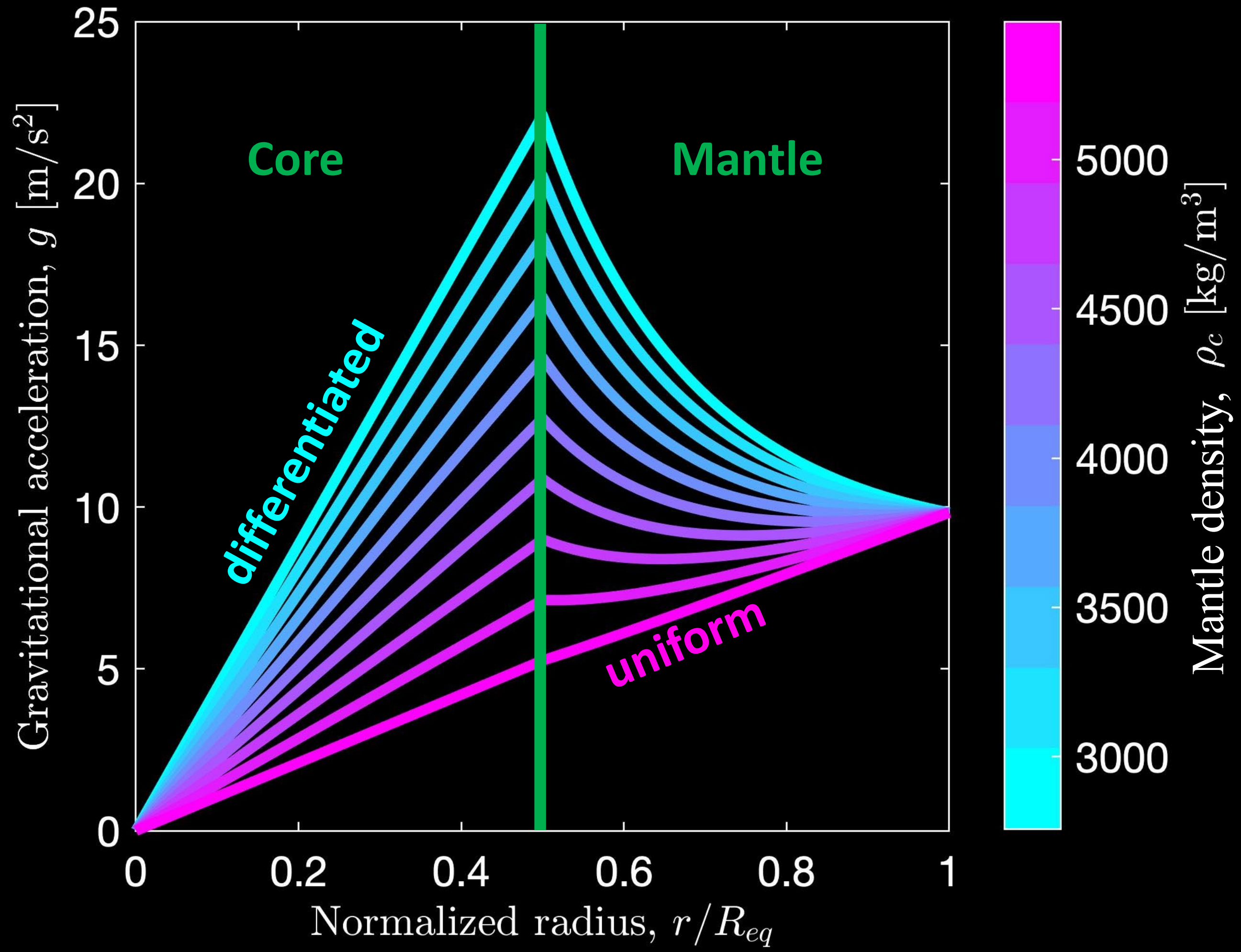
Newton's result: $f = \frac{5}{4} \times \frac{1}{290} = \frac{1}{230}$

Modern observation: $f = \frac{1}{297}$

see Chandrasekhar (1967) for more details

Was Newton wrong?

Not really… Newton assumed the Earth is homogeneous



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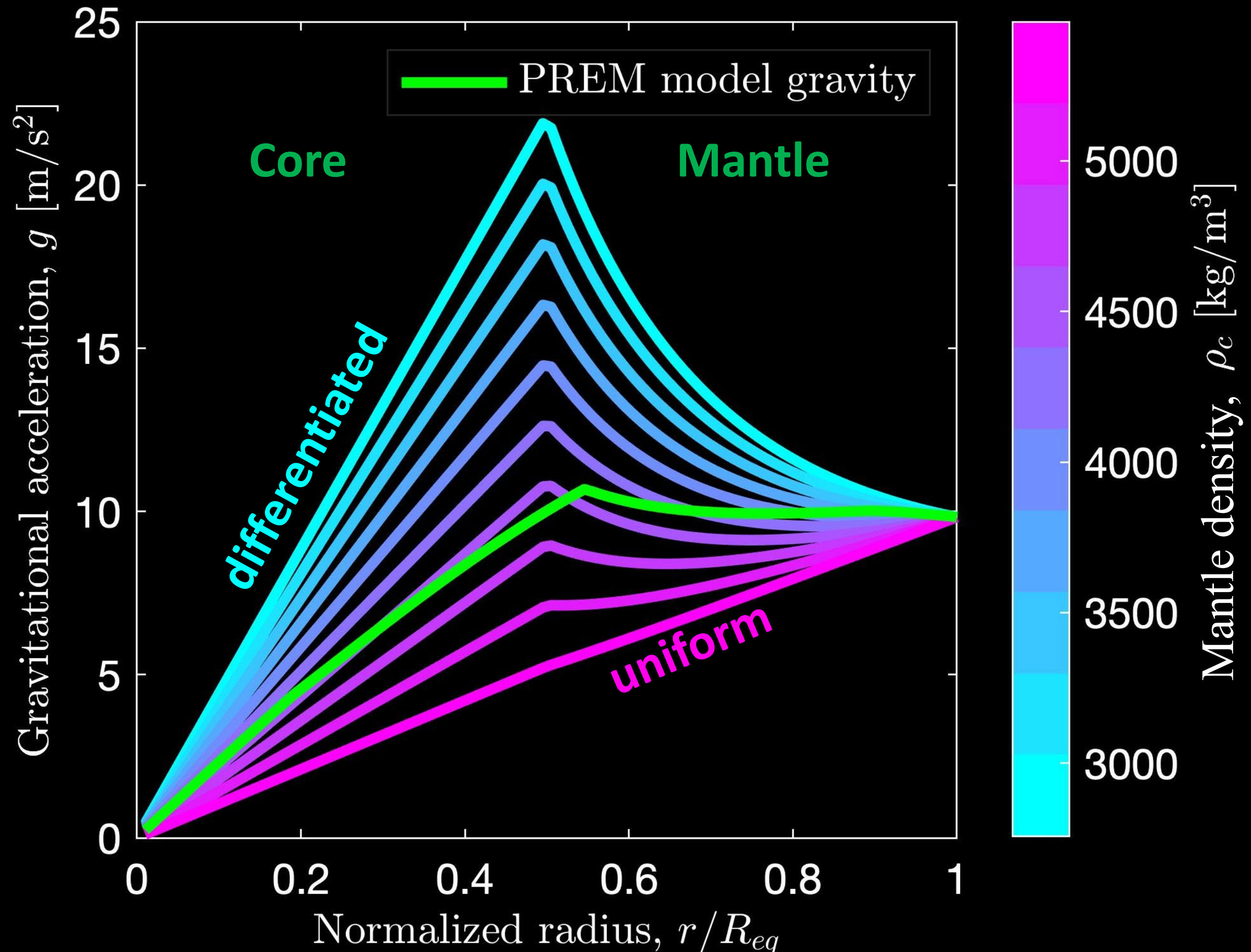
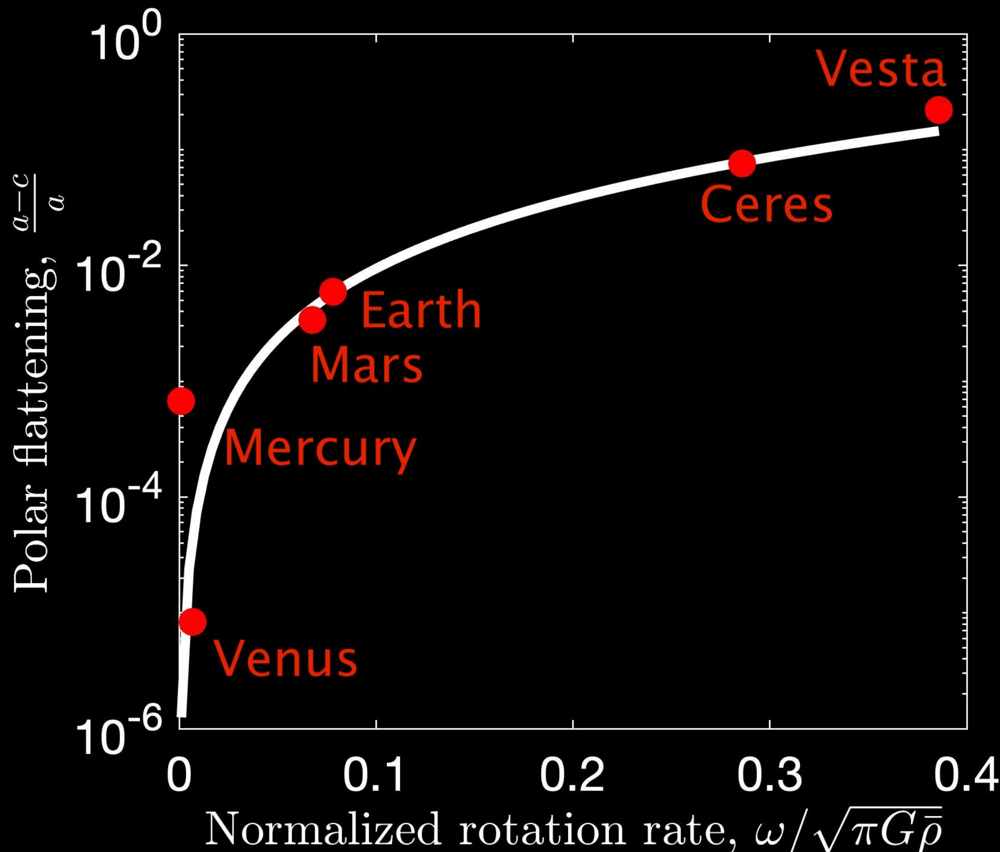
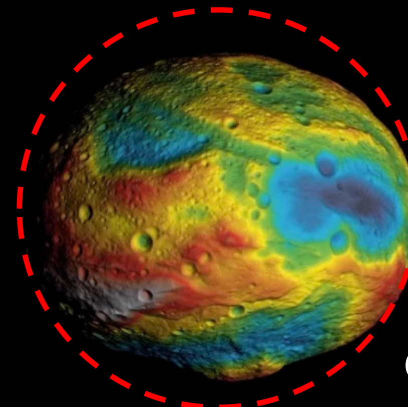


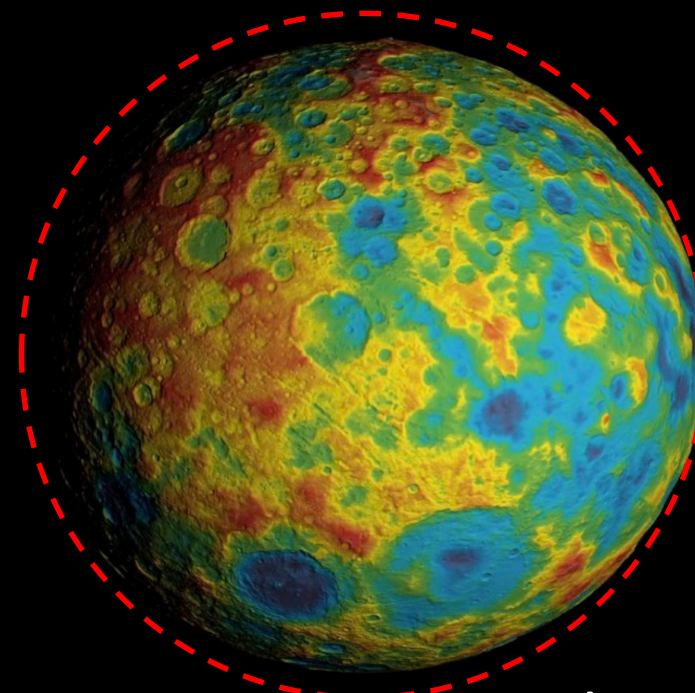
Effect of rotation and density structure on the Earth's shape

Shape response to rotation





Gaskell, 2012



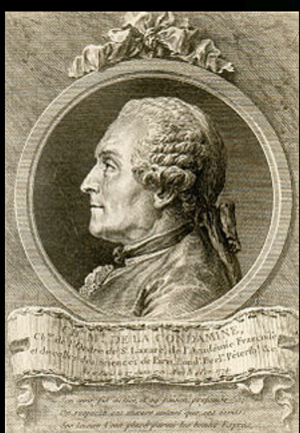
Park et al., 2019

Useful references for computing hydrostatic equilibrium

- **Easy hydrostatic equilibrium for a two-layer body:**
  - *Dermott, S. F. (1979). Shapes and gravitational moments of satellites and asteroids. Icarus, 37(3), 575-586.*
- **Easy hydrostatic equilibrium using ellipsoidal approximation:**
  - *Tricarico, P. (2014). Multi-layer hydrostatic equilibrium of planets and synchronous moons: theory and application to Ceres and to solar system moons. The Astrophysical Journal, 782(2), 99.*
- **Harder, more complete treatment of theory of figures:**
  - *Zharkov, V. N., & Trubitsyn, V. P. (1978). Physics of planetary interiors. Astronomy and Astrophysics Series*
- **Numerical (non-perturbative) way of computing hydrostatic equilibrium**:
  - *Hubbard, W. B. (2013). Concentric Maclaurin spheroid models of rotating liquid planets. The Astrophysical Journal, 768(1), 43.*
  - *Militzer, B., Wahl, S., & Hubbard, W. B. (2019). Models of Saturn's interior constructed with an accelerated concentric Maclaurin spheroid method. The Astrophysical Journal, 879(2), 78.*

