PCA data analysis

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August 24, 2016

The problem(s)

Assume you have an image in which you are looking for a planet.

$$T(n) = I_{\psi_0}(n) + \varepsilon A(n).$$

We call ψ the random state of the telescope+instrument at the exposure.

The problem we want to solve is to figure out what are the relative contributions of the light diffracted within the instrument and of an hypothetical astrophysical signal.

Solutions

- We can have a really good model of our instrument.
- We "construct" a really good model of our instrument based on its data history (science frames+telemetry).
- We get more realizations of I_{ψ} for which we are sure that there is no astrophysical signal. We subtract them from T.

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Observing strategies

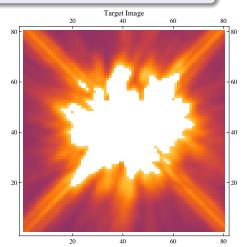
How to get more realizations of the instrument response?

Take images of other sources.

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- The telescope + instrument must be very stable.
- The alignment of the images needs to be very precise (the star needs to be on the same fraction of a pixel).



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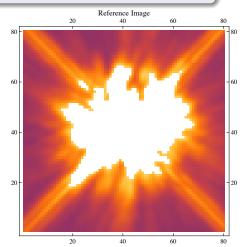
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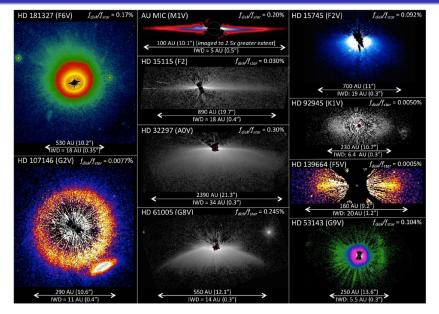
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Observing strategies: PSF subtraction



The data

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- Take images of other sources.
- Take images at other wavelengths/telescope orientations.

$$R(n) = I_{\psi_1}(n) + \varepsilon A(n - \delta n \mathbf{1}_{r,\theta}) \text{ or } R(n) = I_{\psi_1}(n - \delta n \mathbf{1}_{r,\theta}) + \varepsilon A(n)$$

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Solving the least squares problem:

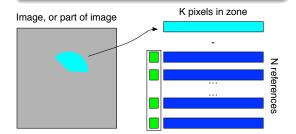
$$\min_{\{c_k\}} \left\{ \sum_{n} \left(T(n) - \sum_{k=1}^{K} c_k R_k(n) \right)^2 \right\}.$$

Equivalent to:

$$E[RR]C = T$$

where E[RR] is the correlation matrix of the ensemble of references over the zone of the image we chose.

- Tweak set up of the inverse problem (geometry, selection of references)
- Regularize of the inverse problem (SVD truncation, PCA)



Solving the least squares problem:

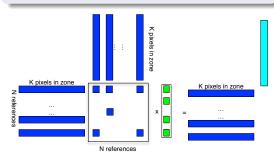
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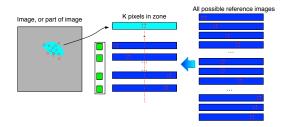
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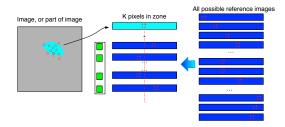
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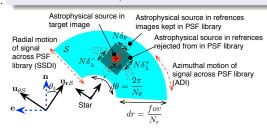
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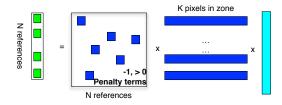
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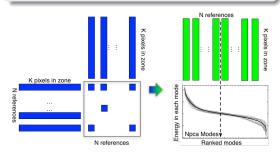
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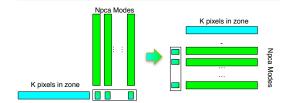
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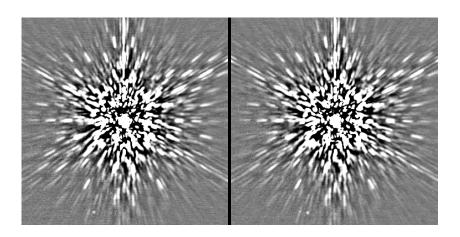
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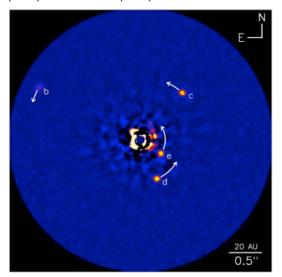
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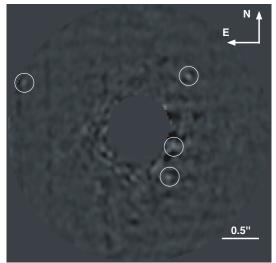
Marois et al. (2008), Marois et al. (2010)



