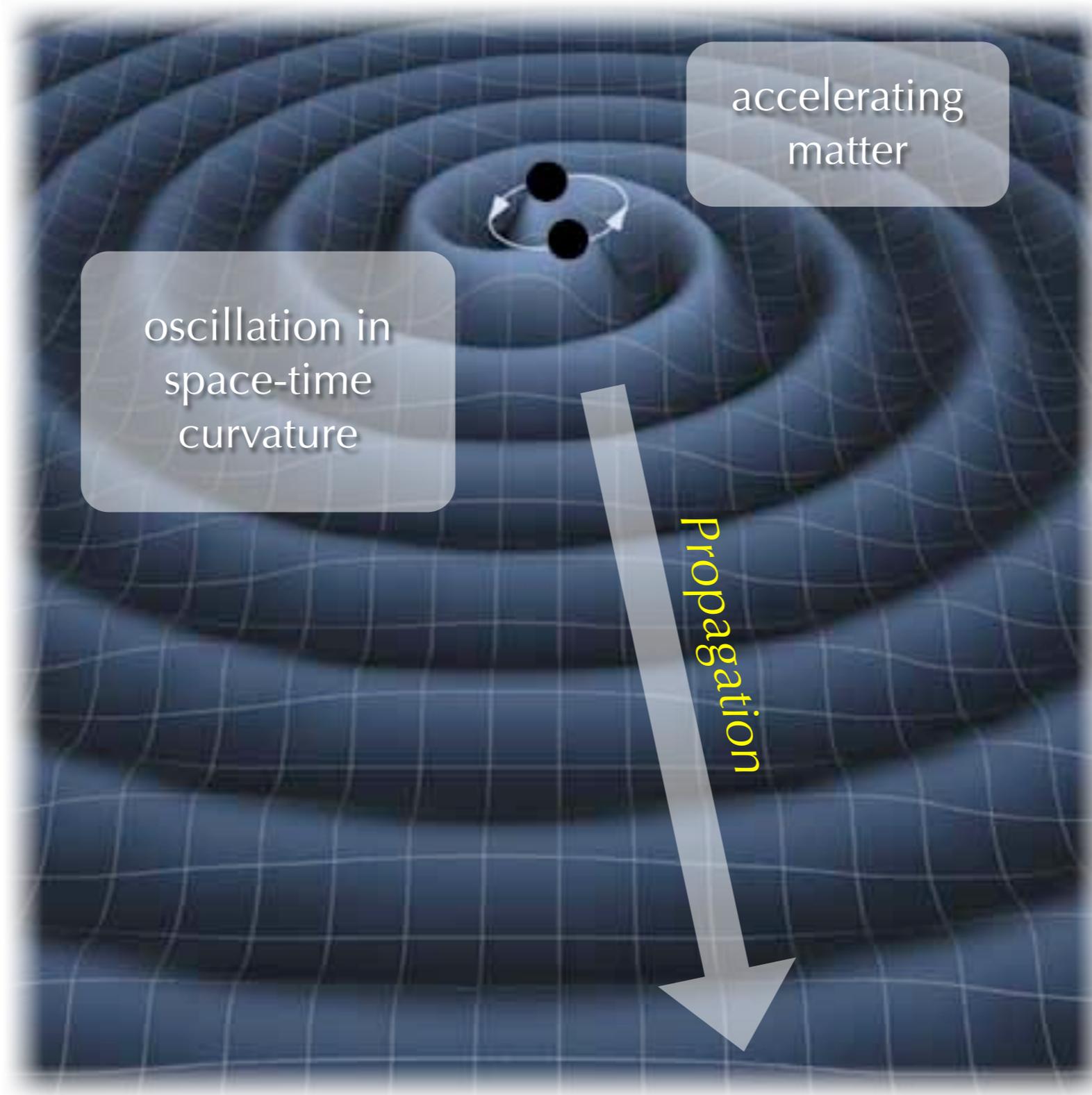


Fundamentals of Optical Interferometry for Gravitational Wave Detection

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California Institute of Technology

Gravitational Waves



A perturbation of \sim Minkowski space-time

Linearized Einstein's Equations

- Near flat spacetime, metric η is corrected by h (relative correction in time² or length²)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

- “trace-reversed perturbation” satisfies wave eqn, sourced by energy and momentum

$$\square \bar{h}_{\mu\nu} = -16\pi T_{\mu\nu}$$

T_{00} : energy density, $T_{01,02,03}$: momentum density, $T_{11,12,\dots,33}$: stress

$$\text{analogous to EM: } \square A_\mu = 4\pi J_\mu$$

- Leading multipole radiation is mass quadrupole (analogous to Electric Quadrupole)

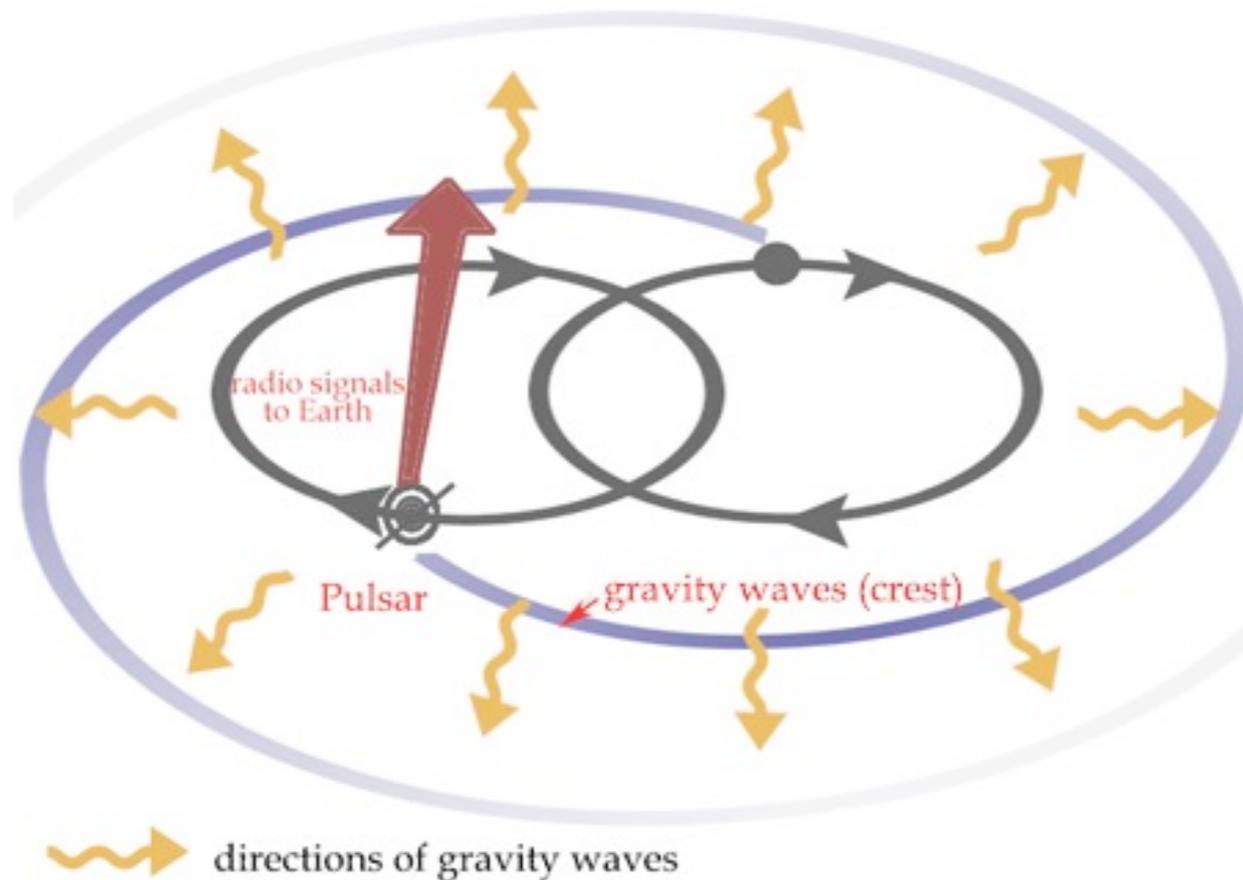
$$h \sim \frac{\ddot{Q}}{d} \sim \frac{ML^2\Omega^2}{d} \sim \frac{Mv^2}{d}$$