Expeditions by Condamine & Maupertuis

Pierre Bouguer



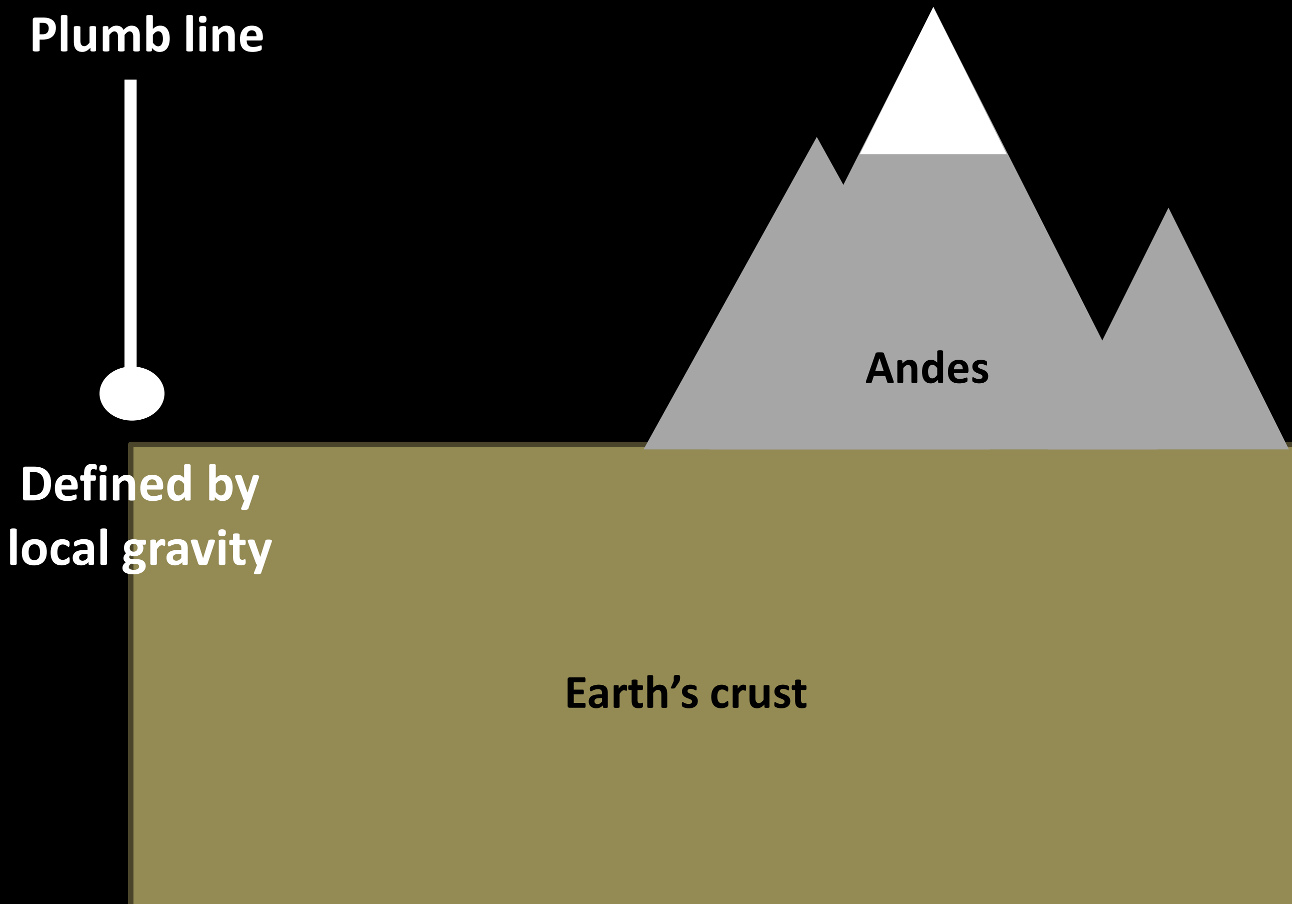
Charles Marie de La Condamine

Image credit: Google Earth

Pierre Louis Maupertuis

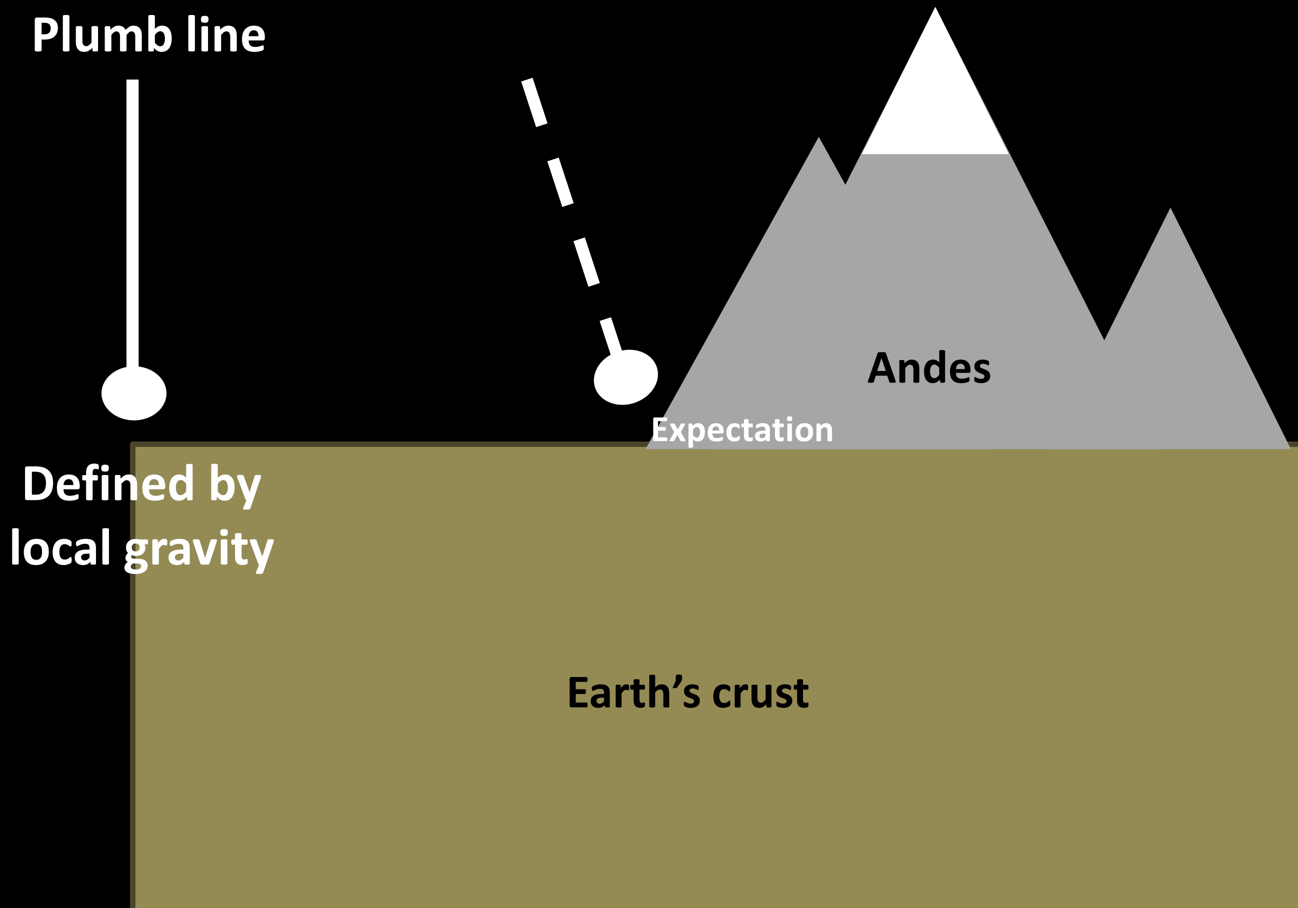
Bouguer measurements

Pierre Bouguer



Bouguer measurements

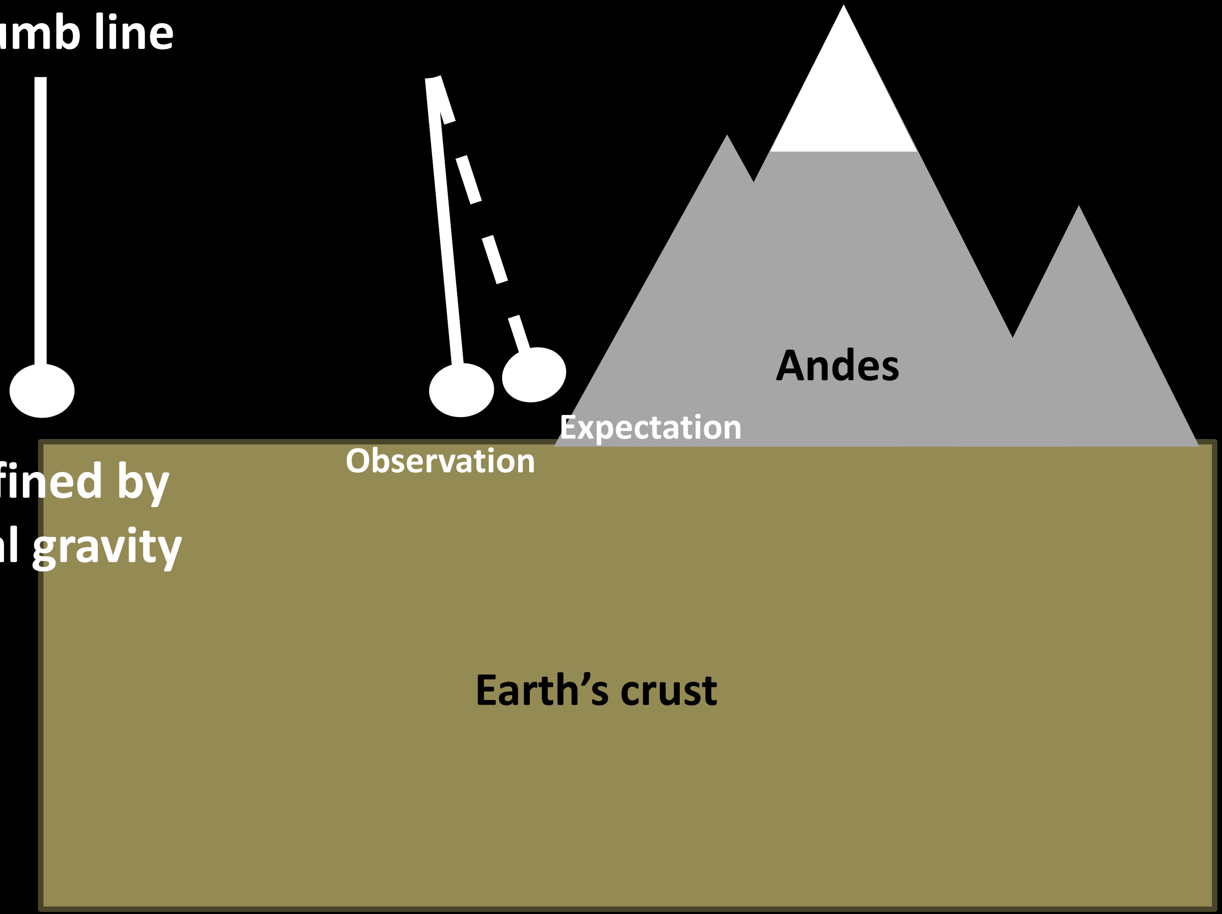
Pierre Bouguer



Bouguer measurements

Pierre Bouguer

Plumb line



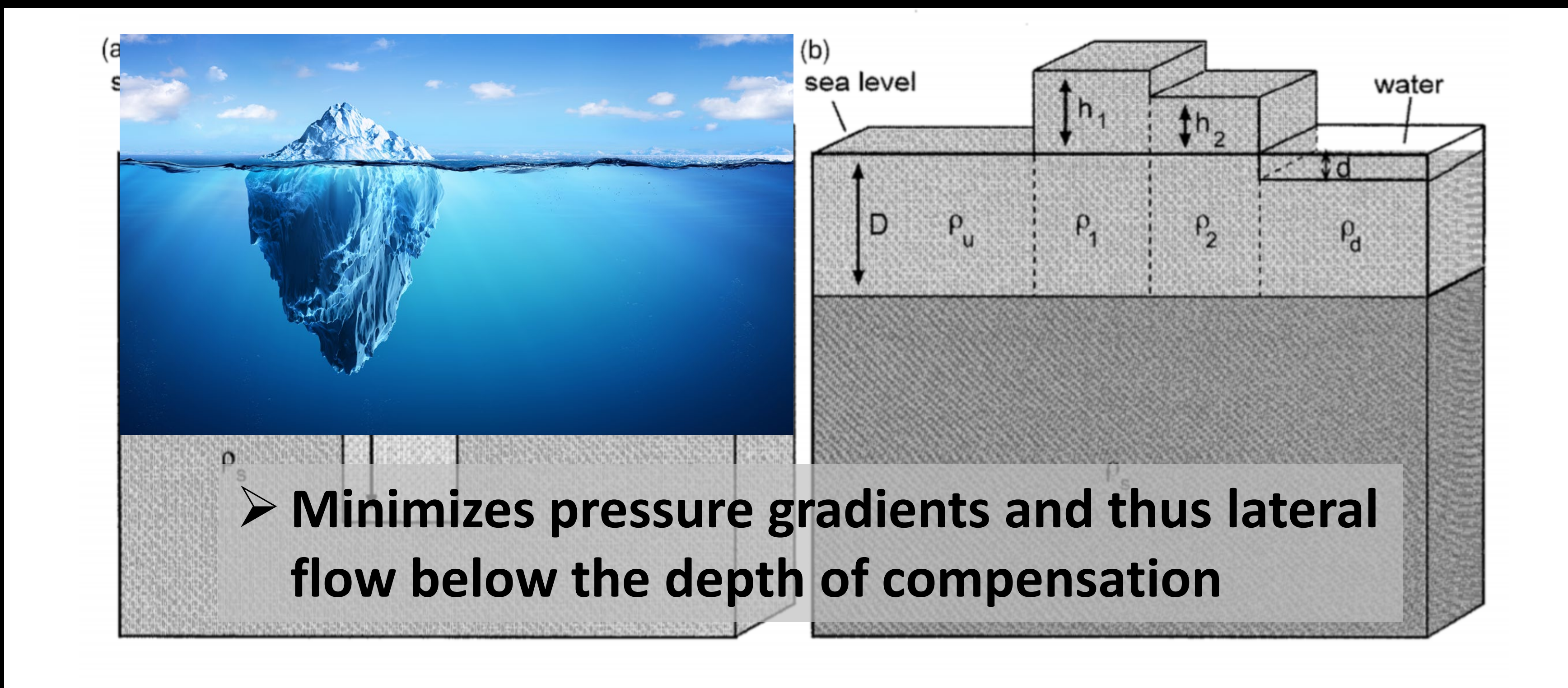
Defined by local gravity

Two models of isostasy

Definition: the equilibrium that exists between parts of the Earth's crust, which behaves as if it consists of blocks floating on the underlying mantle

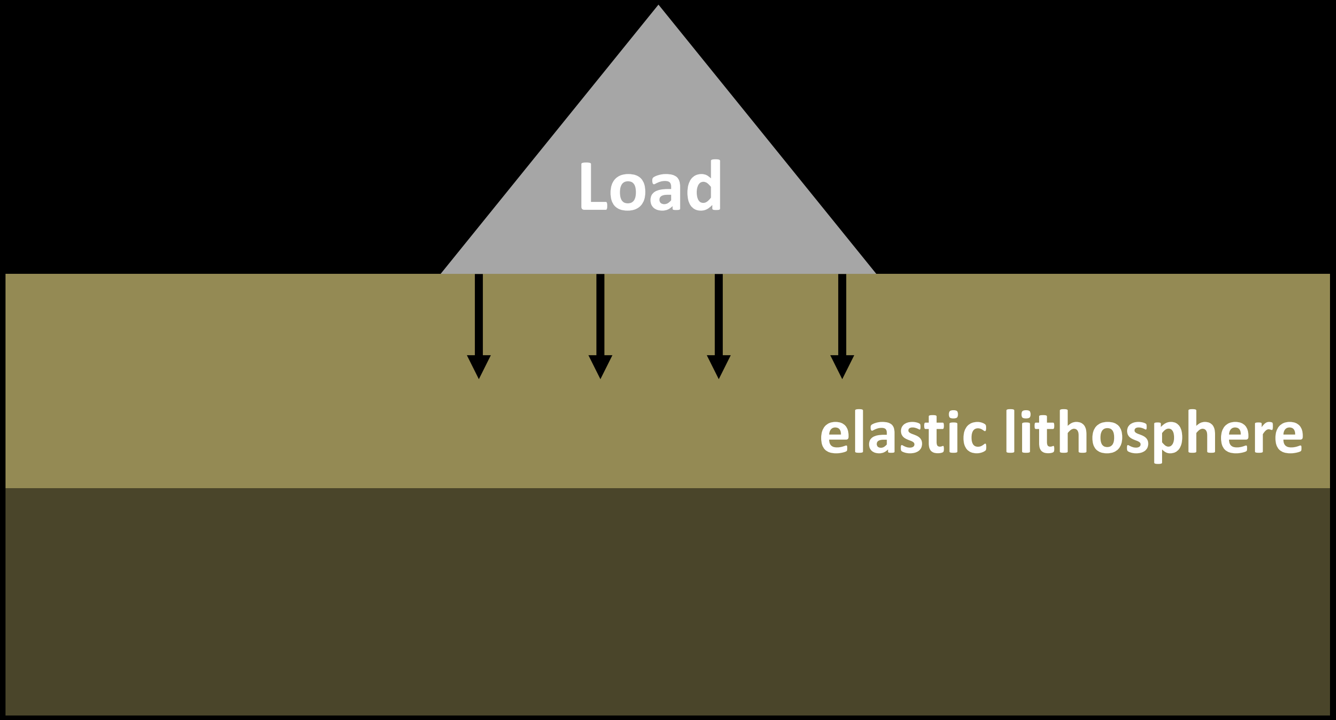
Airy isostasy

Pratt isostasy

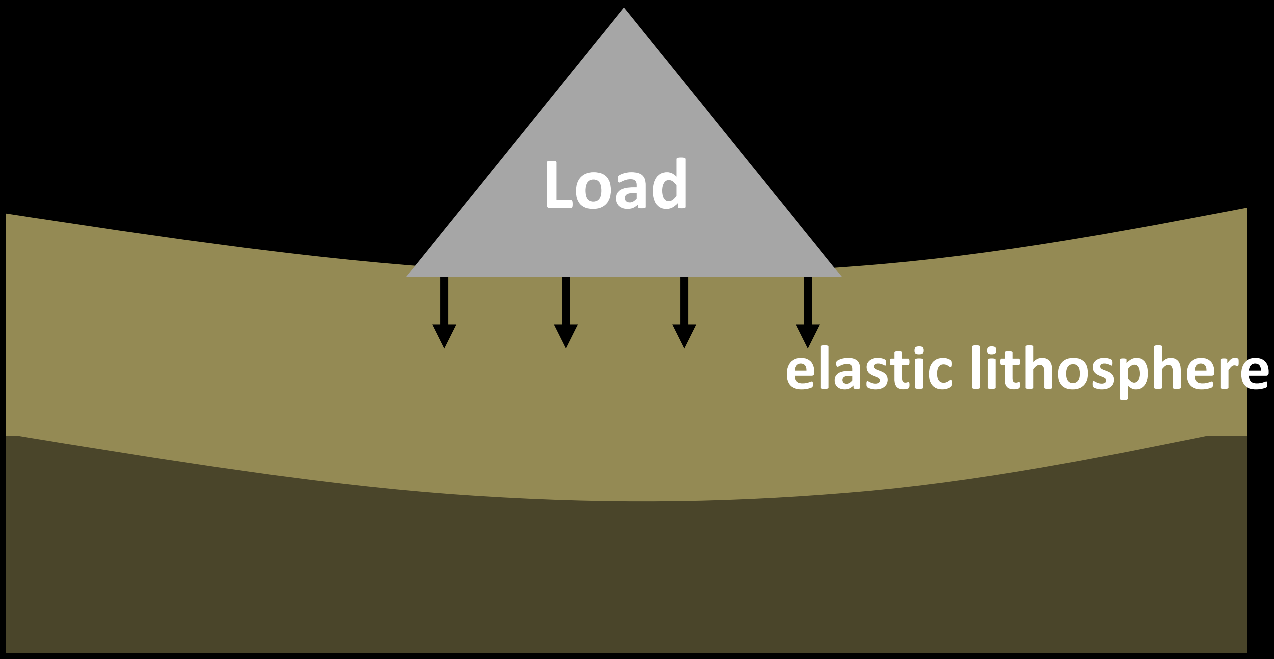


- ➢ **Minimizes pressure gradients and thus lateral flow below the depth of compensation**