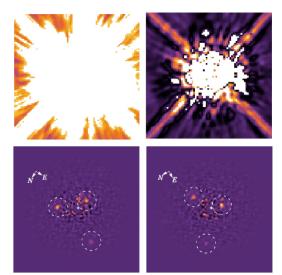
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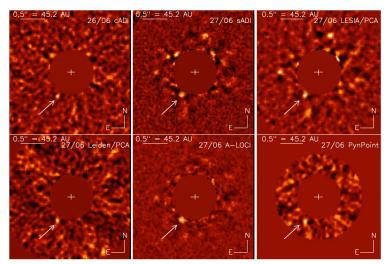
Oppenheimer et al. (2013), Pueyo et al. (2015)

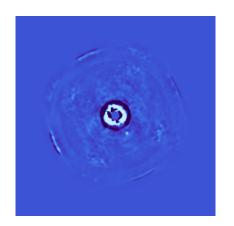


Soummer et al. (2011)



Rameau et al. (2012)

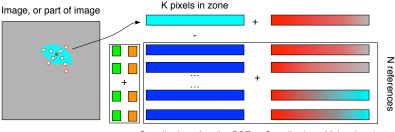






The least squares speckles fitting in the presence of signal can be written as:

$$\min_{\{c_k\}} \left\{ \sum_{n} \left([I_{\psi_0}(n) + A_0(n)] - \sum_{k=1}^{K} (c_k + \delta c_k) [I_{\psi_k}(n) + A_k(n)] \right)^2 \right\}.$$

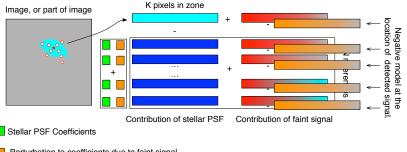


Contribution of stellar PSF Contribution of faint signal

Stellar PSF Coefficients

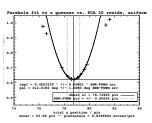
Perturbation to coefficients due to faint signal

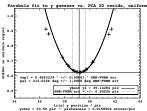
Solution is to inject a negative model of the signal in the entire observing sequence and minimize the residuals over a range of hypothetical astrophysical observables. This can be done in conjunction with any of the algorithms described before. Marois et al. (2010), Lagrange et al. (2012).

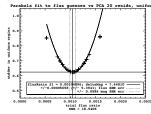


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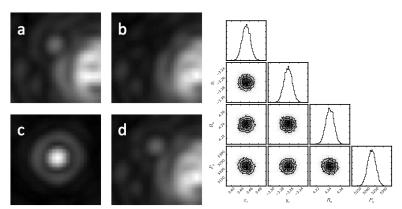
Solution is to inject a negative model of the signal in the entire observing sequence and minimize the residuals over a range of hypothetical astrophysical observables. Example of a grid search for astrometry and photometry, Morzinski et al. (2015).







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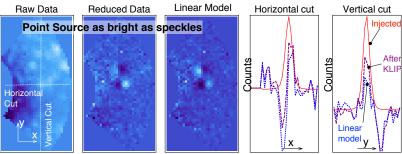
Main drawbacks

- The speckle subtraction algorithm has to be used each time around (involves a matrix inversion).
- There is no guarantee that the cost-function minimized/likelihood explored does not feature local minima. One might get stuck in them.
- In general these are not limiting factors in "small dimensional configurations" (astrometry and photometry = 3 dimensions).
- This becomes a severe limiting factor when trying to get spectrum (astrometry and spectrum = 39 dimensions with GPI).

There is a way to write the influence of the astrophysical signal as:

$$PCA(Speckles + Signal) = PCA(Speckles) + Signal \delta PCA(Speckles)$$

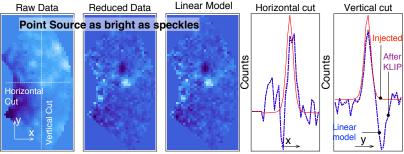
...and this applies to any algorithm relying on covariances. Pueyo (2016).