- Magnitude is very very small

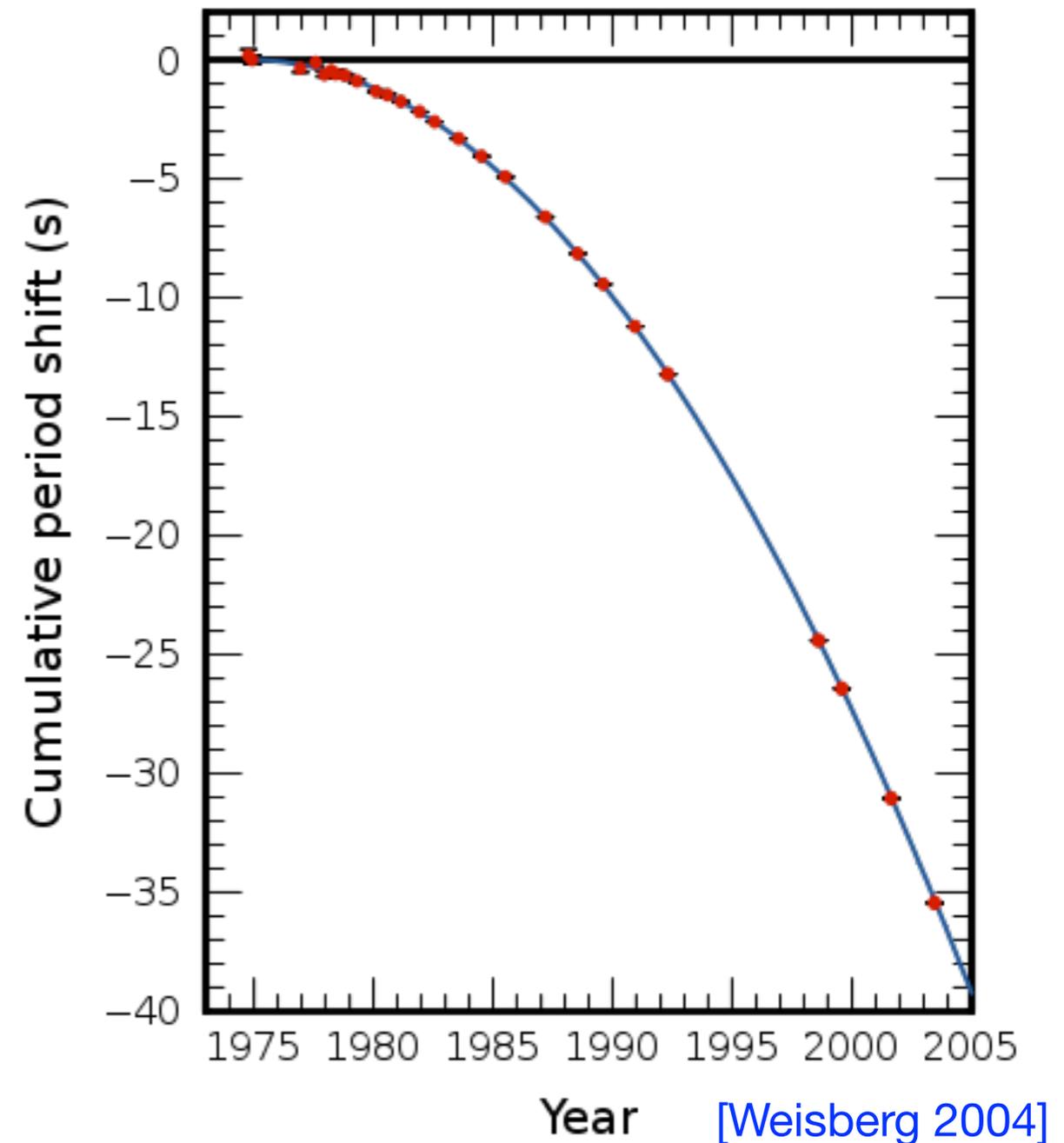
1 m away from the most powerful H-bombs tested (2×10^{17} J): $h \sim 10^{-27}$

Total mass of $5M_\odot$ colliding at $v \sim 0.3c$, at VIRGO cluster: $h \sim 3 \times 10^{-21}$

Evidence of Gravitational Waves

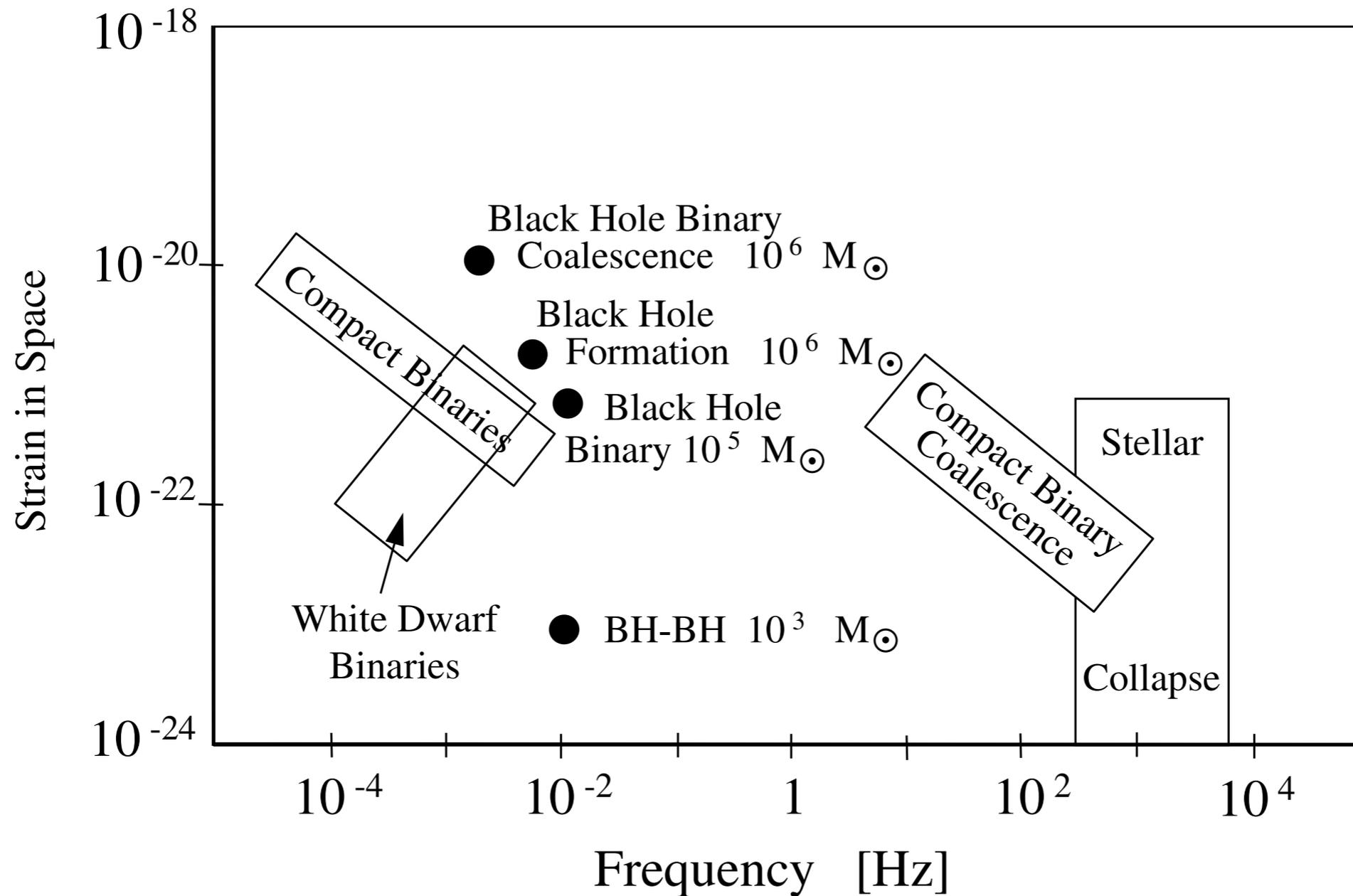


- Hulse-Taylor binary pulsar discovered in 1974
- Two $1.4 M_{\odot}$ neutron stars orbiting around each other with period 7.75h, one emitting radio pulses
- Energy carried away by GW causing orbital period to shrink
- Current GW frequency (twice orbital freq): $71 \mu\text{Hz}$
- Orbital decay will cause merger in 300M years (GW frequency will reach 10Hz - kHz during merger, the final *several minutes*)



estimated merger rate
 20 - 1000/Myr
 in Milky Way
 [e.g., Kalogera et al 2004]

Sources of Gravitational Waves



←→
Space Based Detectors

←→
Ground Based Detectors

merging supermassive black holes
smaller BHs falling into supermassive BHs
binaries with larger separations
stochastic background

merging neutron stars/black holes
rotating aspherical neutron stars
collapsing stars
stochastic background

Ground-Based Detectors

- How do we detect gravitational waves on the earth?
 - Effect of GW in a “small region” (compared with wavelength)
 - Optical Interferometry with short arms
 - Quantum enhancement on the ground
 - Limitations of GW detection on the ground

Plane Gravitational Wave

- Coordinates can be chosen such that a plane wave along z direction can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$h_{ij}^{\text{TT}}(t, x, y, z) = \begin{bmatrix} h_+(t-z) & h_\times(t-z) & 0 \\ h_\times(t-z) & -h_+(t-z) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad i, j = x, y, z$$

This is called the TT gauge because h is transverse, and traceless.

$h_{+, \times}$ are the two polarizations of the plane GW

This coordinate system is convenient in describing wave propagation, but not for describing relative motions of nearby objects

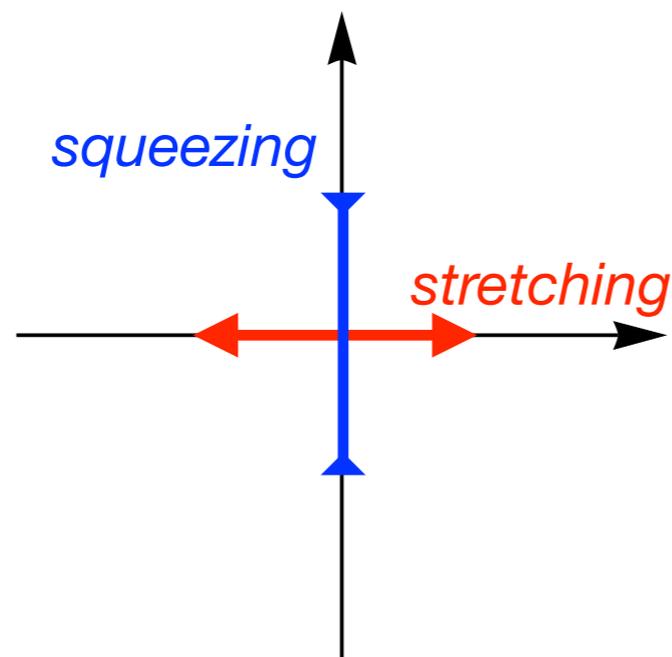
Influence of GW on Light and Matter

... in a region with spatial size much less than GW wavelength
we go to the Local Lorenz Frame