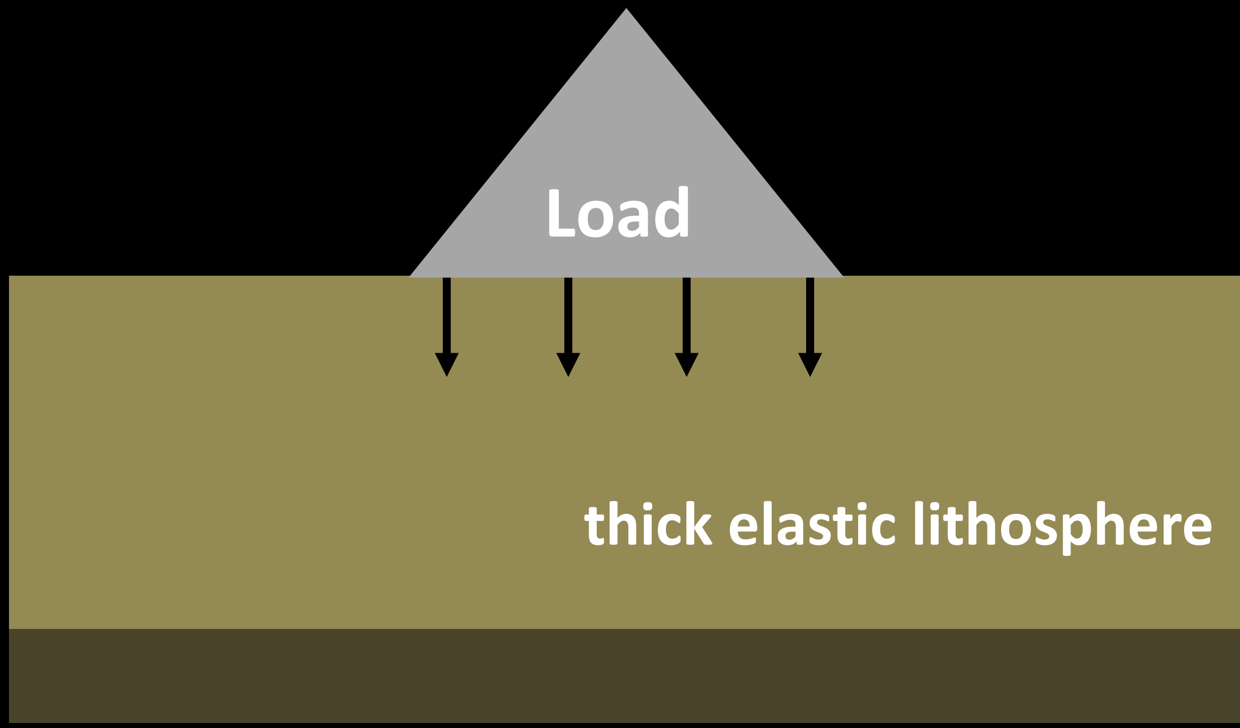
Flexure



Flexure

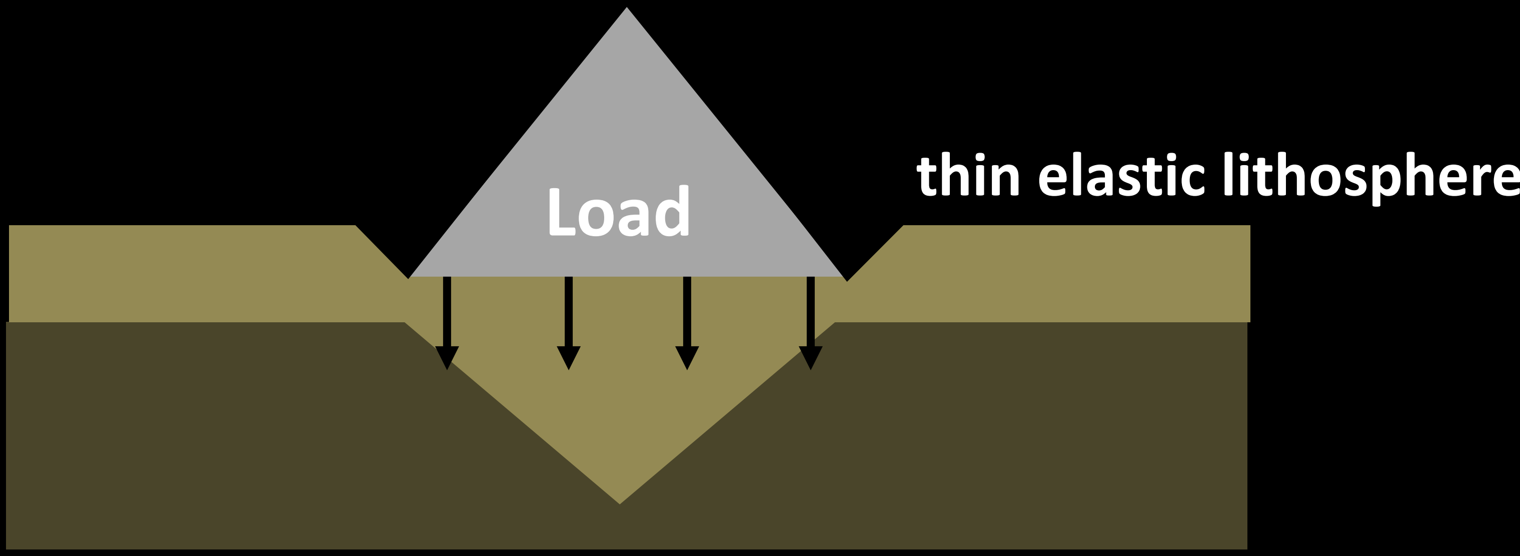


Flexure



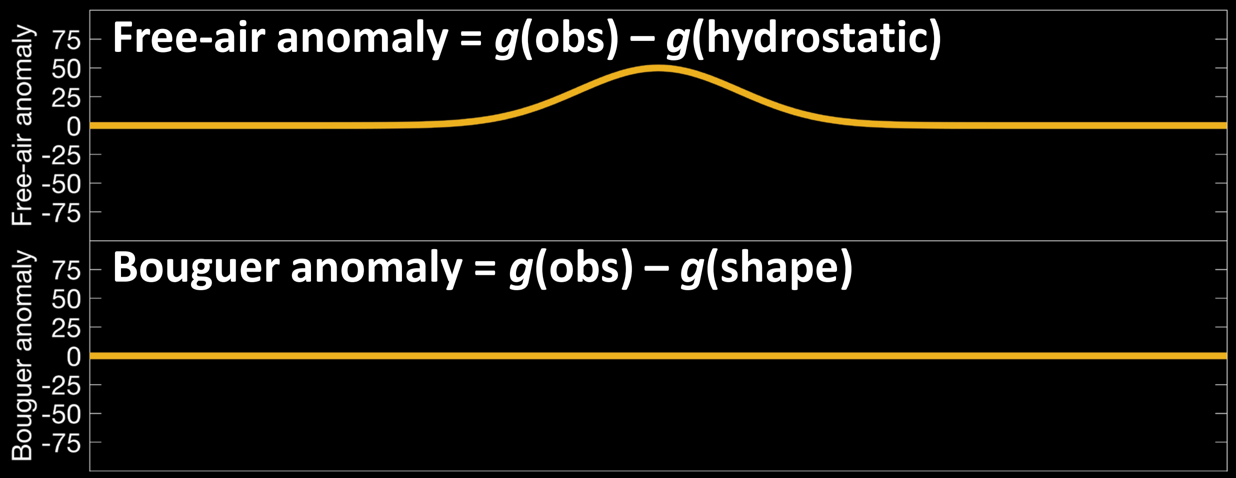
- Thick elastic lithosphere => uncompensated limit

Flexure



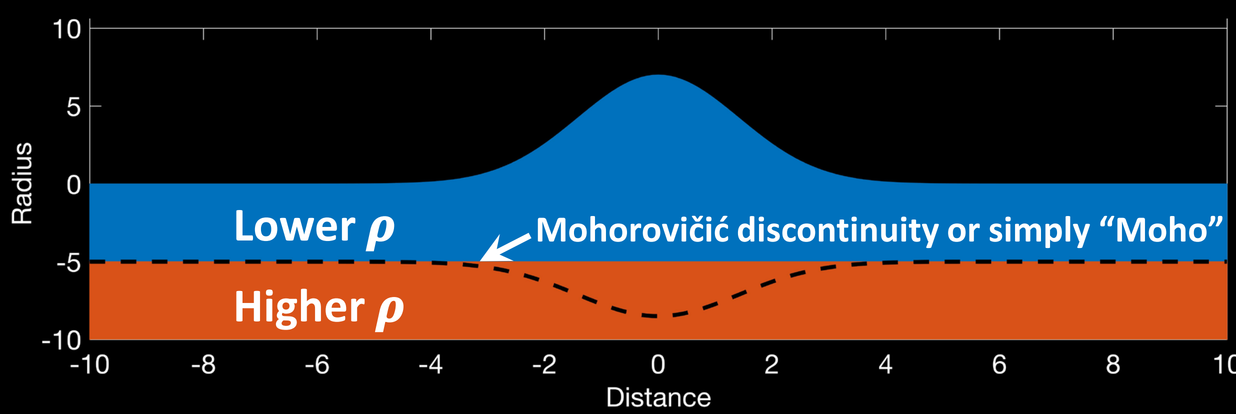
- Thick elastic lithosphere => uncompensated limit
- Thin elastic lithosphere => isostatic compensation limit

Example: uncompensated topography

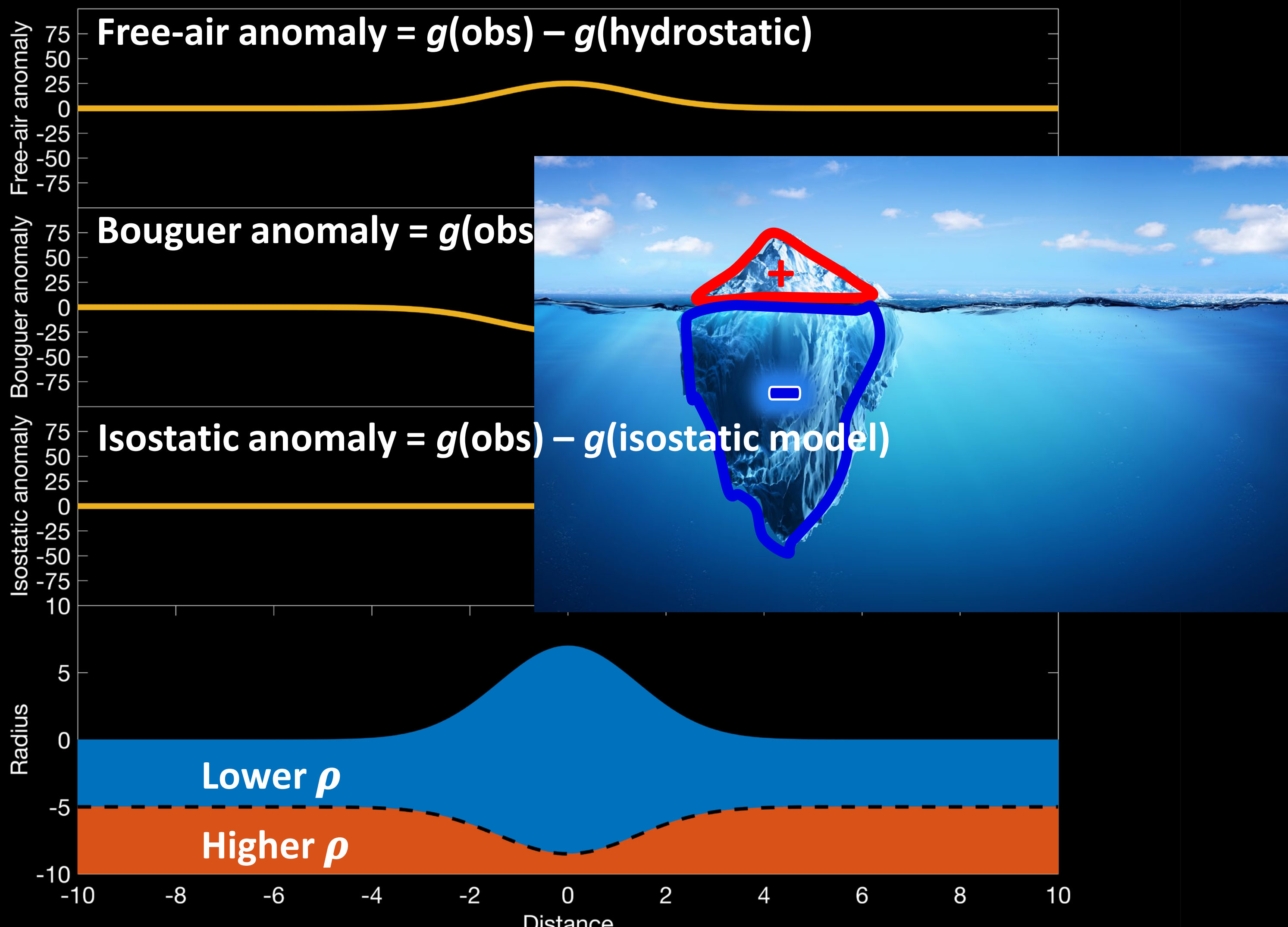


Gravitational acceleration unit = Gal = 0.01 m/s^2 = 1 cm/s^2

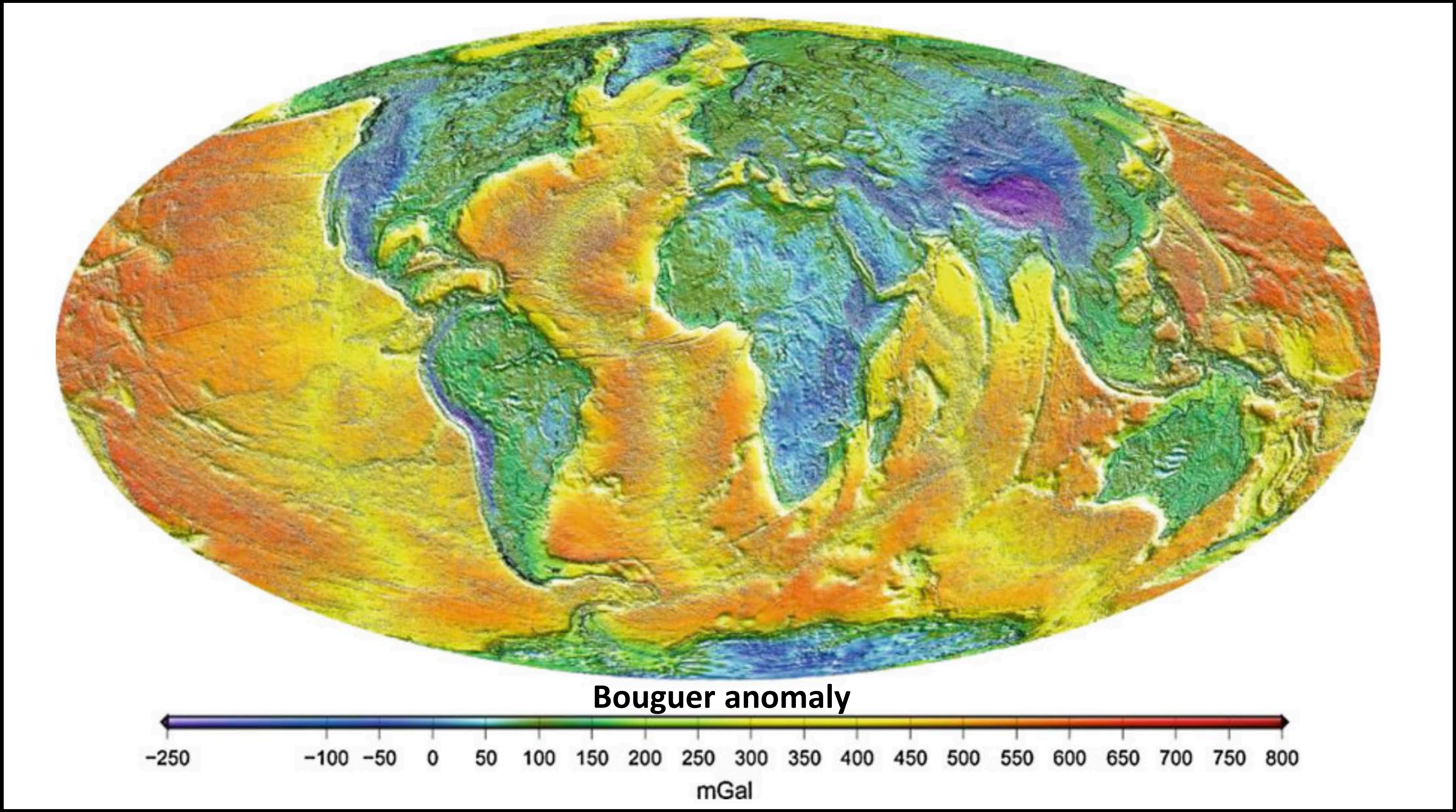
mGal = 10^{-5} m/s^2



Example: compensated topography



Earth's Bouguer gravity anomaly



Balmino 2012

Earth's geoid

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  equi
  (a.k.
- Red :
- Blue
- Noti
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Image Cre