Aggressive reduction: $N_{\mathbf{r}}=5,\ N_{\phi}=4,\ N_{\mathbf{Corr}}=50,\ K_{\mathbf{Klip}}=50,\ N_{\delta}=0.6.$

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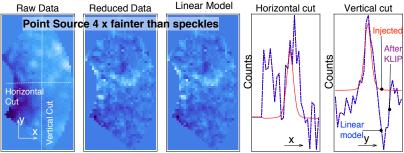


Non aggressive reduction: $N_{\mathbf{r}}=5,\ N_{\phi}=4,\ N_{\mathbf{Corr}}=30,\ K_{\mathbf{Klip}}=30,\ N_{\delta}=0.8.$

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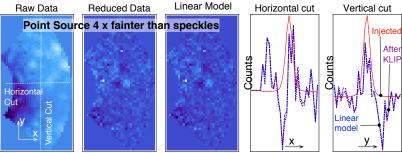


Non aggressive reduction: $N_{\mathbf{r}}=5,~N_{\phi}=4,~N_{\mathbf{Corr}}=30,~K_{\mathbf{Klip}}=30,~N_{\delta}=1.$

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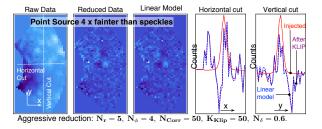
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 ...and this applies to any algorithm relying on covariances. Pueyo (2016).



The linear model works:

- If the astrophysical source is faint when compared to the speckles.
- If the astrophysical source as bright as the speckles/brighter, and the algorithm parameters are chosen accordingly (not too aggressive).

Perturbation of the covariance matrix

$$\begin{split} & C_{RR} & = & R(x) \, R(x)^T \\ & C_{RR} & = & I(x) \, I(x)^T + \epsilon \, a A_{\lambda_{p_0}, \delta}(x) I(x)^T + \epsilon \, I(x) A_{\lambda_{p_0}, \delta}(x)^T a^T + \epsilon^2 a A_{\lambda_{p_0}, \delta}(x) A_{\lambda_{p_0}, \delta}(x)^T a^T \\ & C_{RR} & = & C_{II} + \epsilon \, C_{A_{\delta I}} + \mathscr{O}(\epsilon^2). \end{split}$$

Perturbation of the eigenvalues/vectors of covariance matrix

$$\Gamma_{k} = \Lambda_{k} + \varepsilon V_{k}^{T} \mathbf{C}_{\mathbf{A}_{\delta} \mathbf{I}} V_{k}
U_{k} = V_{k} + \varepsilon \sum_{j=1, j \neq k}^{N_{\mathscr{R}}} \frac{V_{j}^{T} \mathbf{C}_{\mathbf{A}_{\delta} \mathbf{I}} V_{k}}{\Lambda_{k} - \Lambda_{j}} V_{j}$$

Perturbation of the Principal Components

$$\begin{aligned} \mathbf{Y}_{k}(\mathbf{x}) &= & Z_{k}(\mathbf{x}) + \varepsilon \Delta Z_{k}(\mathbf{x}) \\ \varepsilon \Delta Z_{k}(\mathbf{x}) &= & \varepsilon a_{\lambda}^{T} \Delta \mathbf{Z}_{k}^{\lambda}(\mathbf{x}) = f_{\lambda}^{T} \Delta \mathbf{Z}_{k}^{\lambda}(\mathbf{x}) \\ \Delta \mathbf{Z}_{k}^{\lambda}(\mathbf{x}) &= & \frac{\mathbf{S}_{\lambda}}{\sqrt{\Lambda_{k}}} \left[\mathbf{V}_{k} \mathbf{A}_{\delta}(\mathbf{x}) \mathbf{I}(\mathbf{x})^{T} V_{k} Z_{k}(\mathbf{x}) + \mathbf{V}_{k} \mathbf{A}_{\lambda_{\mathbf{p}_{0}} \delta}(\mathbf{x}) + \dots \right. \\ &\left. \qquad \qquad \sum_{j=1, j \neq k}^{N_{\mathscr{R}}} \frac{\sqrt{\Lambda_{j}}}{\Lambda_{k} - \Lambda_{j}} (\mathbf{V}_{k} \mathbf{A}_{\delta}(\mathbf{x}) \mathbf{I}(\mathbf{x})^{T} V_{j} + \mathbf{V}_{j} \mathbf{A}_{\delta}(\mathbf{x}) \mathbf{I}(\mathbf{x})^{T} V_{k}) Z_{j}(\mathbf{x}) \right] \end{aligned}$$