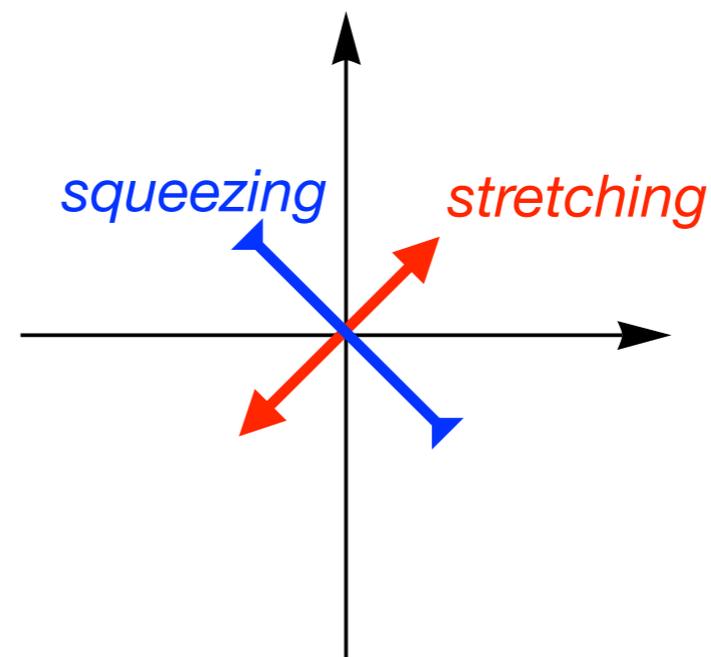
- low-velocity objects feel tidal gravity force:

$$M\ddot{x}^j = \frac{1}{2} M \ddot{h}_{jk}^{TT} x^k + F^j \quad h_{jk}^{TT} = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- array of free masses will be distorted with strain $\sim h$



+ polarization



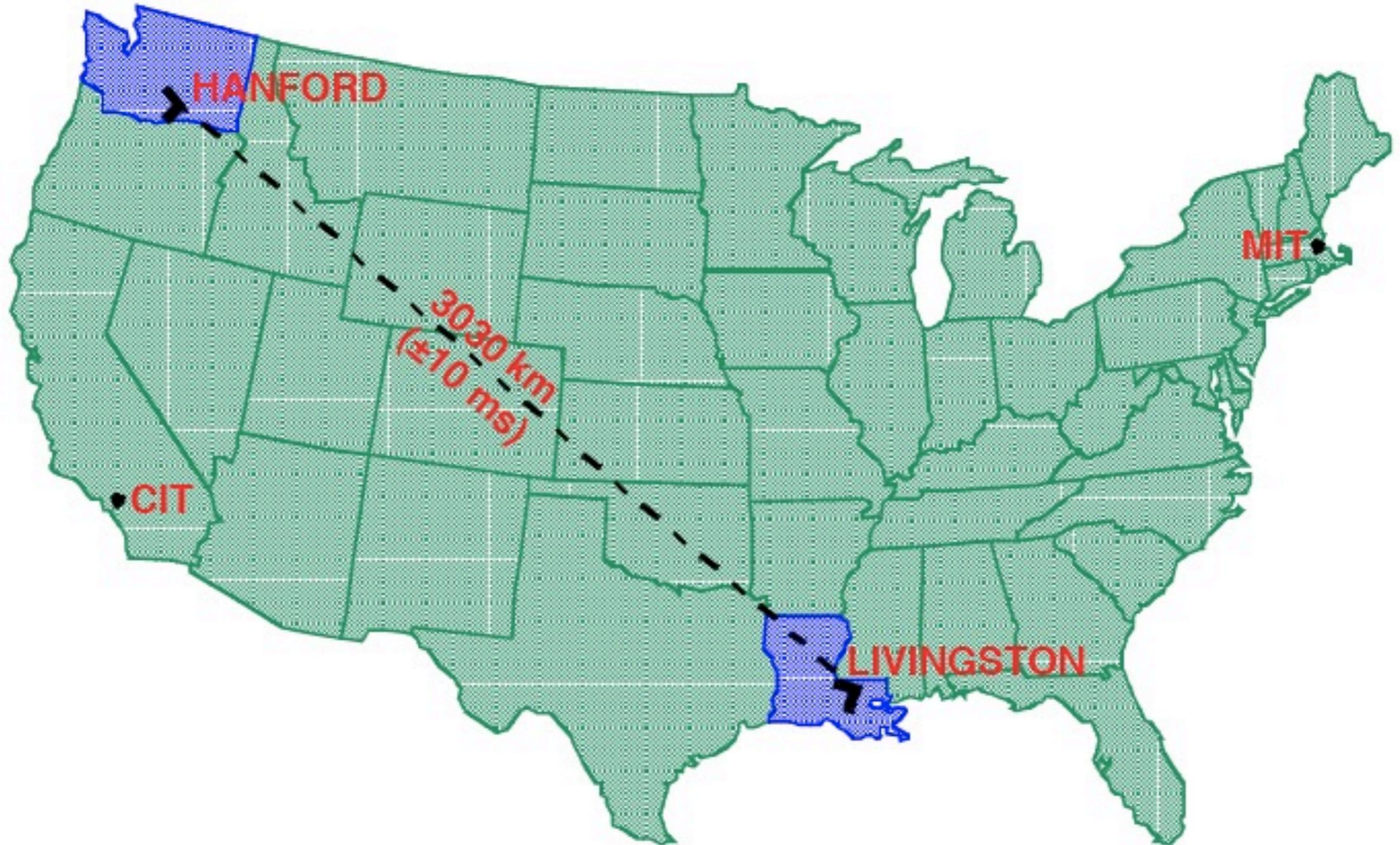
× polarization

- Light propagation is unaffected by gravitational wave
- Problem reduced to the measurement of a (very weak) classical force field

A Global Network



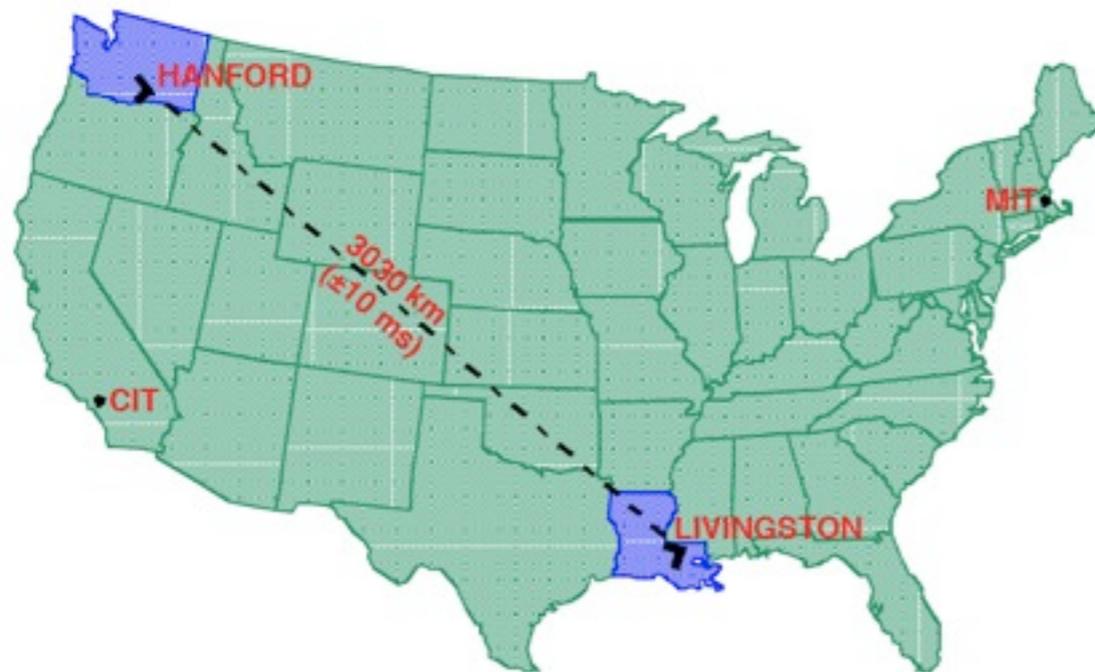
Laser Interferometer Gravitational-wave Observatory (LIGO) ¹⁰