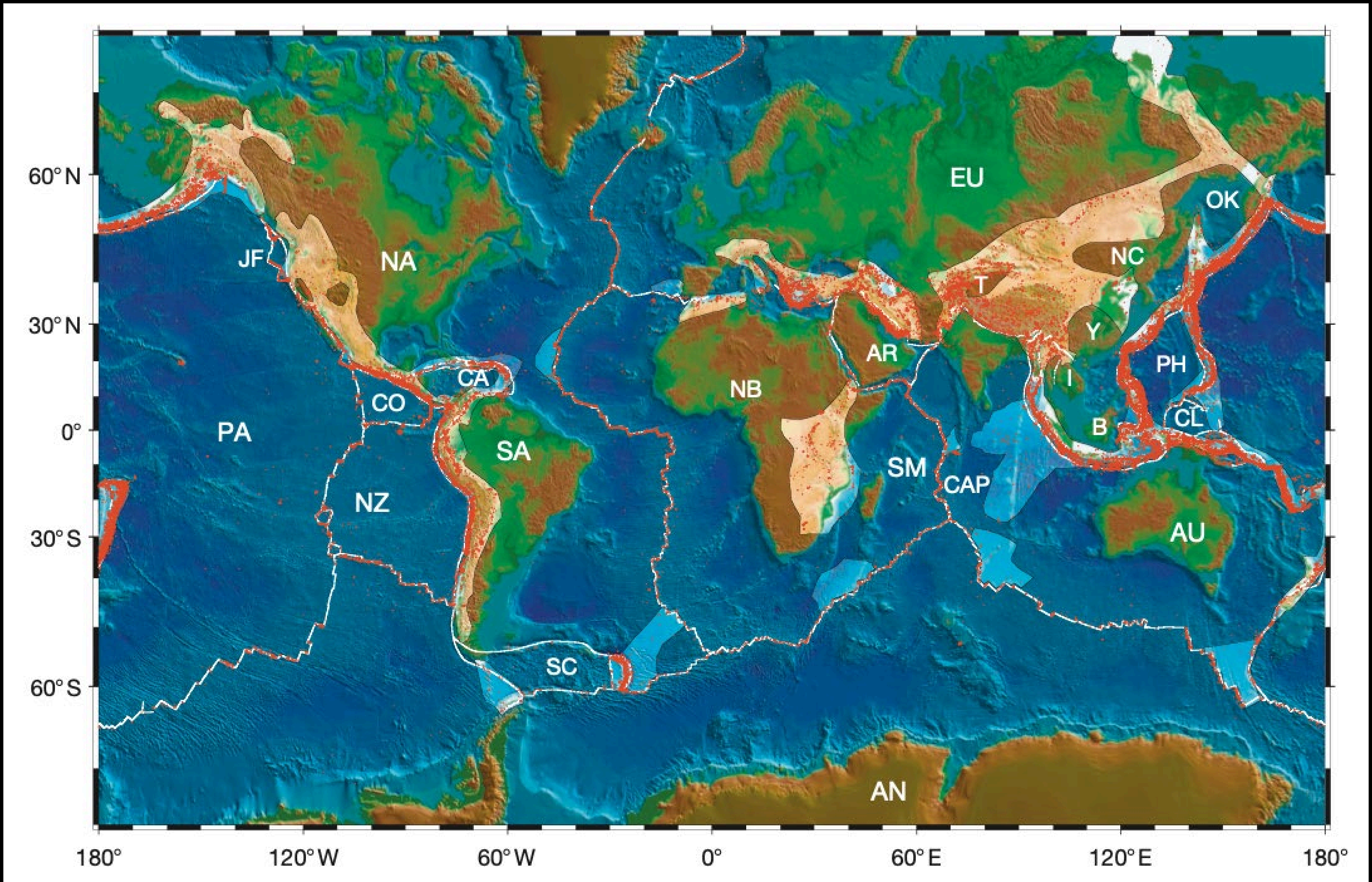
Useful references for isostasy and flexure

- **A very good review**
  - Watts, A. B. (2001). *Isostasy and Flexure of the Lithosphere*. Cambridge University Press

- **Isostasy and flexure on a sphere:**
  - *Turcotte, D. L., Willemann, R. J., Haxby, W. F., & Norberry, J. (1981). Role of membrane stresses in the support of planetary topography. Journal of Geophysical Research: Solid Earth, 86(B5), 3951-3959*
  - *Dahlen, F. A. (1982). Isostatic geoid anomalies on a sphere. Journal of Geophysical Research: Solid Earth, 87(B5), 3943-3947*

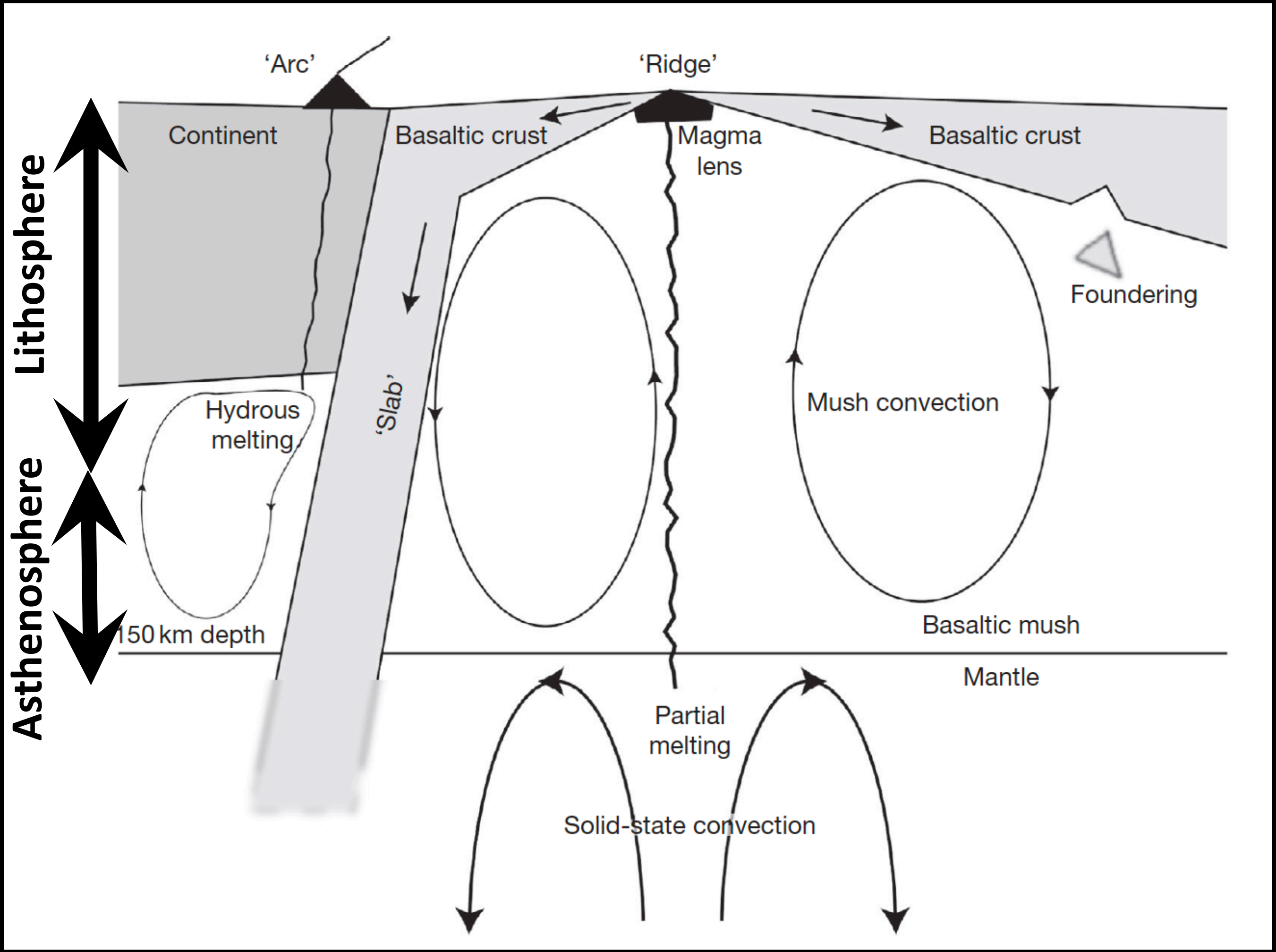
- **Isostasy for the icy shells of ocean worlds**
  - *Čadek, O., Souček, O., & Běhounková, M. (2019). Is Airy Isostasy Applicable to Icy Moons?. Geophysical Research Letters, 46(24), 14299-14306.*

Plate tectonics



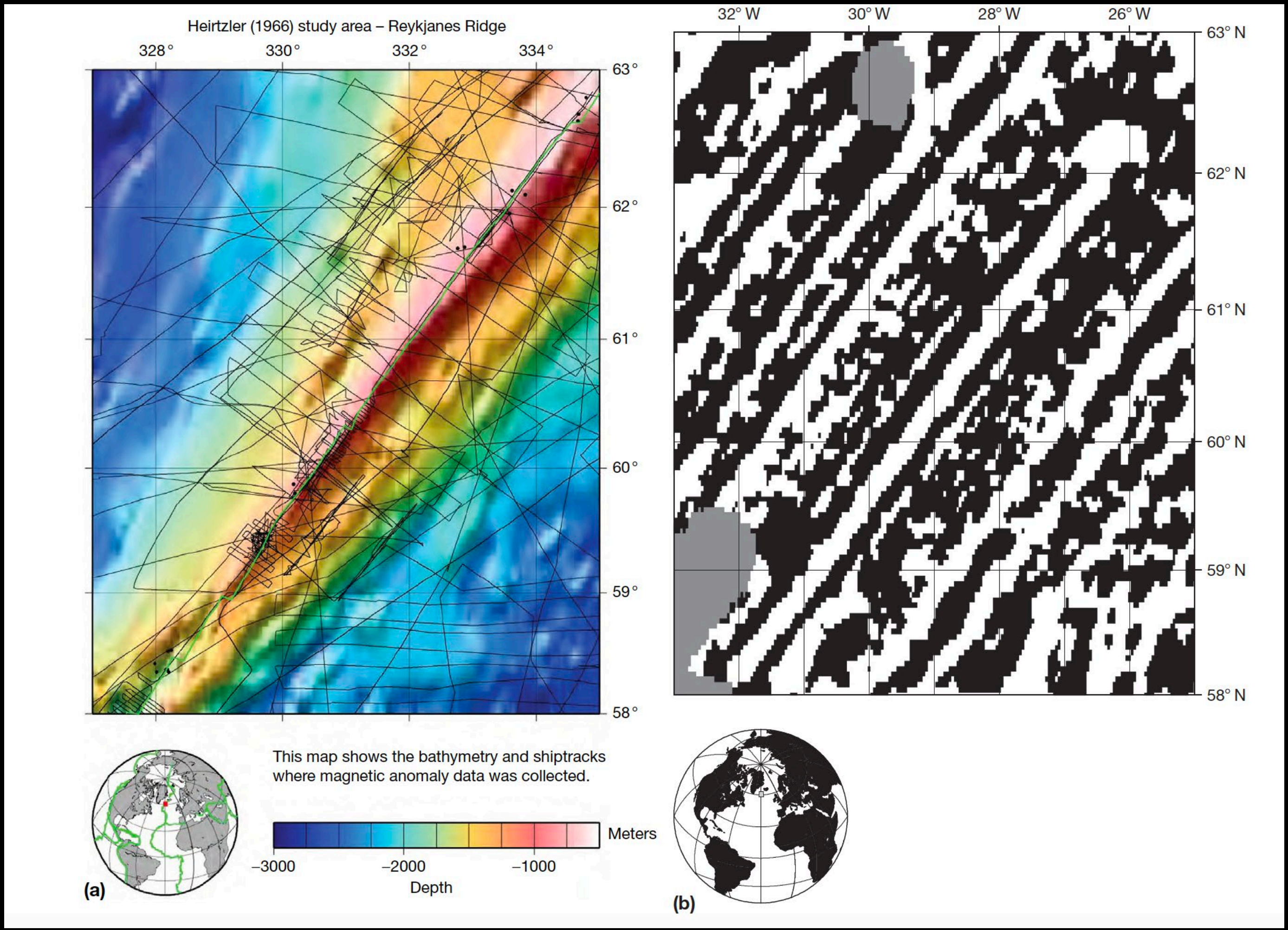
Wessel 2015

Plate tectonics



Sleep 2008

Magnetic striping on the sea floor



Wessel et al., 2015

Surface deformation techniques

Image credit: Google Earth

Surface deformation techniques

- Very long-base interferometry (VLBI)

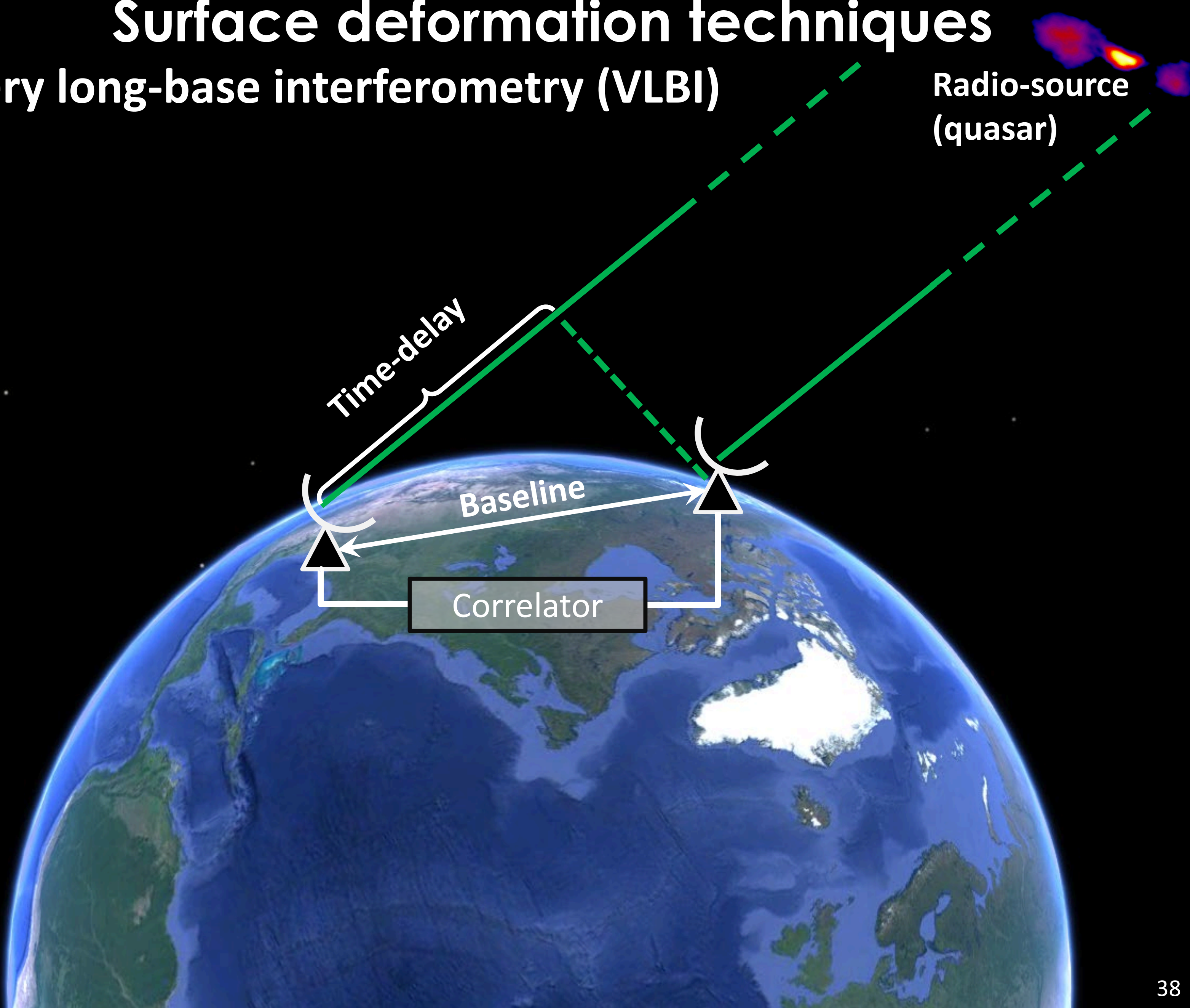


Image credit:
Google Earth

Surface deformation techniques

- Satellite Laser Ranging (SLR)