The details

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Perturbation of the Principal Components

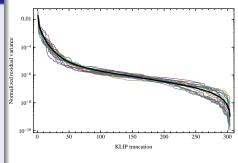
$$\begin{aligned} Y_{k}(\mathbf{x}) &= Z_{k}(\mathbf{x}) + \varepsilon \Delta Z_{k}(\mathbf{x}) \\ \varepsilon \Delta Z_{k}(\mathbf{x}) &= \varepsilon a_{\lambda}^{T} \Delta Z_{k}^{\lambda}(\mathbf{x}) = f_{\lambda}^{T} \Delta Z_{k}^{\lambda}(\mathbf{x}) \\ \Delta Z_{k}^{\lambda}(\mathbf{x}) &= \frac{S_{\lambda}}{\sqrt{\Lambda_{k}}} \left[V_{k} \mathbf{A}_{\delta}(\mathbf{x}) \mathbf{I}(\mathbf{x})^{T} V_{k} Z_{k}(\mathbf{x}) + V_{k} \mathbf{A}_{\lambda_{\mathbf{p}_{0}} \delta}(\mathbf{x}) + \dots \right. \\ &\left. \sum_{j=1, j \neq k}^{N_{\mathscr{R}}} \frac{\sqrt{\Lambda_{j}}}{\Lambda_{k} - \Lambda_{j}} (\mathbf{V}_{k} \mathbf{A}_{\delta}(\mathbf{x}) \mathbf{I}(\mathbf{x})^{T} V_{j} + \mathbf{V}_{j} \mathbf{A}_{\delta}(\mathbf{x}) \mathbf{I}(\mathbf{x})^{T} V_{k}) Z_{j}(\mathbf{x}) \right] \end{aligned}$$

What does it mean?

$$Y_k(\mathbf{x}) = Z_k(\mathbf{x}) + \varepsilon \Delta Z_k(\mathbf{x})$$
. We can rank them in order of $||\varepsilon \Delta Z_k(\mathbf{x})/Z_k(\mathbf{x})||$.

Three main terms:

- over-subtraction: unperturbed Principal Components $Z_k(\mathbf{x})$. Scales as $||Z_k(\mathbf{x})|| = 1$.
- direct self-subtraction: presence of an astrophysical source at various parallactic angles and wavelengths in the observing sequence multiplied by LOCI coefficient. Scales as $\varepsilon/\sqrt{\Lambda_k}$.
- indirect self-subtraction: perturbation in the LOCI coefficient. Scales as ε/Λ_k .



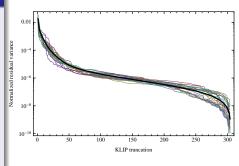
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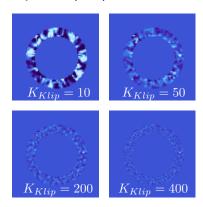
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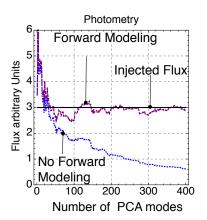
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Pueyo et al. (2015). Rescaled cube Raw cube Point Source , o o o o o o Speckle λ_{k_0} , 0 0.0.0.0 φ r_S

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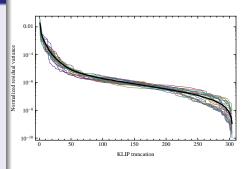


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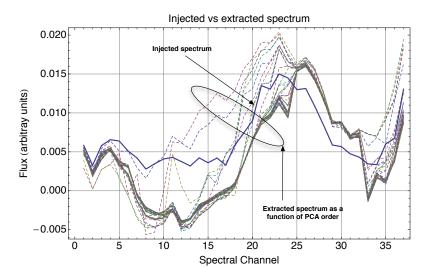
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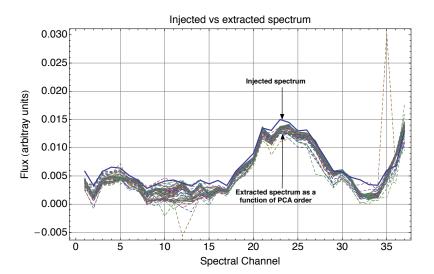
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- direct self-subtraction: presence of an astrophysical source at various parallactic angles and wavelengths in the observing sequence multiplied by LOCI coefficient. Scales as $\varepsilon/\sqrt{\Lambda_k}$. Esposito et al. (2012)
- indirect self-subtraction: perturbation in the LOCI coefficient. Scales as ε/Λ_k. Brandt et al. (2013).