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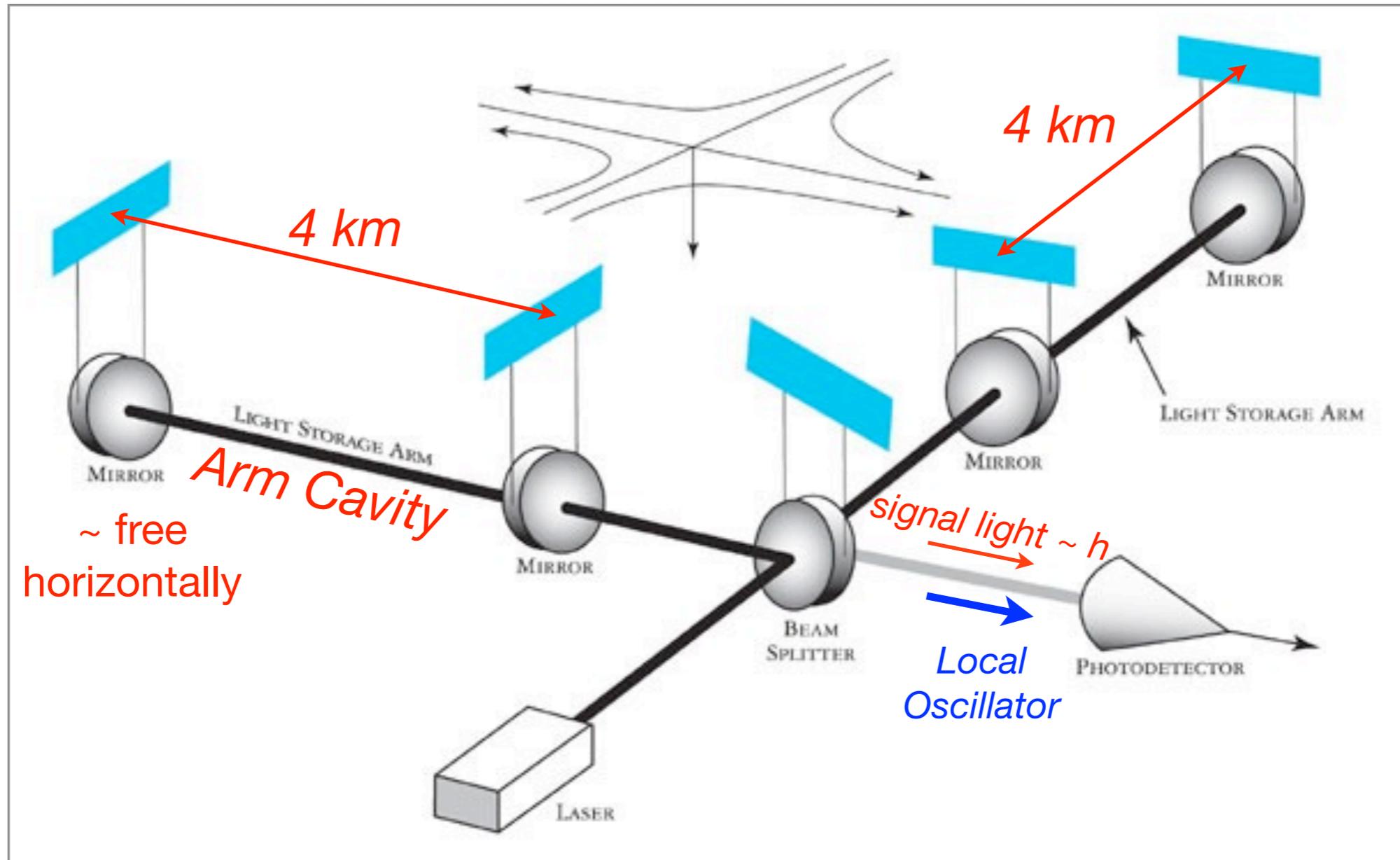


LIGO Hanford, WA site



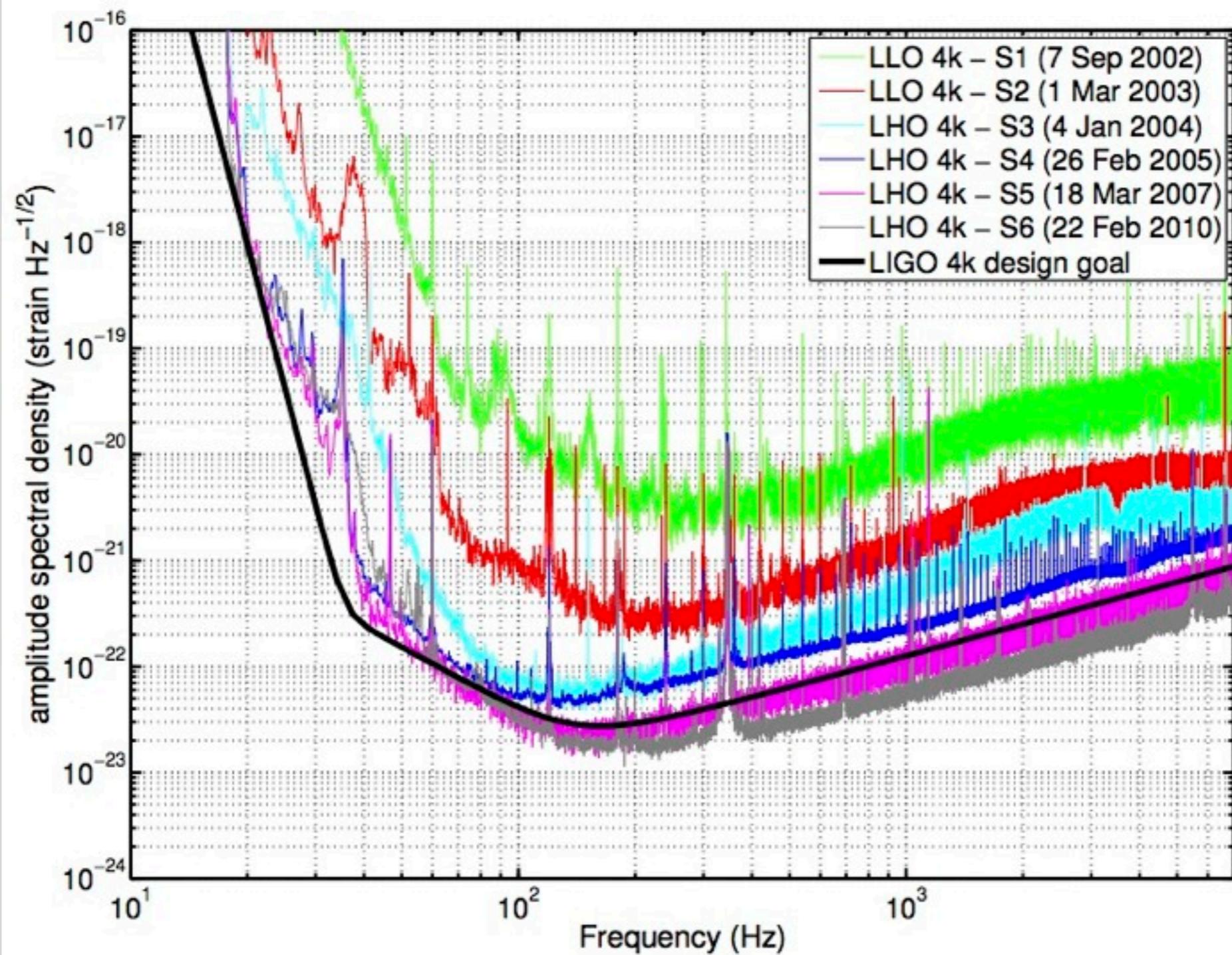
LIGO Livingston, LA site

Ground-Based Laser Interferometer GW Detector ¹¹



Schematic drawing of LIGO Detectors

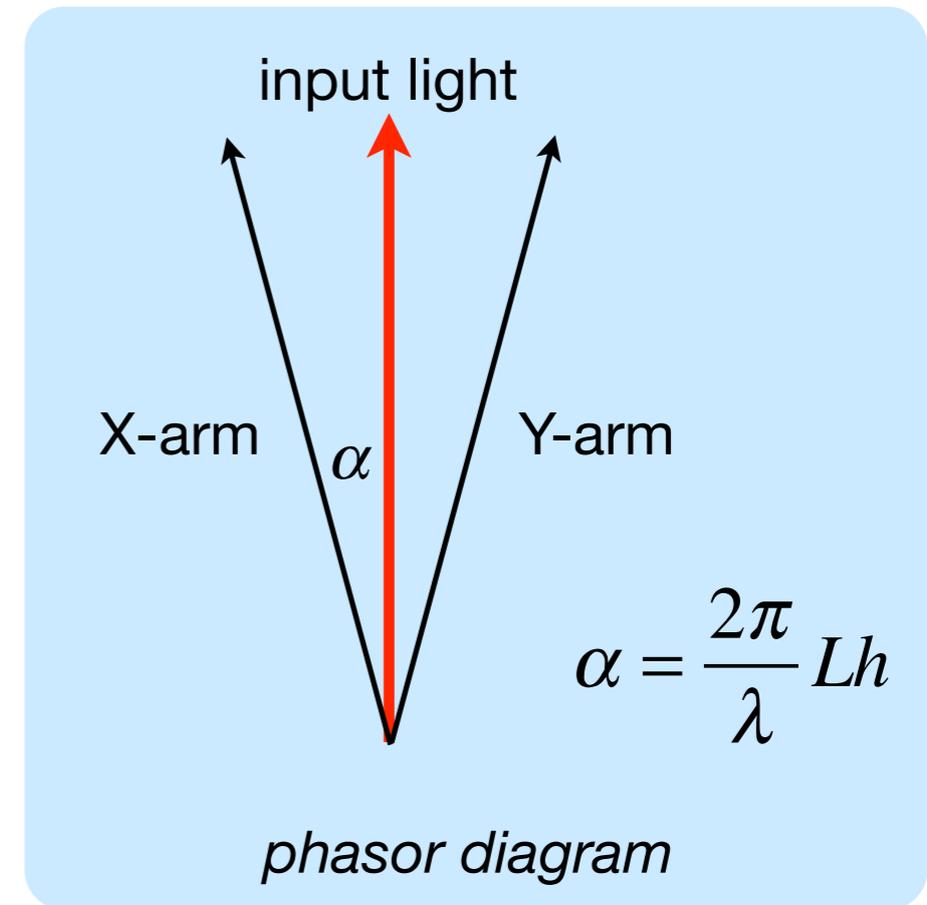
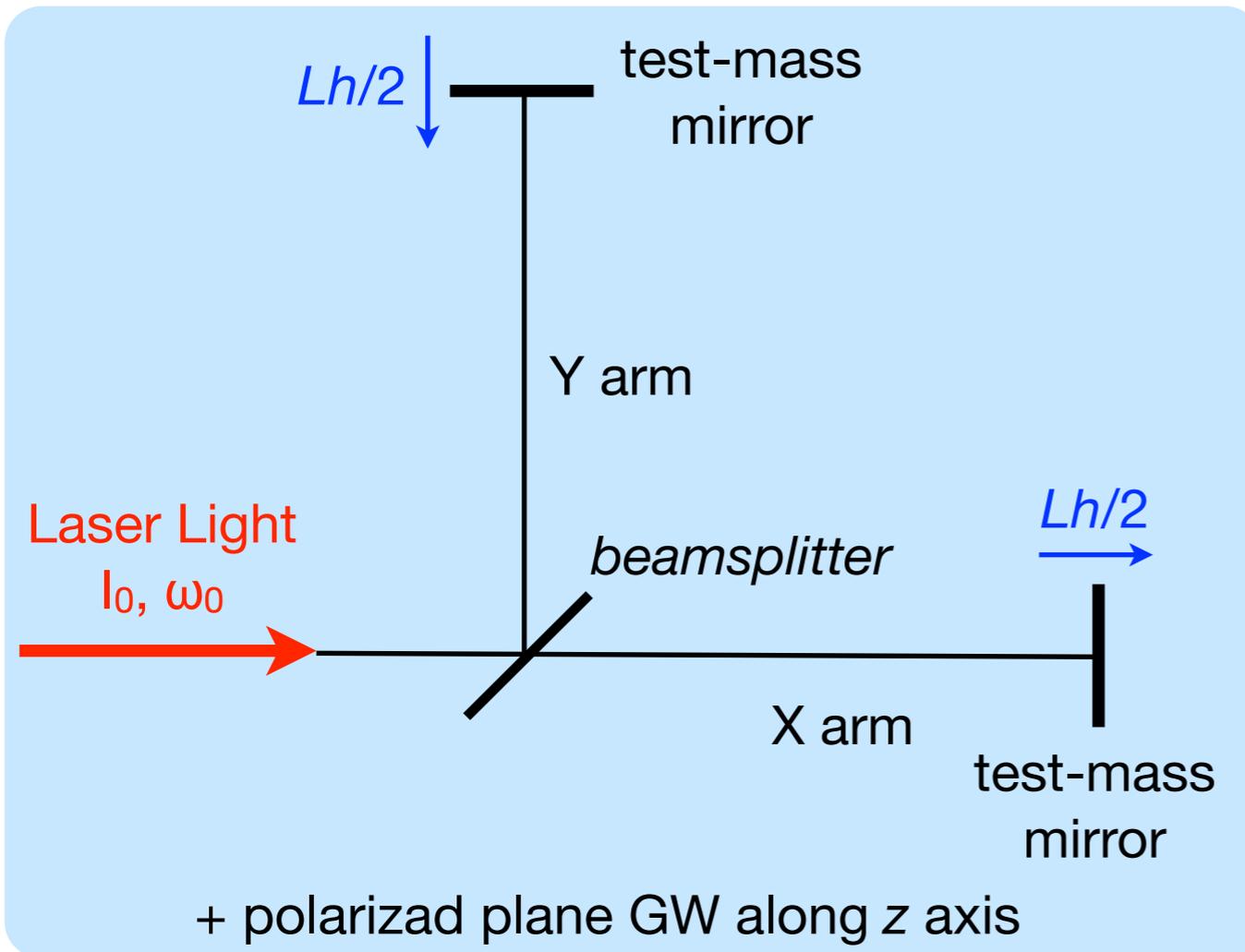
Sensitivity achieved in first-generation LIGO