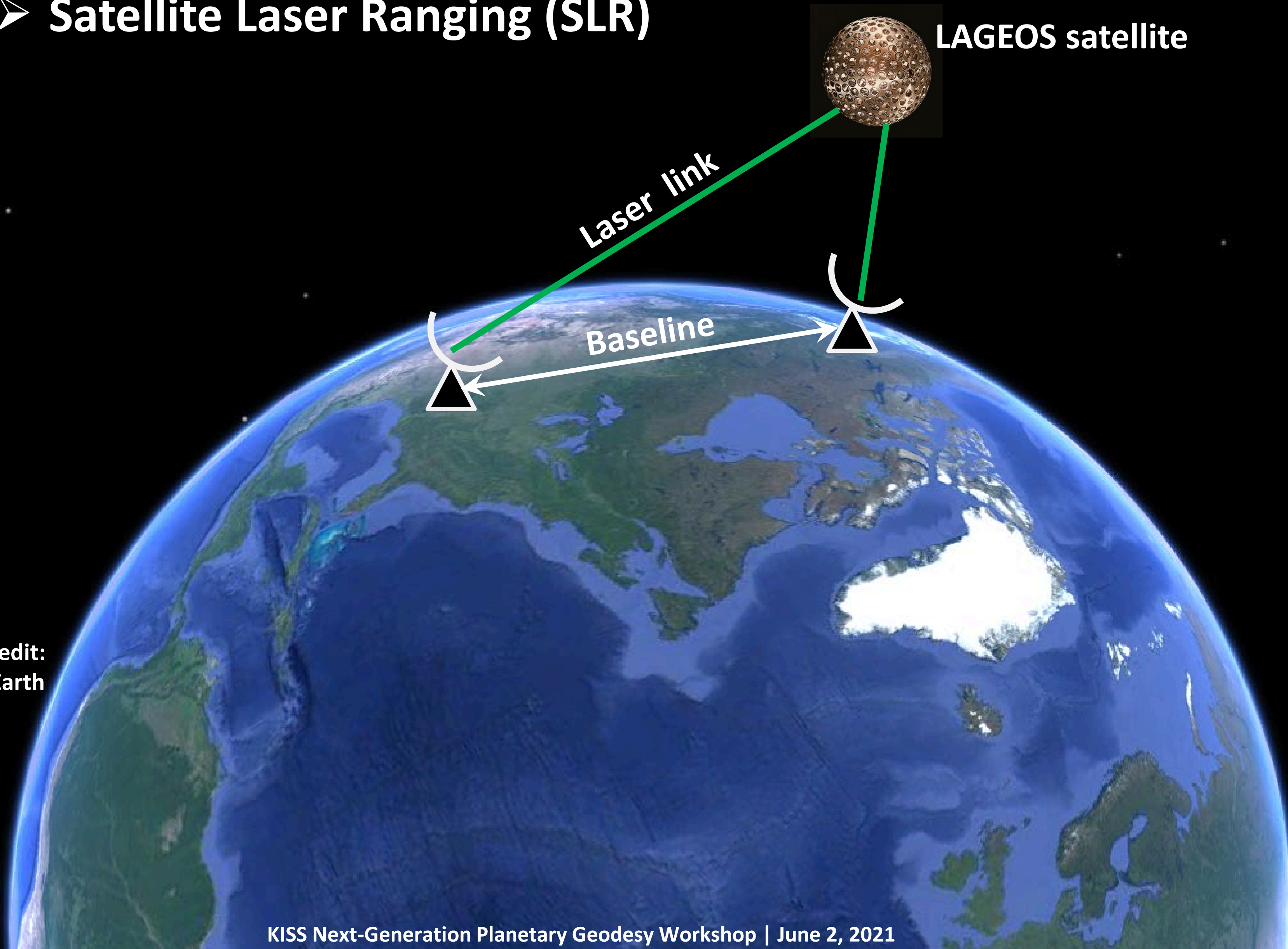


Image credit: Google Earth

Surface deformation techniques

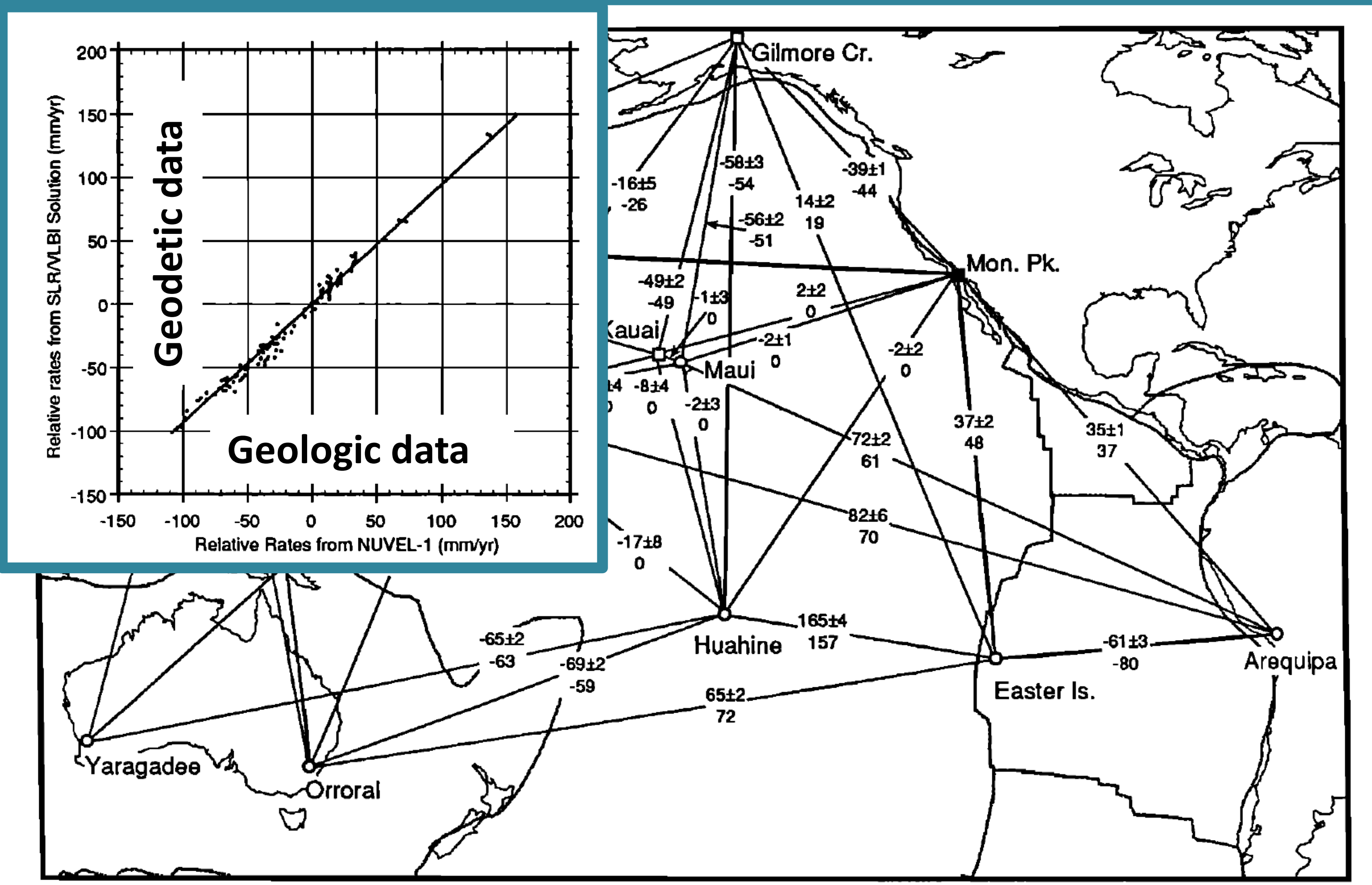


Fig. 12. Relative geodesic rates in mm/yr for selected lines across the Pacific basin. The top values are from the combined solution and the bottom value as predicted by NUVEL-1. Notation and symbols are as in Figure 5.

Robbins et al., 1993

Surface deformation techniques

- Global Positioning System (GPS)

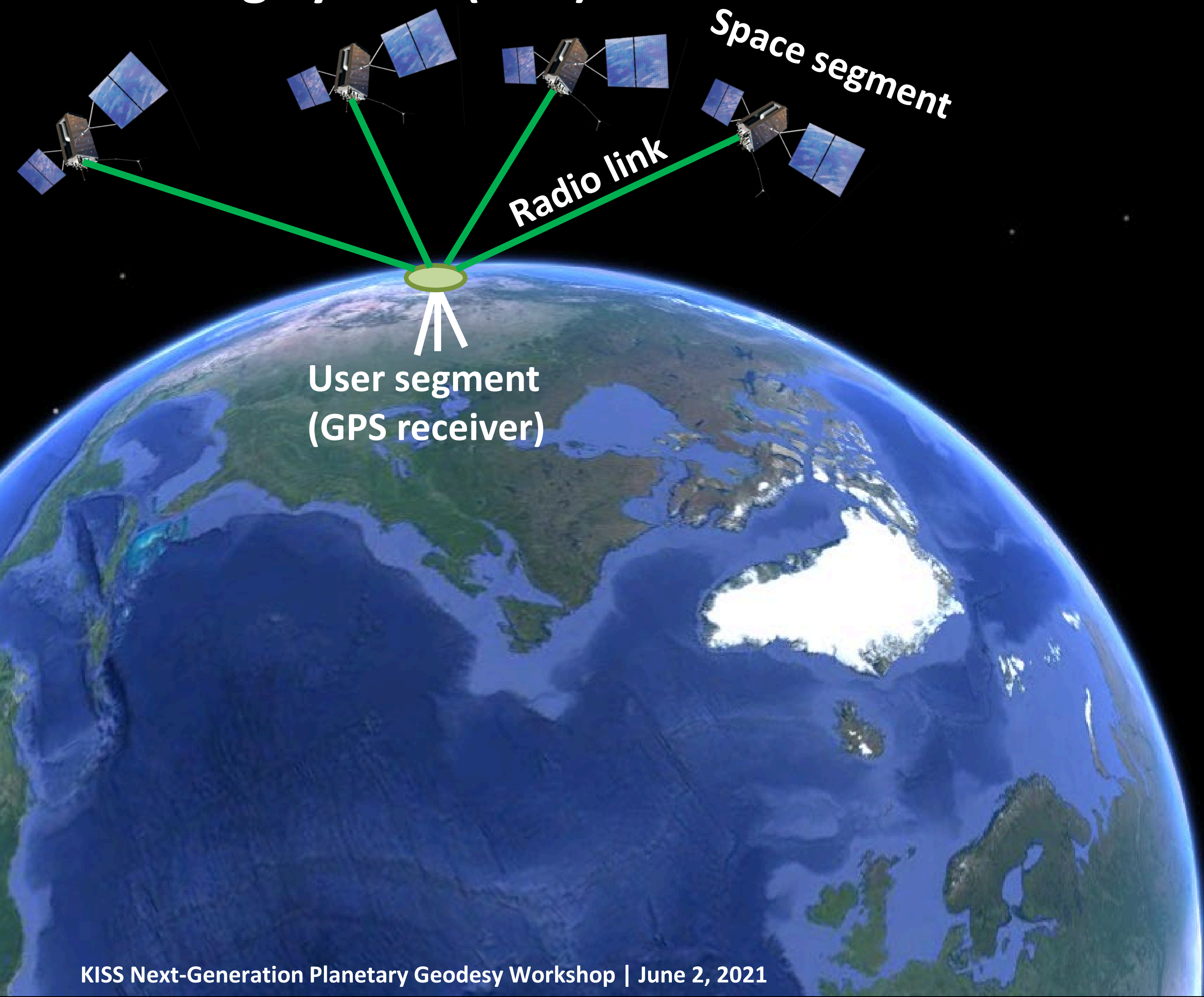


Image credit: Google Earth

Surface deformation techniques

- Global Positioning System (GPS)

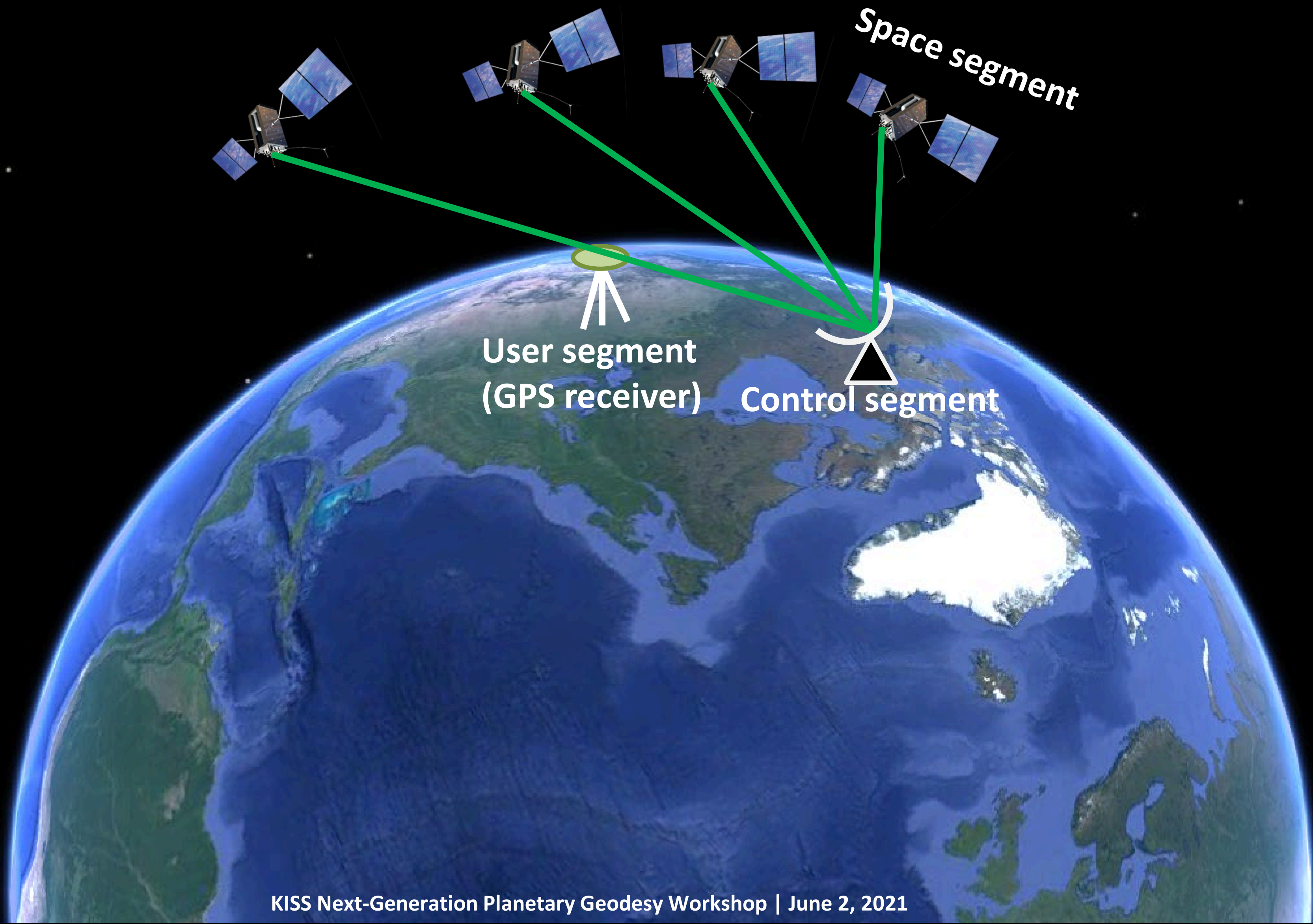
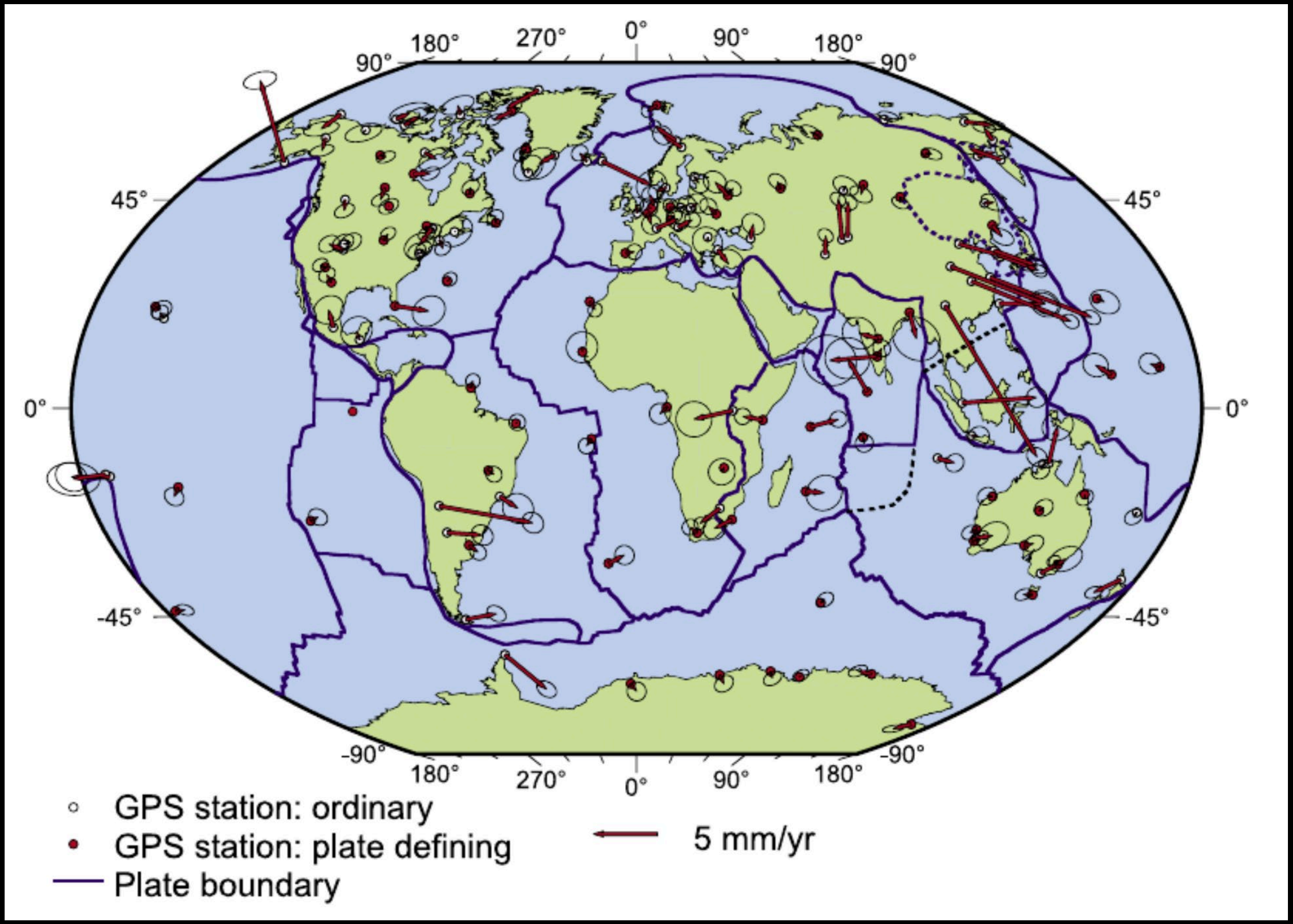