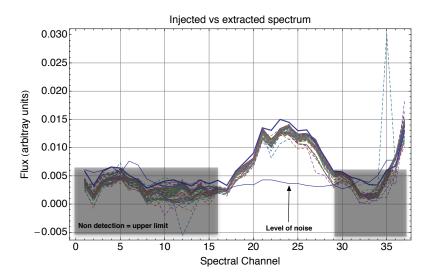
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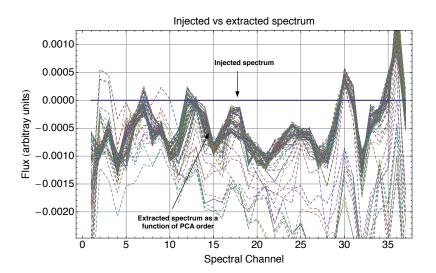




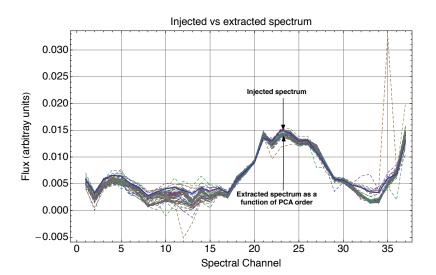








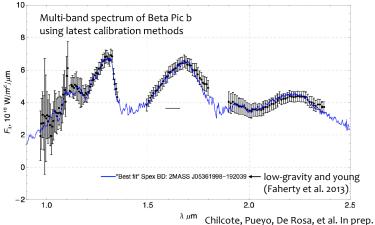


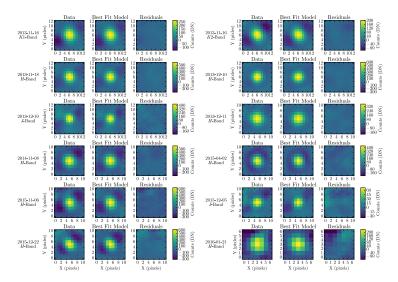


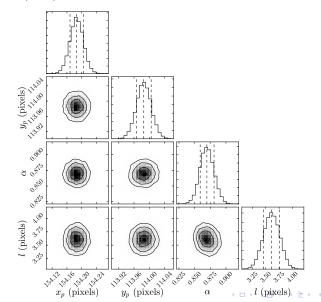


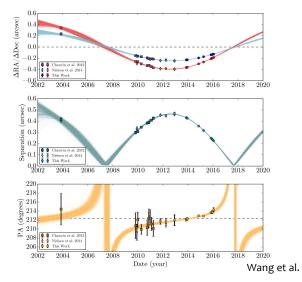
Application: YJHK Spectrum of β Pic b

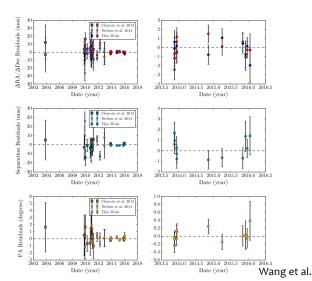


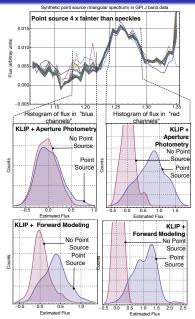






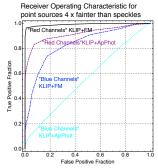




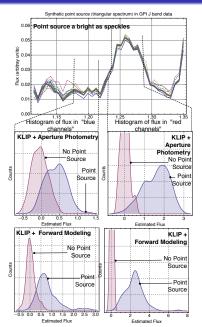


Forward Modeling for the detection problem

- Forward Modeling does not change the False Positive Fraction (= does not change the post KLIP speckles statistics).
- Forward Modeling changes the True Positive Fraction (= does change the post KLIP astrophysical flux).