achieving 2×10^{-23} /rtHz at ~ 200 Hz

Spectral Density: Noise Power Per Frequency Band $h_{rms} \sim \sqrt{f \cdot S_h}$

Michelson Interferometer: Sensitivity Estimate



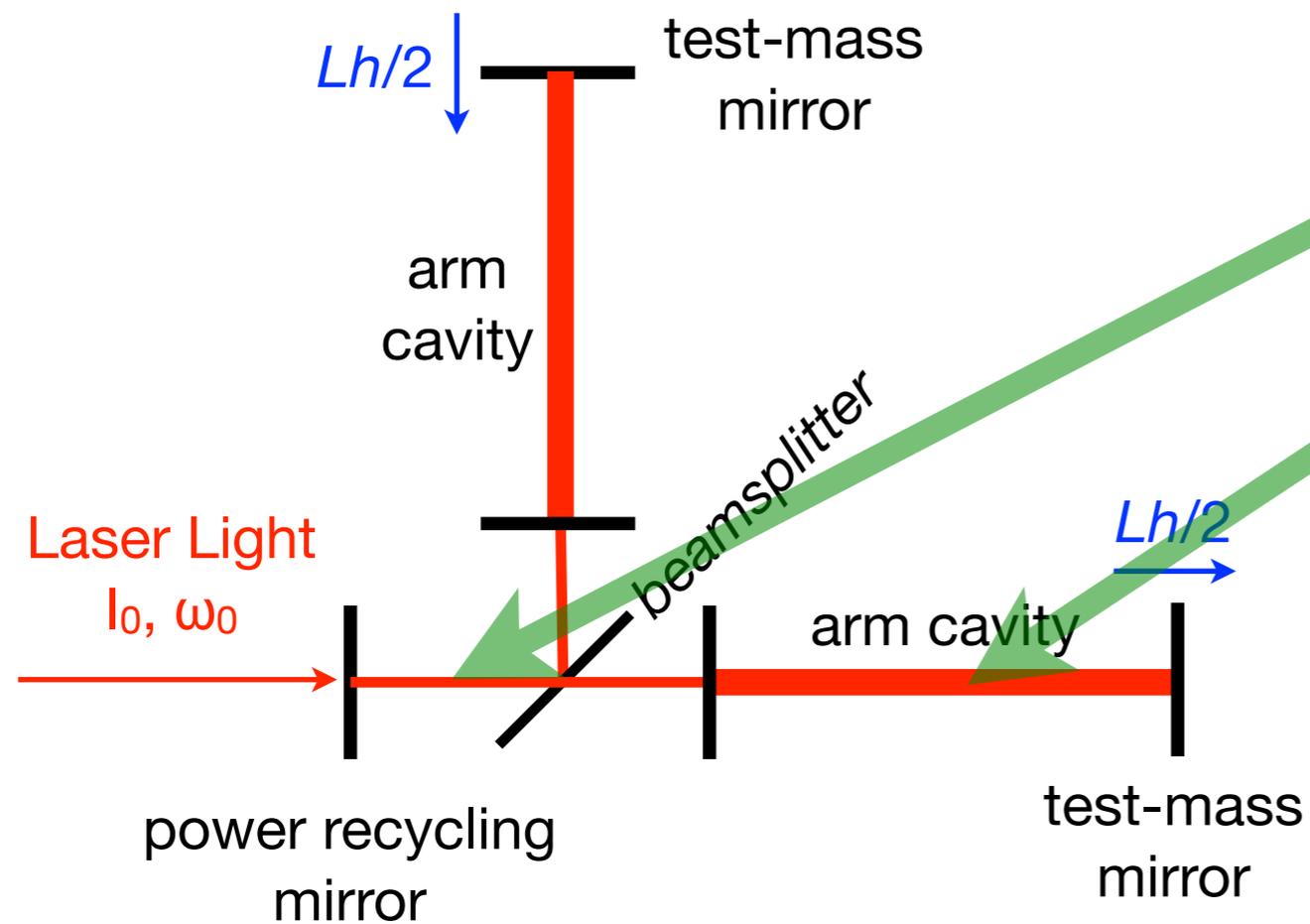
- Resolvable phase: $\sim 1/(\text{Number of Photons})^{1/2}$
- Photon Number: $\text{Power} \times \text{Duration} / (\text{Energy of Photon})$

assuming $\lambda = 1 \mu\text{m}$

$$\delta h = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \omega_0}{I_0 \tau}} \Rightarrow \sqrt{S_h} = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \omega_0}{I_0}} = 7.5 \cdot 10^{-21} \left(\frac{4 \text{ km}}{L} \right) \left(\frac{5 \text{ W}}{I_0} \right)^{1/2} \text{ Hz}^{-1/2}$$

δh is labeled as "rms error".
 S_h is labeled as "shot noise" spectral density (noise power/frequency band).
 The final term is labeled as "initial LIGO 2×10^{-23} factor of 300-400 away!".

Resonant Enhancement of Sensitivity



LIGO I:

Power-recycling gain $\sim 50 - 60$
 [noise $\sim 1/(\text{Mich. input power})^{1/2}$]

of bounces in arm cavity ~ 40

Total factor of improvement ~ 300

Improvement is only below the *bandwidth* of the cavity.

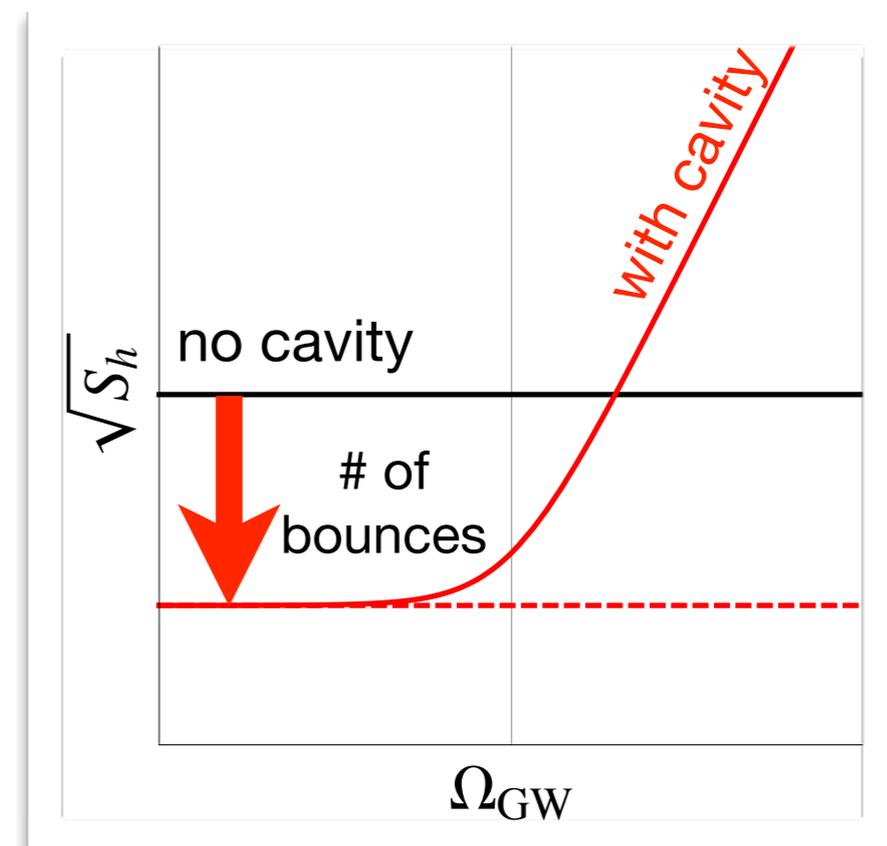
Above bandwidth:

$$\Omega_{\text{GW}} > \gamma \Leftrightarrow \tau_{\text{GW}} < \tau_{\text{storage}}$$