Plate motions from GPS measurements



Kogan et al., 2008

Useful references for plate tectonics and space geodesy

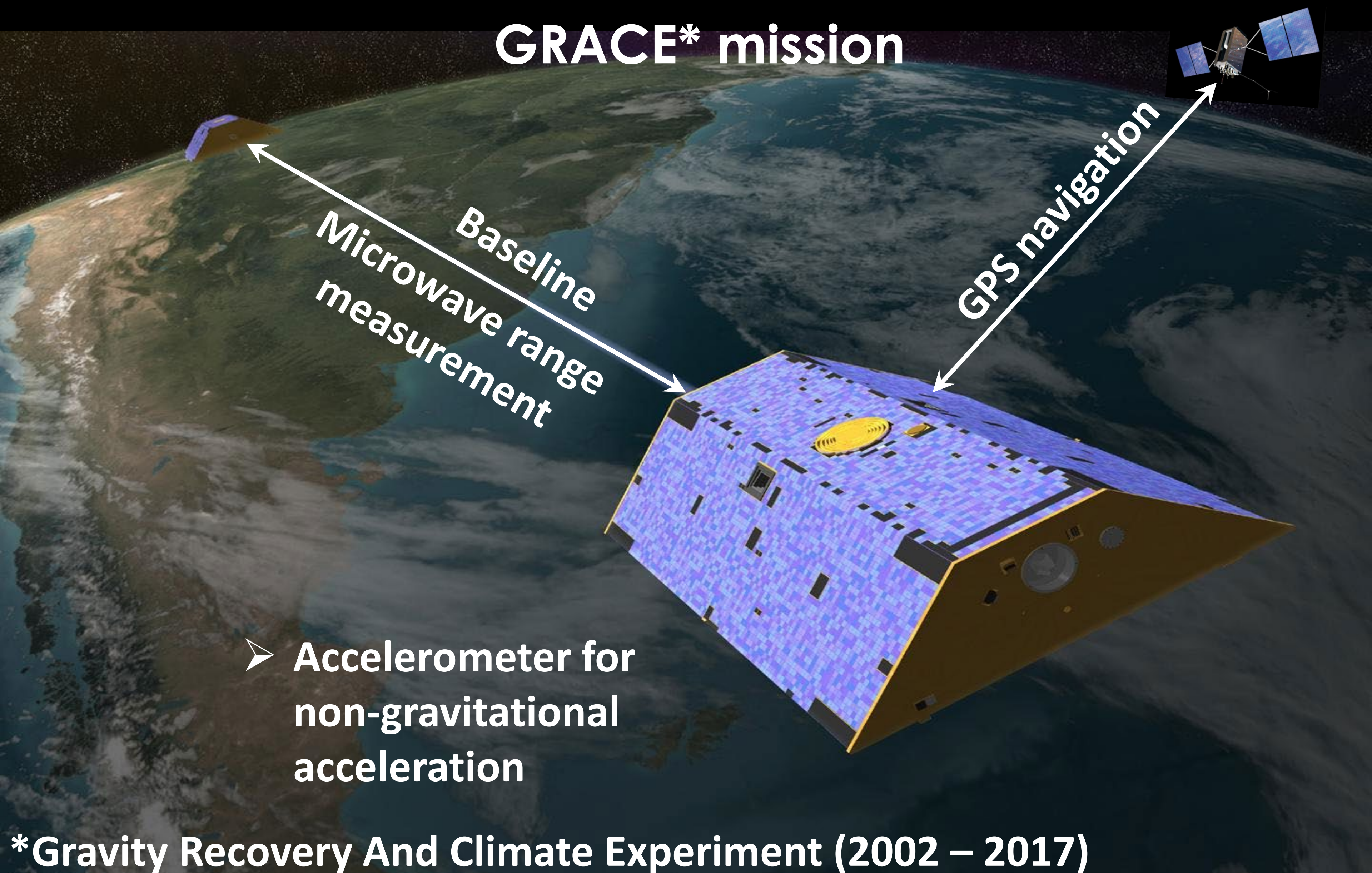
- **Classical textbook**
  - *Turcotte, D. L., & Schubert, G. (2002). Geodynamics. Cambridge university press.*

- ***Treatise on Geophysics:***
  - *https://www.sciencedirect.com/referencework/9780444538031/treatise-on-geophysics*

- ***Specifically the following chapters of the treatise:***
  - *Wessel, P., & Müller, R. D. (2015). Plate tectonics. Treatise on Geophysics*
  - *Sleep, N. H. (2015). Evolution of the earth: plate tectonics through time. Treatise on Geophysics.*
  - *Bercovici, D., Tackley, P., & Ricard, Y. (2015). 7.07- The generation of plate tectonics from mantle dynamics. Treatise on Geophysics.*

- **NASA's Space Geodesy archive (see references there for VLBI, SLR and GPS data)**
  - *https://cddis.nasa.gov/index.html*

GRACE* mission



- Accelerometer for non-gravitational acceleration

*Gravity Recovery And Climate Experiment (2002 – 2017)

Image credit: NASA/GSFC

Time-variable gravity from GRACE

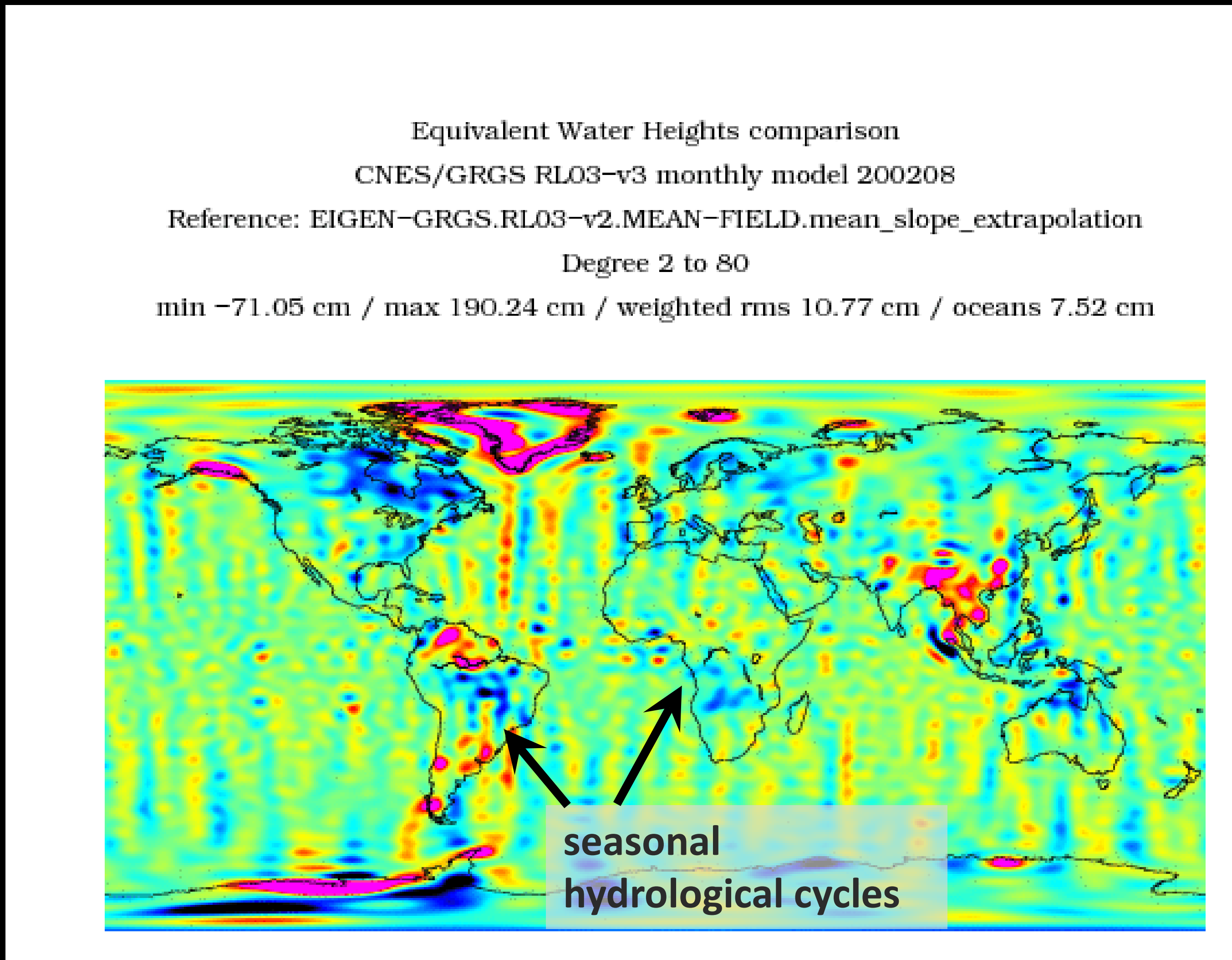


Image credit: CNRS

Ice loss in Greenland

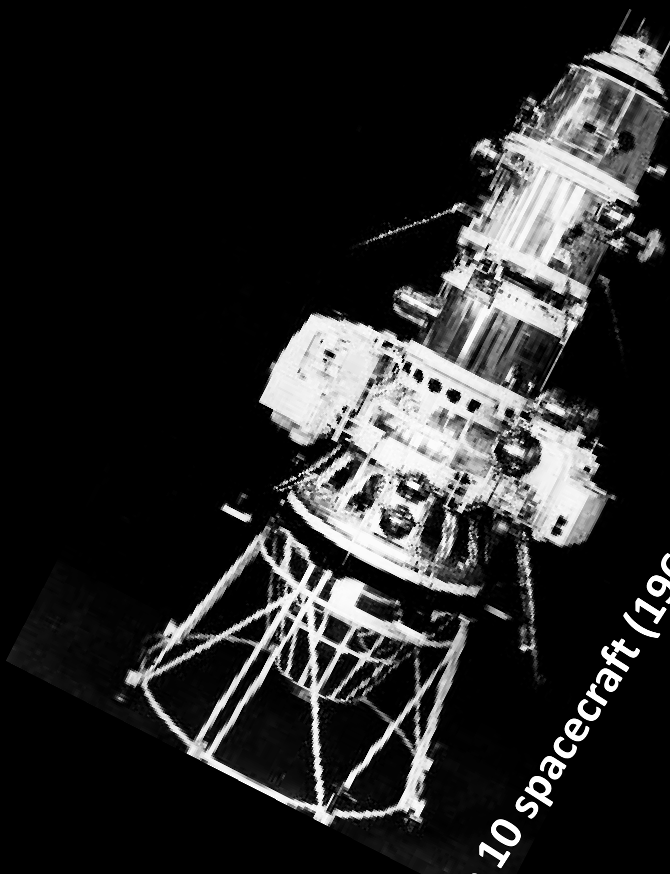
Ice loss in Antarctica

NASA Scientific Visualization Studio

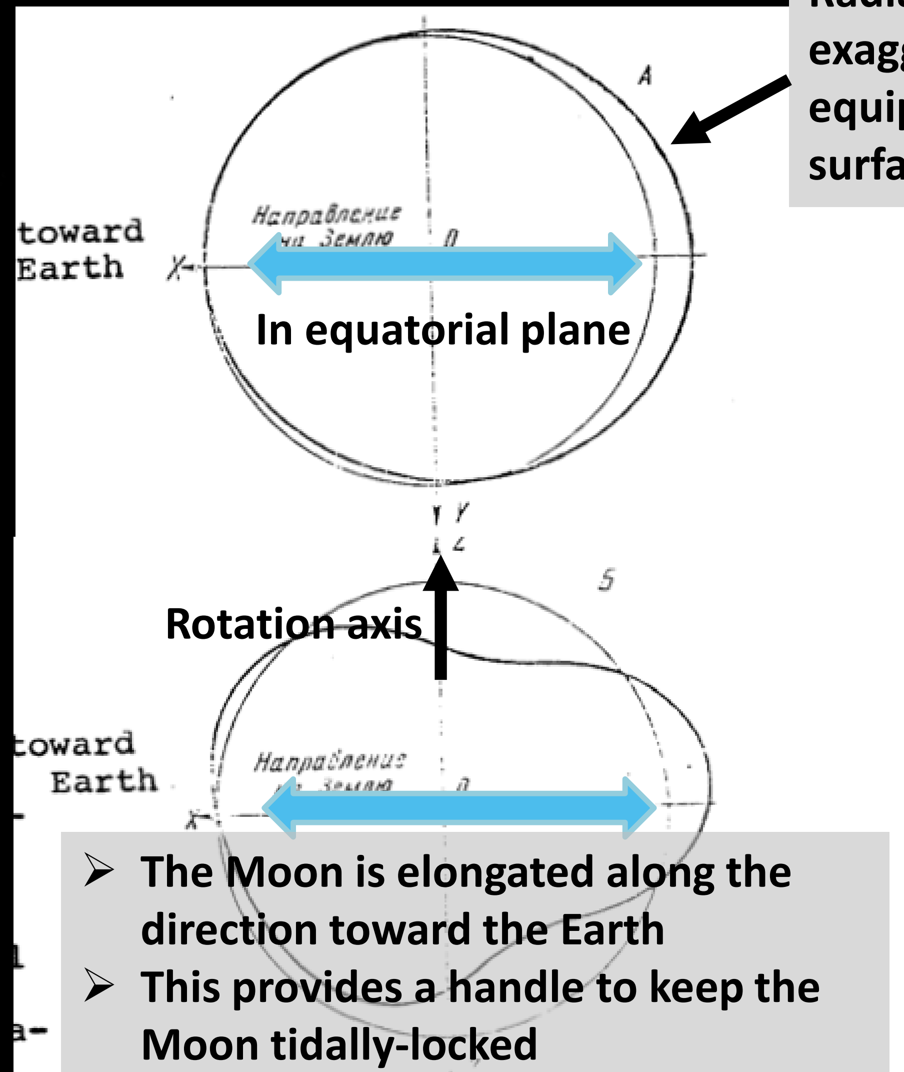
Useful references for Earth's shape and gravity studies

- **Good resource for the theory of potential**
  - *MacMillan, W. D. (1930). The Theory of the Potential https://catalog.hathitrust.org/Record/000584021*
- **Review of gravity and topography studies**
  - *Wieczorek, M. A. (2007). Gravity and topography of the terrestrial planets. Treatise on geophysics, 10, 165-206.*
- **Useful paper for modeling gravity of a sphere**
  - *Wieczorek, M. A., & Phillips, R. J. (1998). Potential anomalies on a sphere: Applications to the thickness of the lunar crust. Journal of Geophysical Research: Planets, 103(E1), 1715-1724.*
- **GRACE gravity data**
  - *https://grace.jpl.nasa.gov/data/get-data/*
- **High-resolution Earth gravity and shape models**
  - *http://geodesy.curtin.edu.au/research/*
- **Code for working with gravity and shape data in spherical harmonics**
  - *https://shtools.github.io/SHTOOLS/index.html*
  - *https://github.com/fjsimons*

Luna-10 – first lunar gravity map



Luna 10 spacecraft (1966)

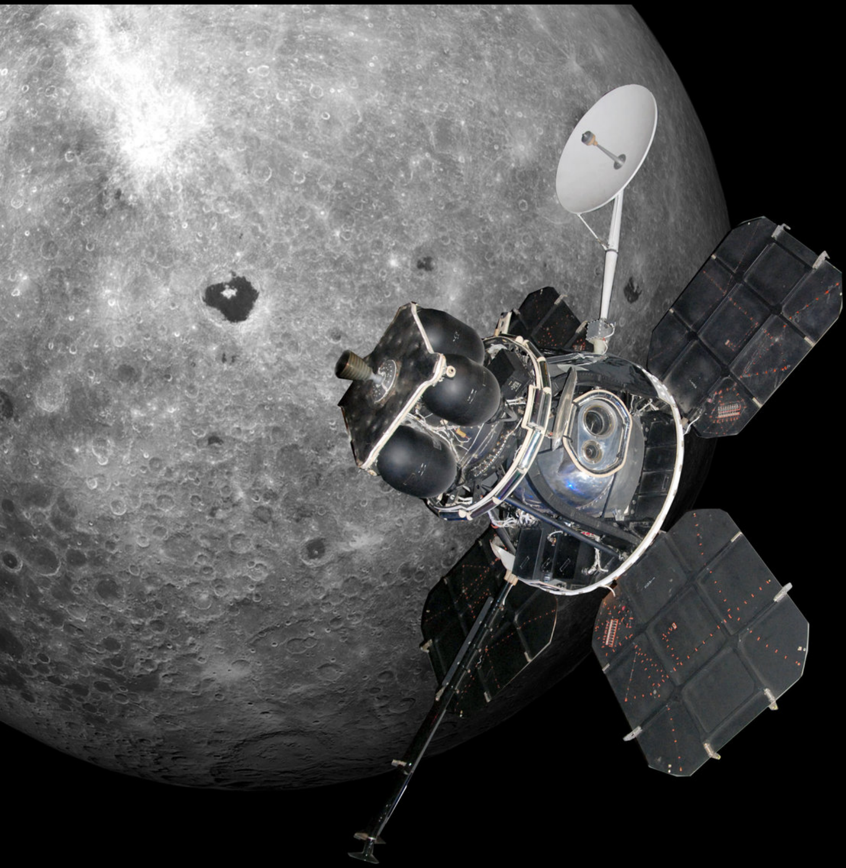


Radially–exaggerated equipotential surface

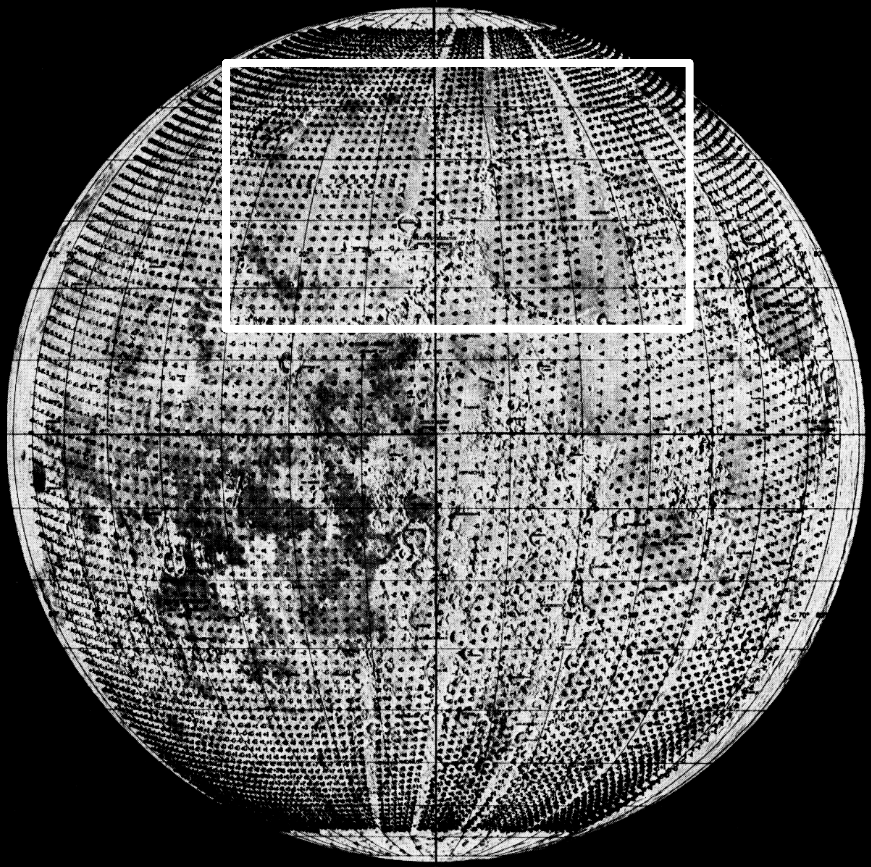
- The Moon is elongated along the direction toward the Earth
- This provides a handle to keep the Moon tidally-locked

Akim 1966

Discovery of mass concentrations (mascons)