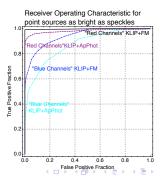






Forward Modeling for the detection problem

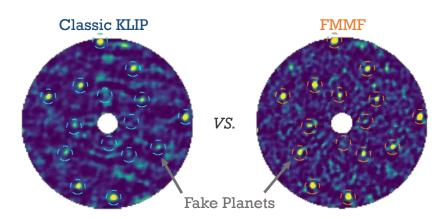
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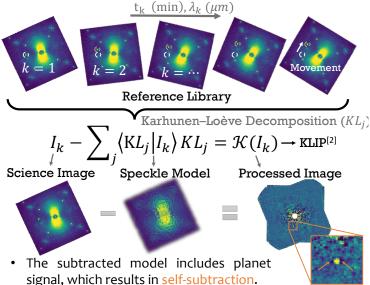




Ruffio et al., in prep.



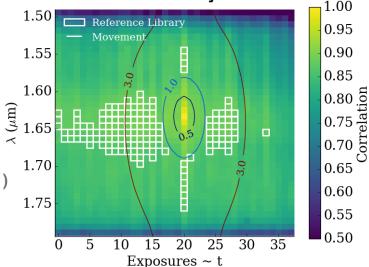
Ruffio et al., in prep.





Ruffio et al., in prep.

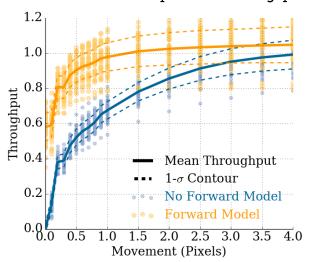
Reference Library Selection





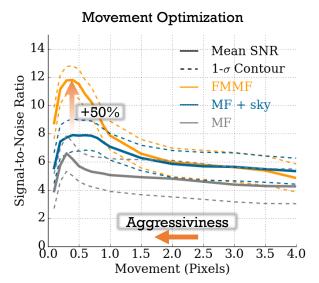
Ruffio et al., in prep.

The Forward Model improves the throughput



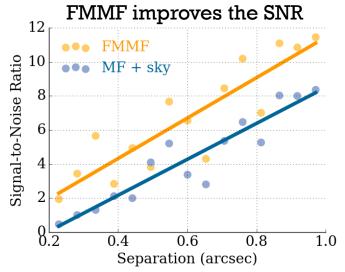


Ruffio et al., in prep.



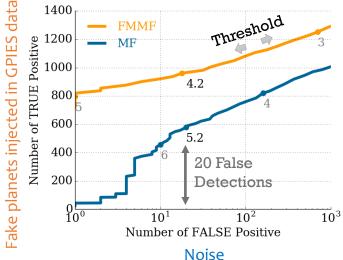


Ruffio et al., in prep.

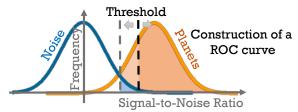


Ruffio et al., in prep.

ROC for Different Metrics



Ruffio et al., in prep.

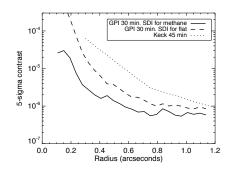


The Receiver Operating Characteristic (ROC) indicates the cost of a true detection in term of false positives. It is the right tool to compare detection metrics.

Contrast curves from different metrics should be drawn at the same false positive rate, which is not necessarily 5σ .

Contrast curves and completeness

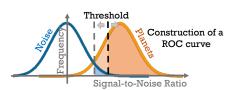
Macintosh et al. (2015)



How are survey results presented

- Pick the "right" contrast curve for each star. Delta mag vs separation.
- Convert into Mass vs SMA using your favorite model for mass-luminosity and Monte Carlo simulations to explore all possible orbits.
- Convert into Mass vs SMA using your favorite model for mass-luminosity and analytical propagation of priors.
- Sum over all stars in survey.

Contrast curves and completeness



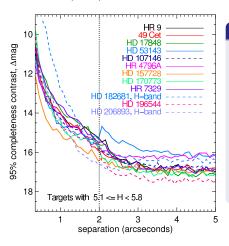
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Contrast curves and completeness

Wahhaj et al. (2013)



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Other methods

Moving forward with data analysis

By and large most of the community is using "blind" Principal Component Analysis to analyze high-contrast imaging data. This is an ancient method! There is room to do better:

- Use correlation between telemetry and images (Vogt et al., 2010).
- Use the images (and maybe telemetry) a physical model of the complex field at the telescope entrance (Ygouf et al., 2012).
- Give up on the L2 norm (L1 norm?).
- Use only positive modes and positive coefficients (Non Negative Matrix Factorization).
- "Track" the motion of the planet in the data (low rank sparse decomposition, LLSG, Gomez et al., 2016).

Thank you