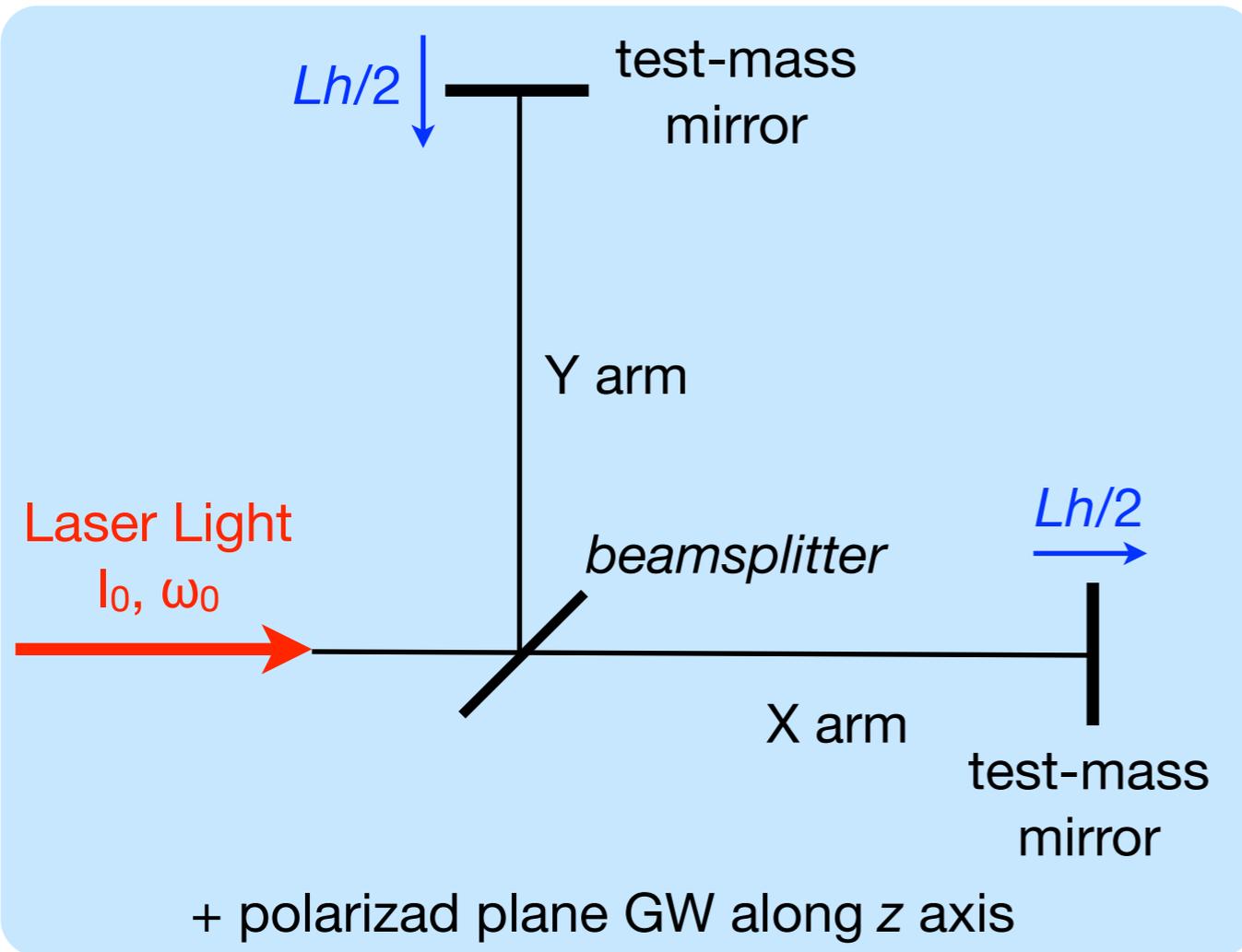
within light storage time, GW already changes sign. Resonant enhancement deteriorates!

$$\text{Cavity Gain} = \frac{2/T}{\sqrt{1 + (\Omega/\gamma)^2}}$$

$$\sqrt{S_h^{\text{shot}}} = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar\omega_0}{I_0}} \frac{\sqrt{1 + (\Omega/\gamma)^2}}{2/T}$$



Radiation Pressure Noise

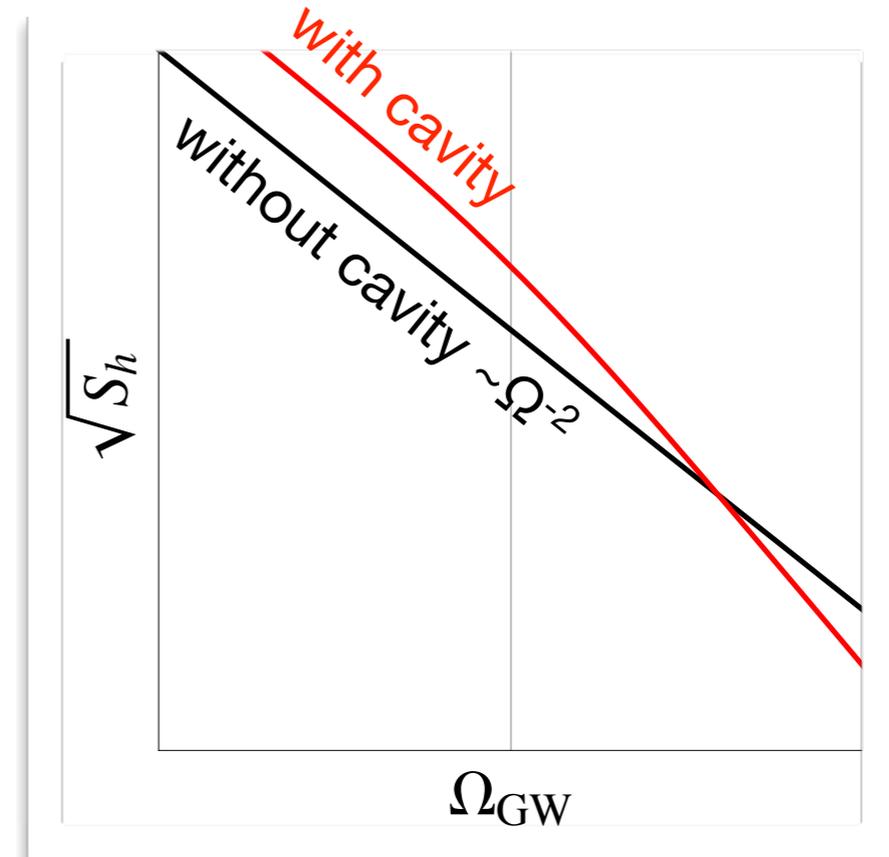


without cavity ...

$$\delta P = \delta N \cdot \frac{2\hbar\omega_0}{c} = \sqrt{N} \frac{2\hbar\omega_0}{c} = \sqrt{\frac{I_0\tau}{\hbar\omega_0}} \frac{2\hbar\omega_0}{c}$$

rms momentum of mirror
given by photon # fluctuation

$$\delta F = \delta P / \tau = \sqrt{\frac{4\hbar\omega_0 I_0}{c^2 \tau}}$$



$$\Rightarrow \sqrt{S_F} = \sqrt{\frac{4\hbar\omega_0 I_0}{c^2}}$$

$$\Rightarrow \sqrt{S_x} = \frac{1}{m\Omega^2} \sqrt{\frac{4\hbar\omega_0 I_0}{c^2}}$$

$$\Rightarrow \sqrt{S_h^{\text{rad pres}}} = \frac{1}{m\Omega^2 L} \sqrt{\frac{4\hbar\omega_0 I_0}{c^2}}$$

with cavity gain

$$\sqrt{S_h^{\text{rad pres}}} = \frac{1}{m\Omega^2 L} \sqrt{\frac{4\hbar\omega_0 I_0}{c^2}} \frac{2/T}{\sqrt{1 + (\Omega/\gamma)^2}}$$