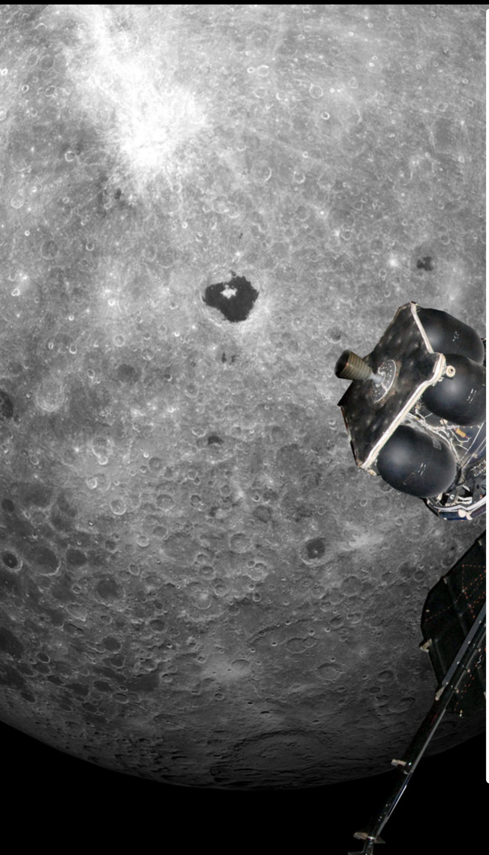


Lunar Orbiter V
Image Credit: NASA



Muller & Sjorgen 1968

Discovery of mass concentrations



Lunar Orbiter V
Image Credit: NASA

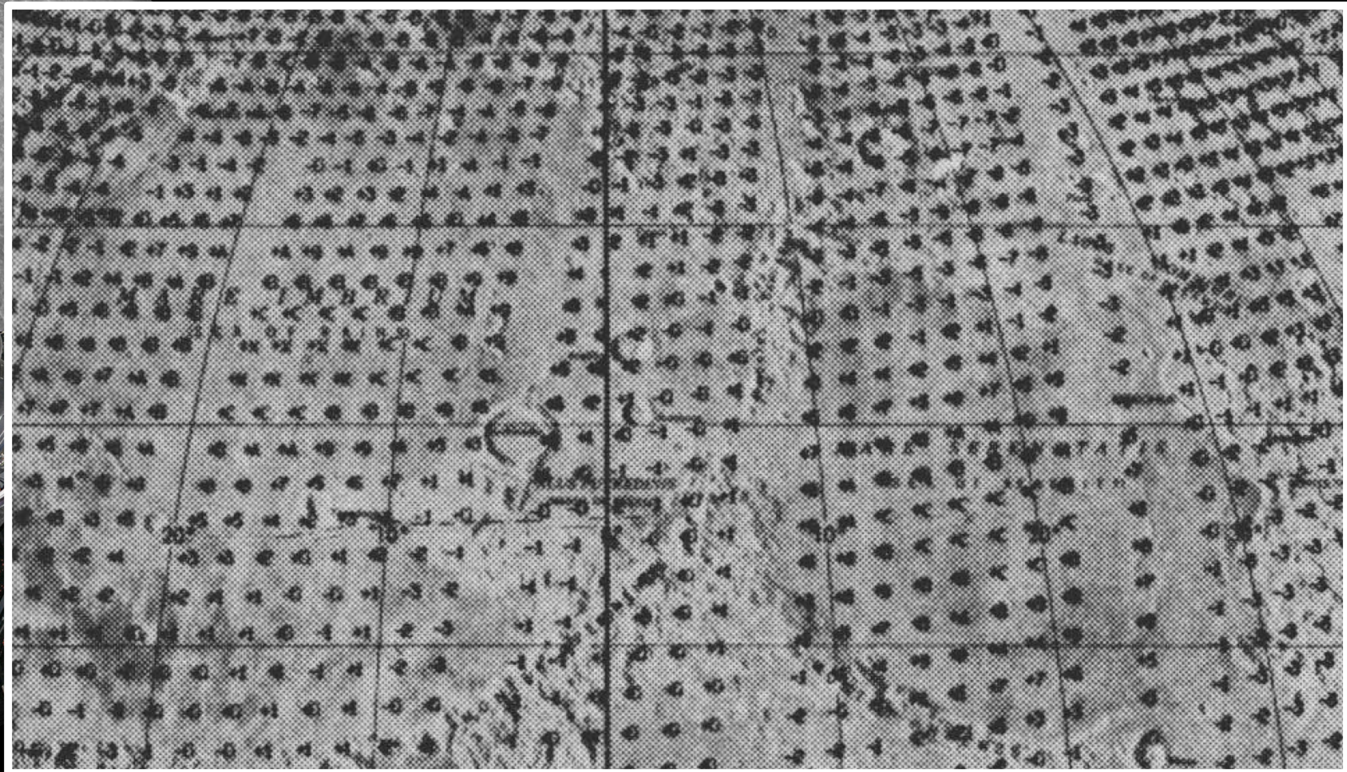


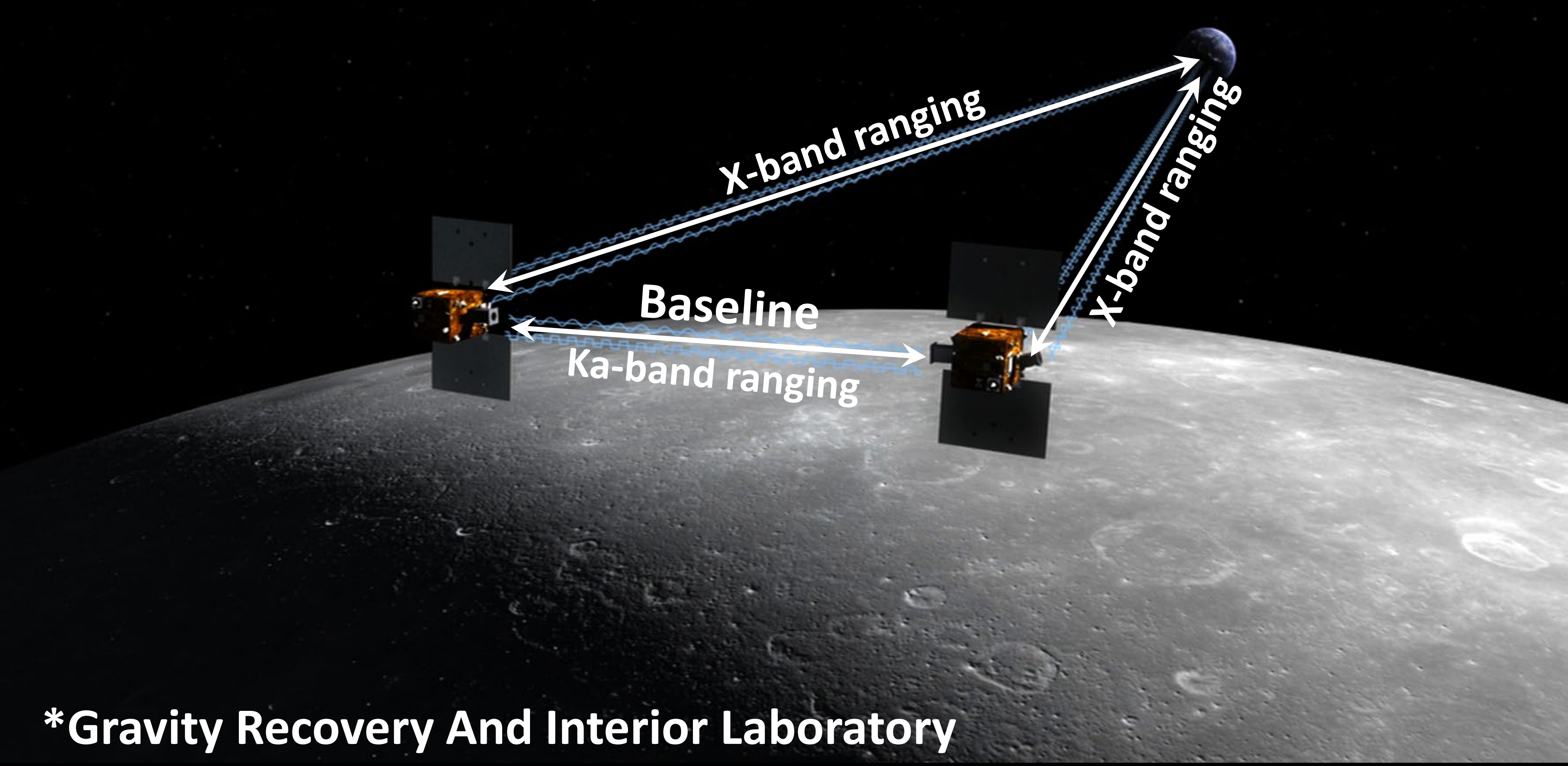
Fig. 1. Gravimetric and acceleration map of the lunar nearside. Ranges are indicated below.

Range *	Symbol	Range*	Symbol	Range*	Symbol
Beyond ±20.	±X	± 7.5 to ± 8.5	±8	± 2.5 to ± 3.5	±3
±15. to ±20.	±C	± 6.5 to ± 7.5	±7	± 1.5 to ± 2.5	±2
±11.5 to ±15.	±B	± 5.5 to ± 6.5	±6	± 0.5 to ± 1.5	±1
± 9.5 to ±11.5	±A	± 4.5 to ± 5.5	±5	0.0 to + 0.5	+0
± 8.5 to ± 9.5	±9	± 3.5 to ± 4.5	±4	− 0.5 to 0.0	−0

* Above × 0.1 = mm/sec²; above × 10 = milligals; these scaling factors also apply to the cover.