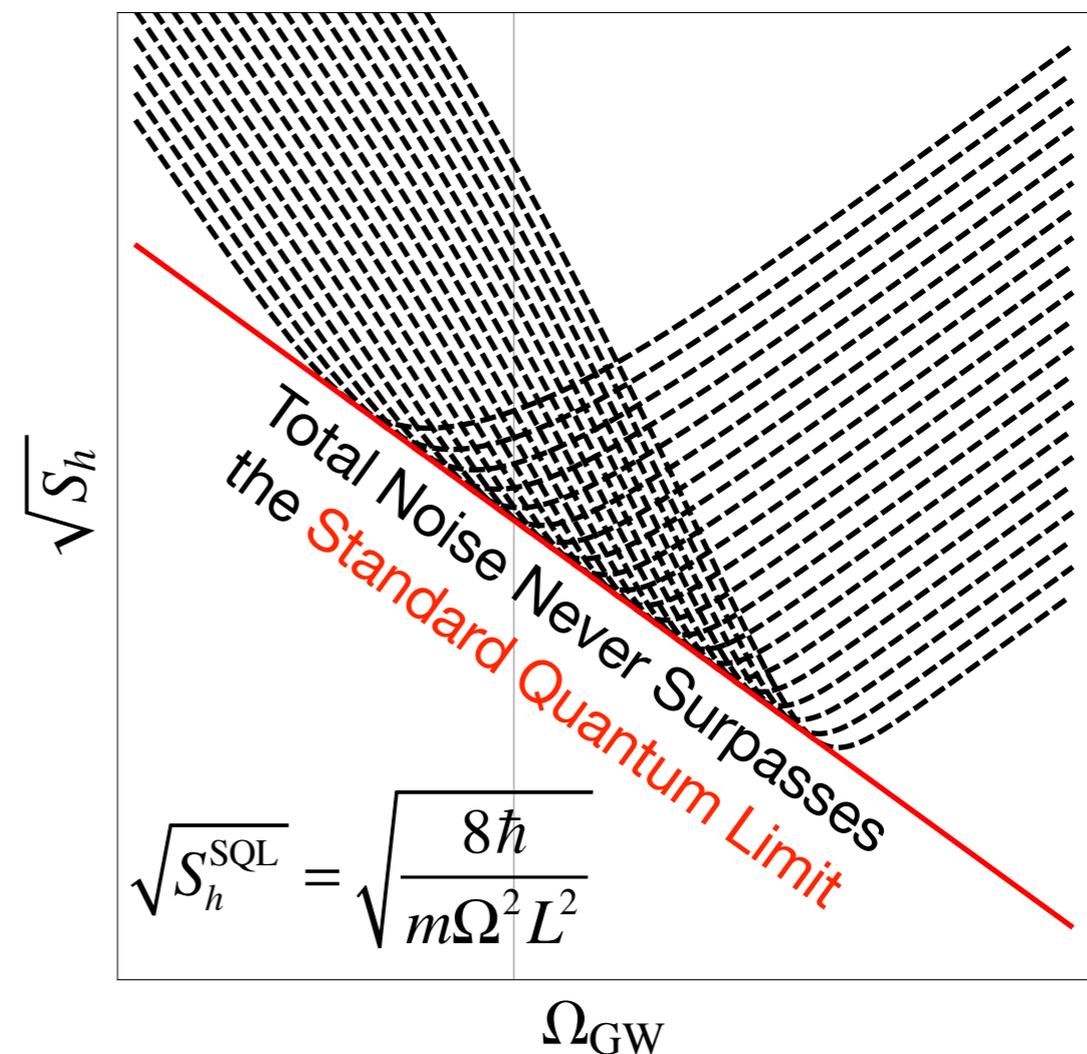
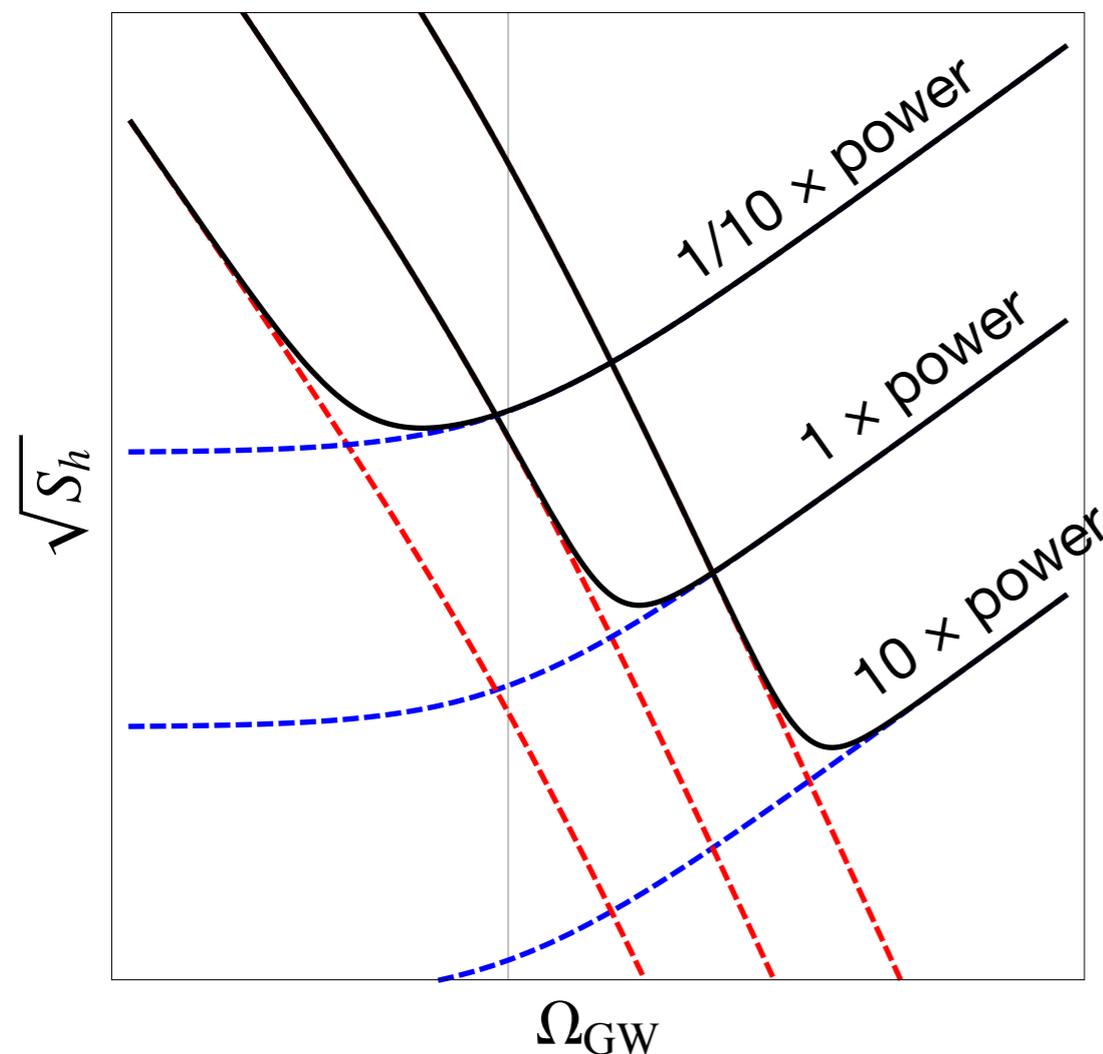
Standard Quantum Limit

If we place the two types of noise together

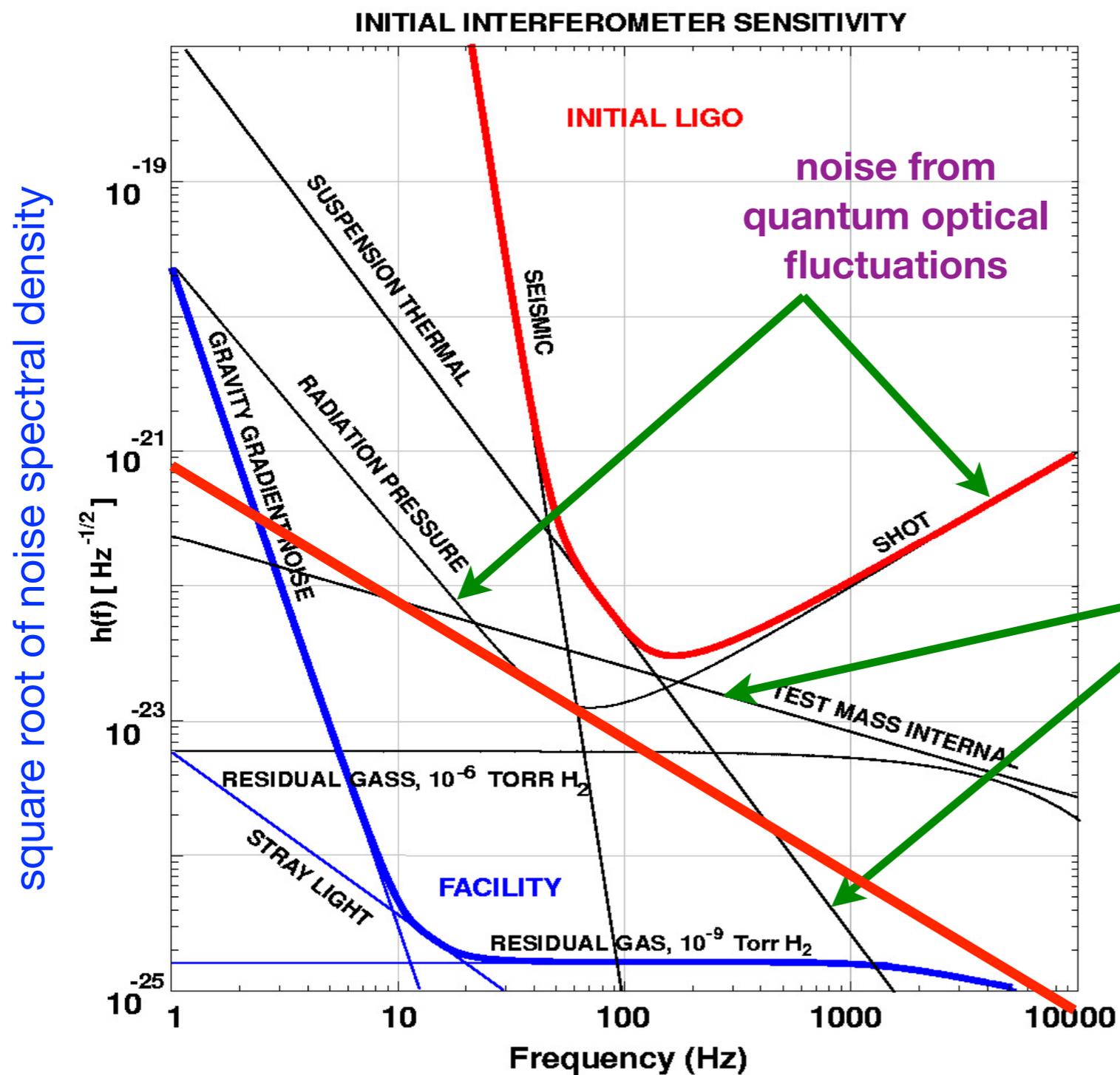
$$\sqrt{S_h^{\text{shot}}} = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar\omega_0}{I_0}} = \frac{1}{L} \sqrt{\frac{\hbar c^2}{I_0\omega_0}} \frac{\sqrt{1+(\Omega/\gamma)^2}}{2/T}$$

$$\sqrt{S_h^{\text{grad pres}}} = \frac{1}{m\Omega^2 L} \sqrt{\frac{4\hbar I_0\omega_0}{c^2}} \frac{2/T}{\sqrt{1+(\Omega/\gamma)^2}} = \frac{2\hbar}{m\Omega^2 L} \sqrt{\frac{I_0\omega_0}{\hbar c^2}} \frac{2/T}{\sqrt{1+(\Omega/\gamma)^2}}$$