Muller & Sjorgen 1968

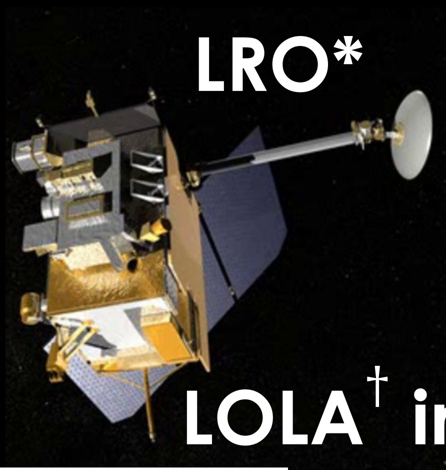
GRAIL* mission



*Gravity Recovery And Interior Laboratory

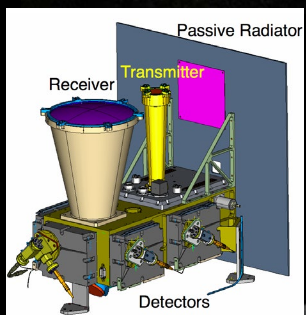
Image credit: NASA, JPL

LRO*

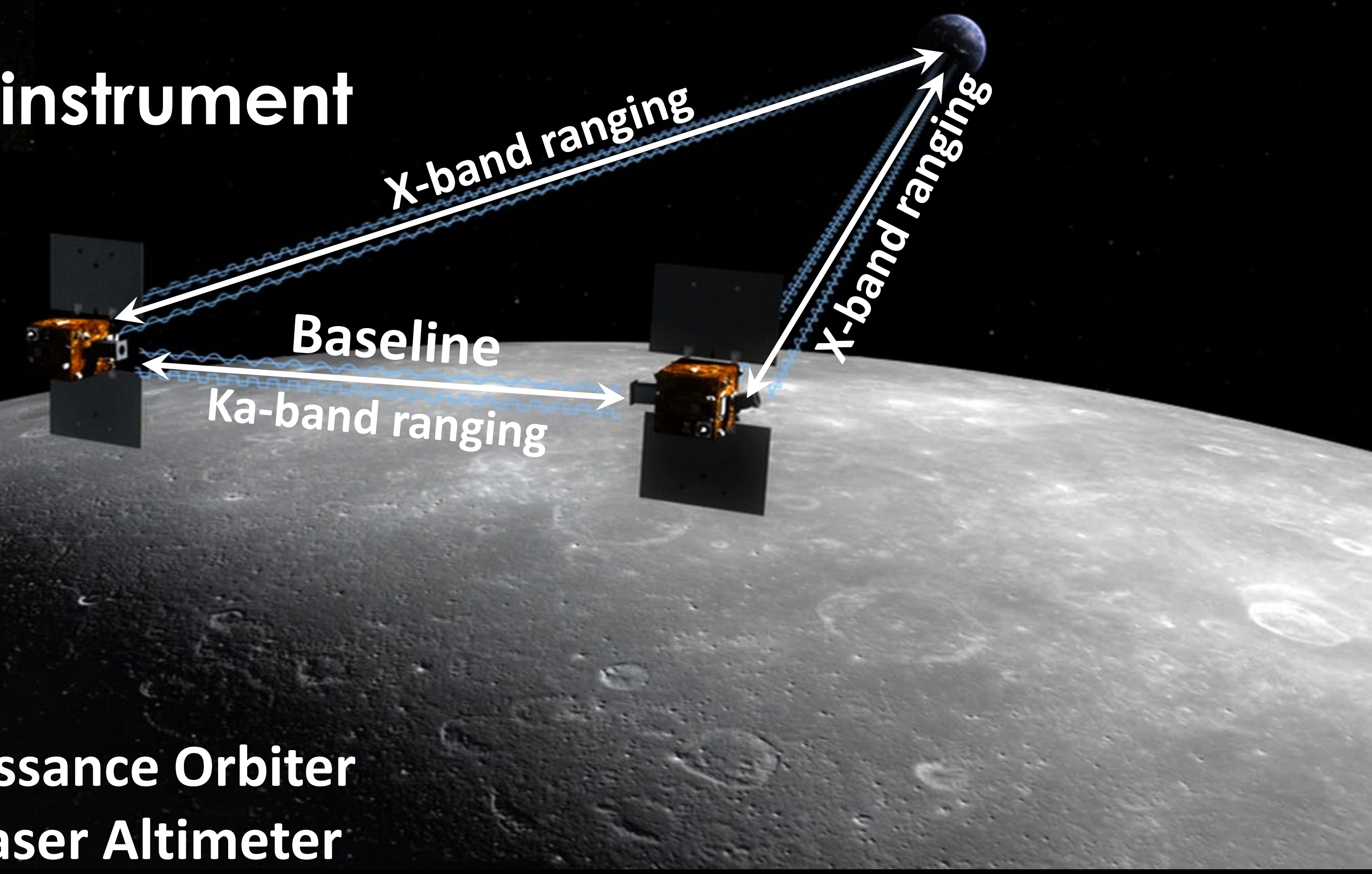


GRAIL mission

LOLA† instrument



NASA/GSFC
Conceptual Image Lab



*Lunar Reconnaissance Orbiter

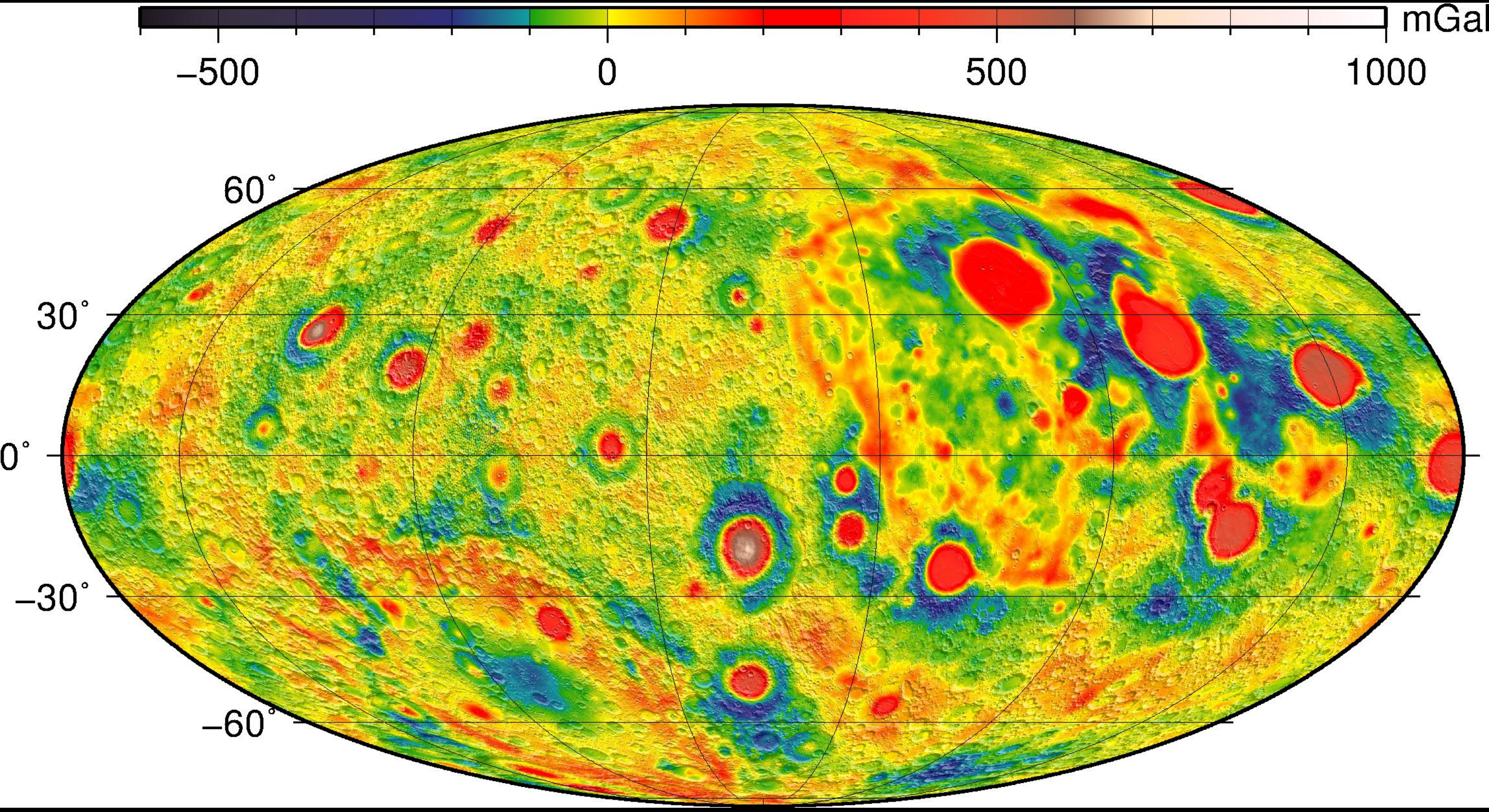
†Lunar Orbiter Laser Altimeter

Image credit: NASA, JPL

Lunar topography and gravity

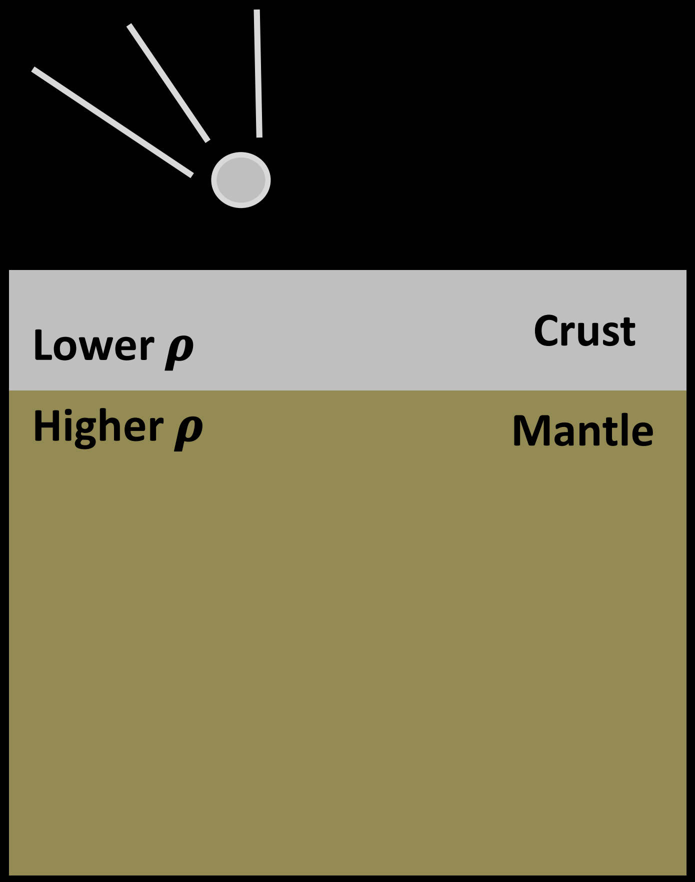
NASA Scientific Visualization Studio

Bouguer gravity anomaly from GRAIL

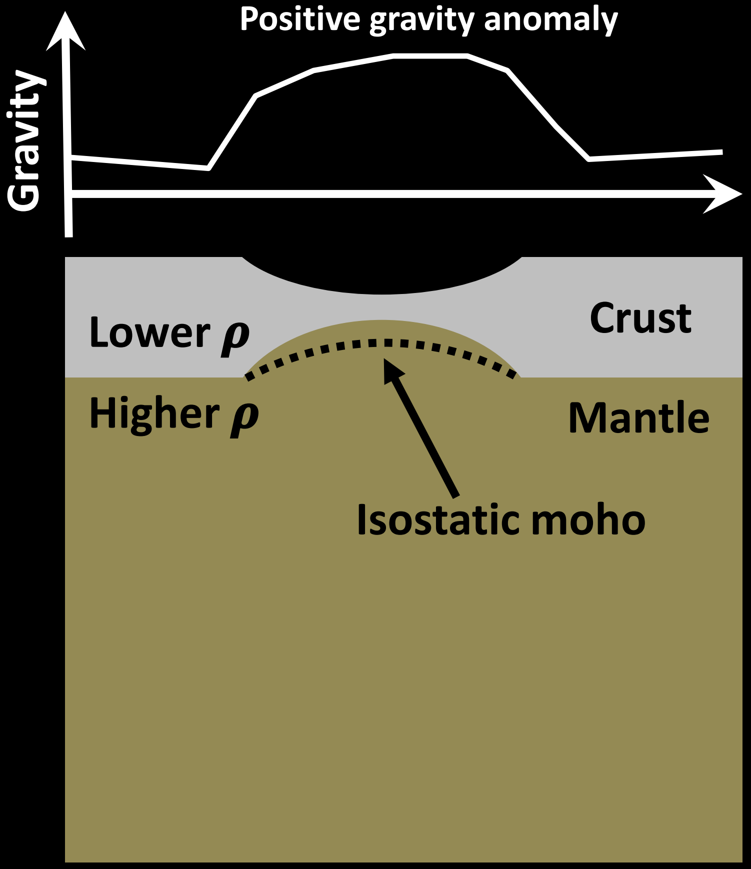


https://pgda.gsfc.nasa.gov/products/50

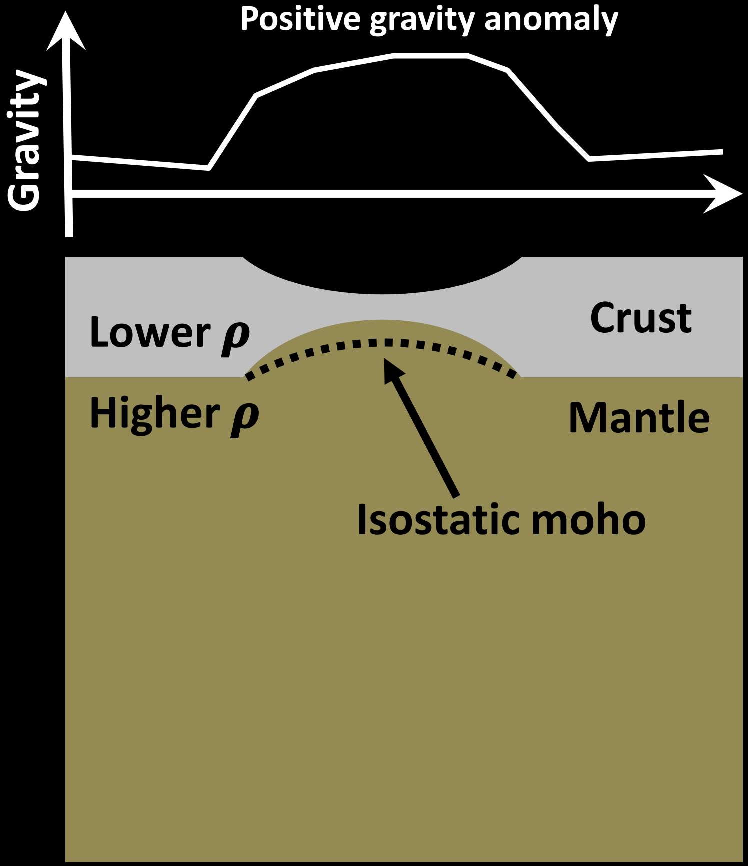
Crater formation modeling

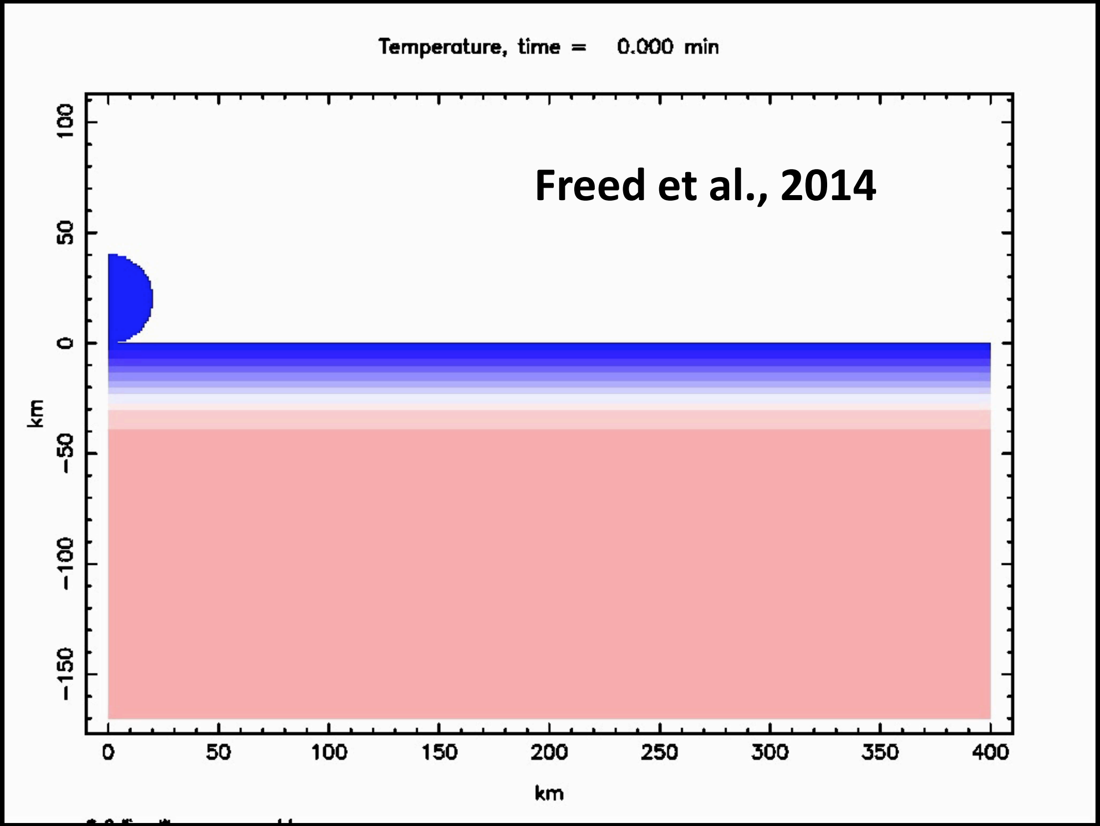


Crater formation modeling



Crater formation modeling





- After the impact, isostatic equilibrium is achieved

Mascon formation mechanism

- ➢ **Two-step modeling approach of Melosh et al., 2013**

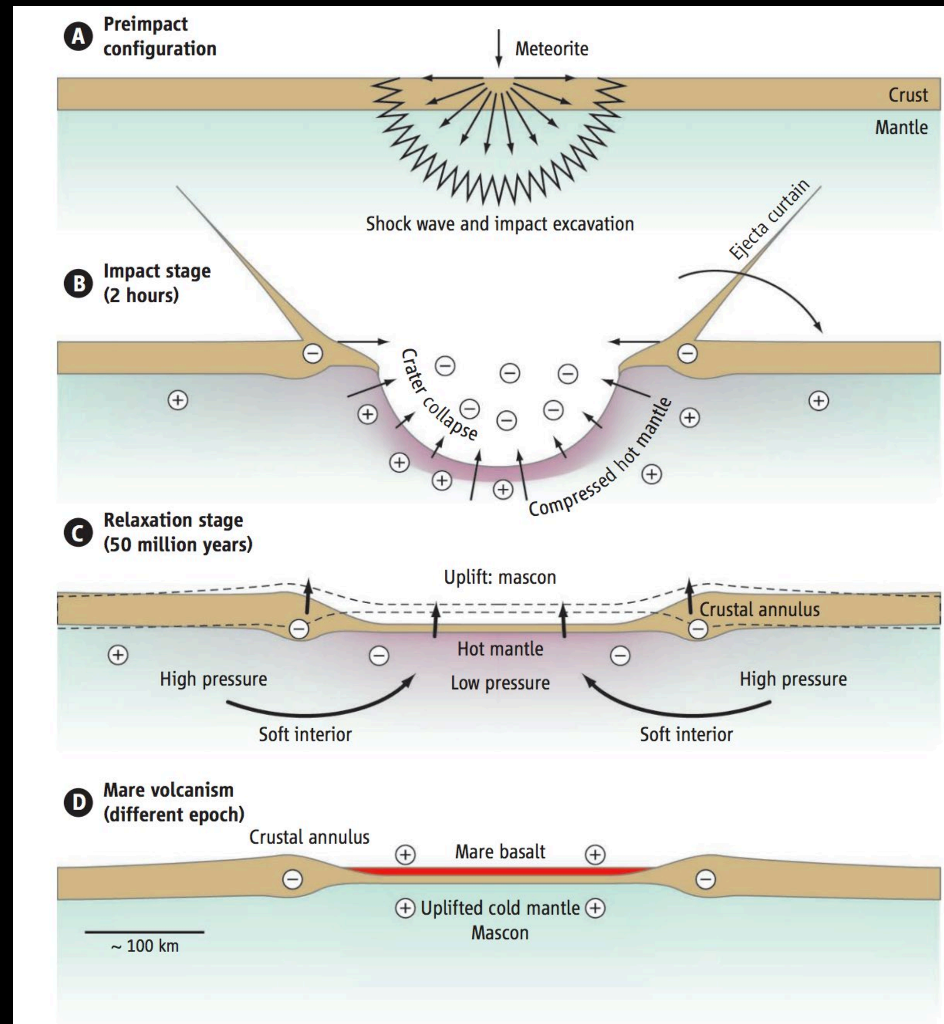
Impact formation modeling with shock physics code — Hours

↓

Viscoelastic relaxation coupled with thermal evolution using finite elements — 10^7 years

↓

Topography, Crust-mantle boundary, Gravity anomaly

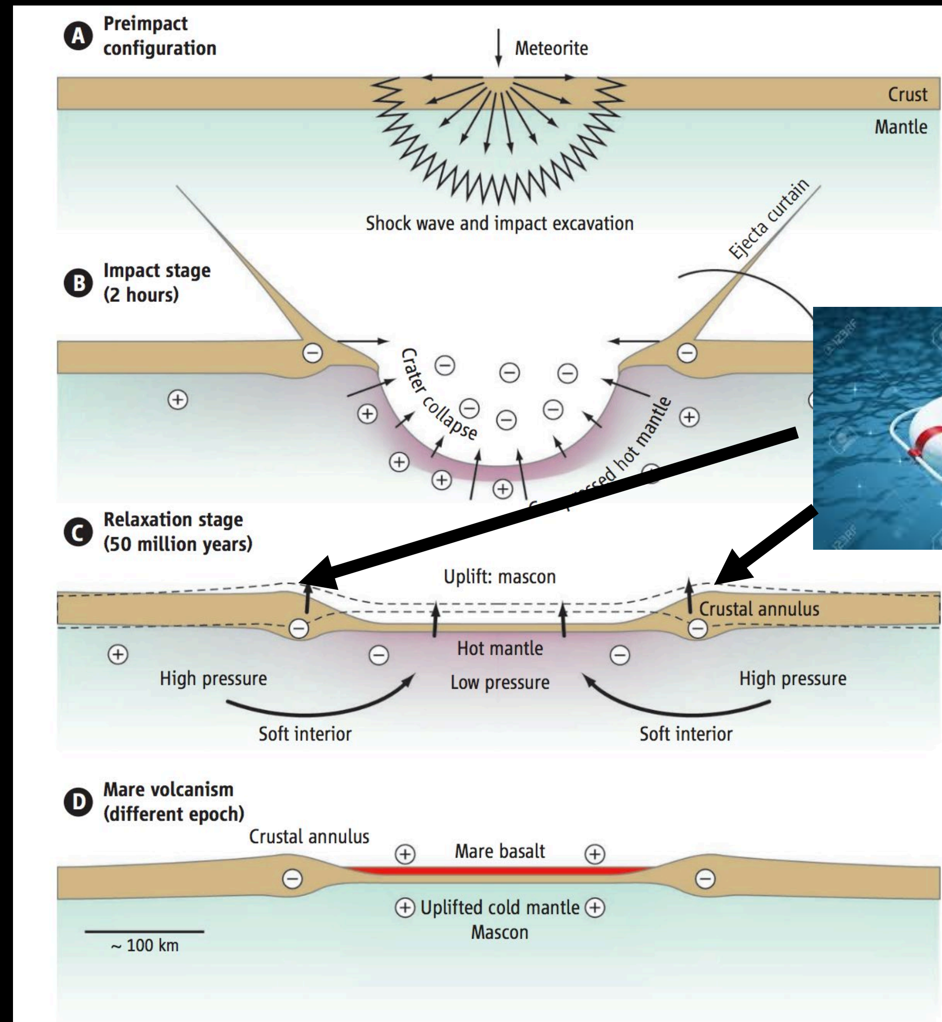


Montesi, 2013; Melosh et al., 2013; Freed et al., 2014

Mascon formation mechanism

- **Two-step modeling approach of Melosh et al., 2013**

Step	Timescale
Impact formation modeling with shock physics code	Hours
↓	
Viscoelastic relaxation coupled with thermal evolution using finite elements	10^7 years
↓	
Topography, Crust-mantle boundary, Gravity anomaly	



Montesi, 2013; Melosh et al., 2013; Freed et al., 2014

Useful references for lunar and planetary geodesy

- **Planetary gravity models on NASA's Planetary Data system:**
  - *https://pds-geosciences.wustl.edu/dataserv/gravity_models.htm*
- **General purpose finite-element codes for geodynamics**
  - *https://www.dealii.org/ (see their great lectures and tutorials)*
  - *https://fenicsproject.org/*
- **Shock physics code (to study planetary impacts)**
  - *https://isale-code.github.io/*
- **Planetary Geodesy data archive (Moon, Mars, Mercury gravity and shape data and more)**
  - *https://pgda.gsfc.nasa.gov/*
- ***Tools for working with geometry of spacecraft observations***
  - *https://naif.jpl.nasa.gov/naif/*
- ***JPL planetary ephemeris system (to know where the planets are)***
  - *https://ssd.jpl.nasa.gov/*

Summary

1. Geodetic data were used to determine the shape of the Earth leading to understanding that the Earth is differentiated.
2. Analysis of geodetic data coupled with gravity modeling led to understanding the Earth crust is in the state of isostasy.
3. Geodesy revolutionized plate tectonics studies confirming that current instantaneous plate motions agree with plate motions inferred from geologic data.
4. Geodetic data from GRACE and GRACE-FO continue to quantify the effect of global warming on the ice sheet loss.
5. High-resolution lunar gravity from GRAIL answered long-standing question on the origin of lunar mascons.