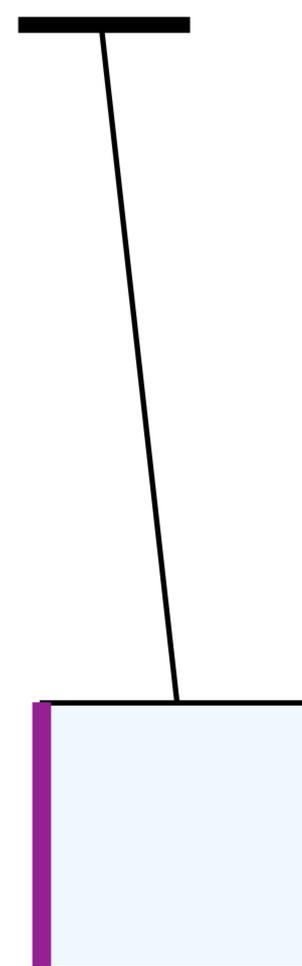
Their dependences on *power* and *cavity gain* are opposite



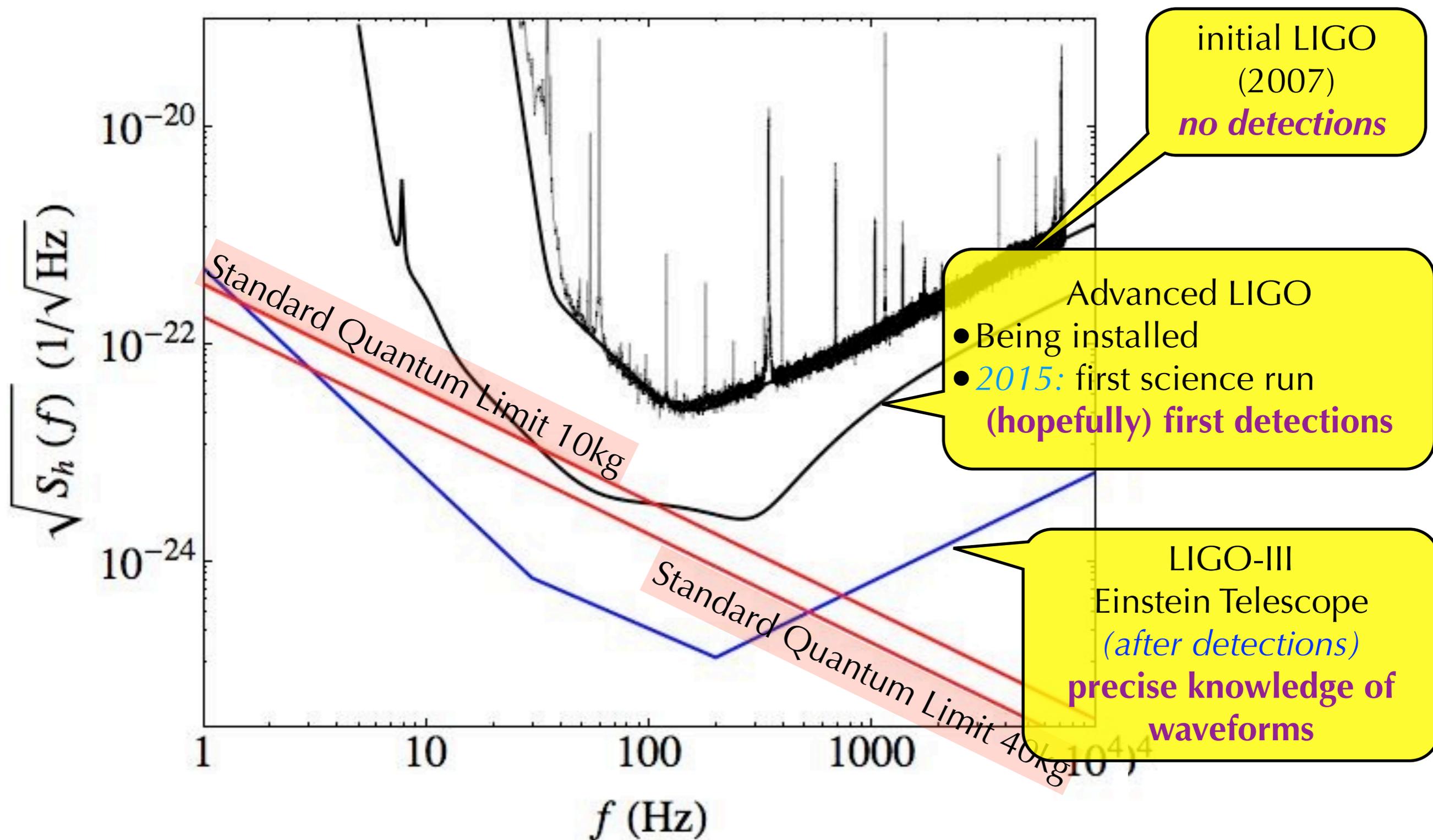
Quantum Optical Noise in LIGO-I



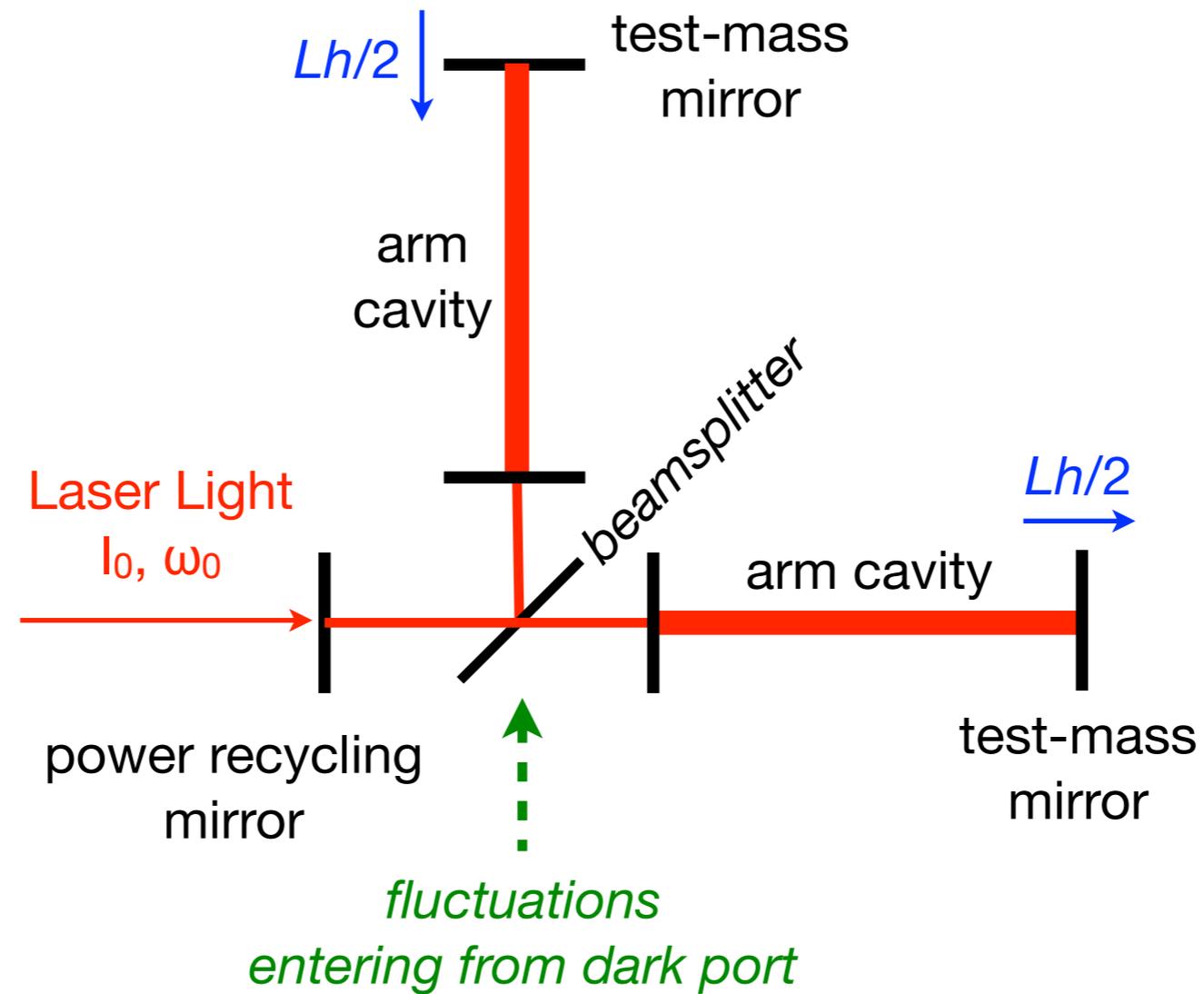
noise from thermal fluctuations



Generations of GW Detectors



Where does quantum noise come from?

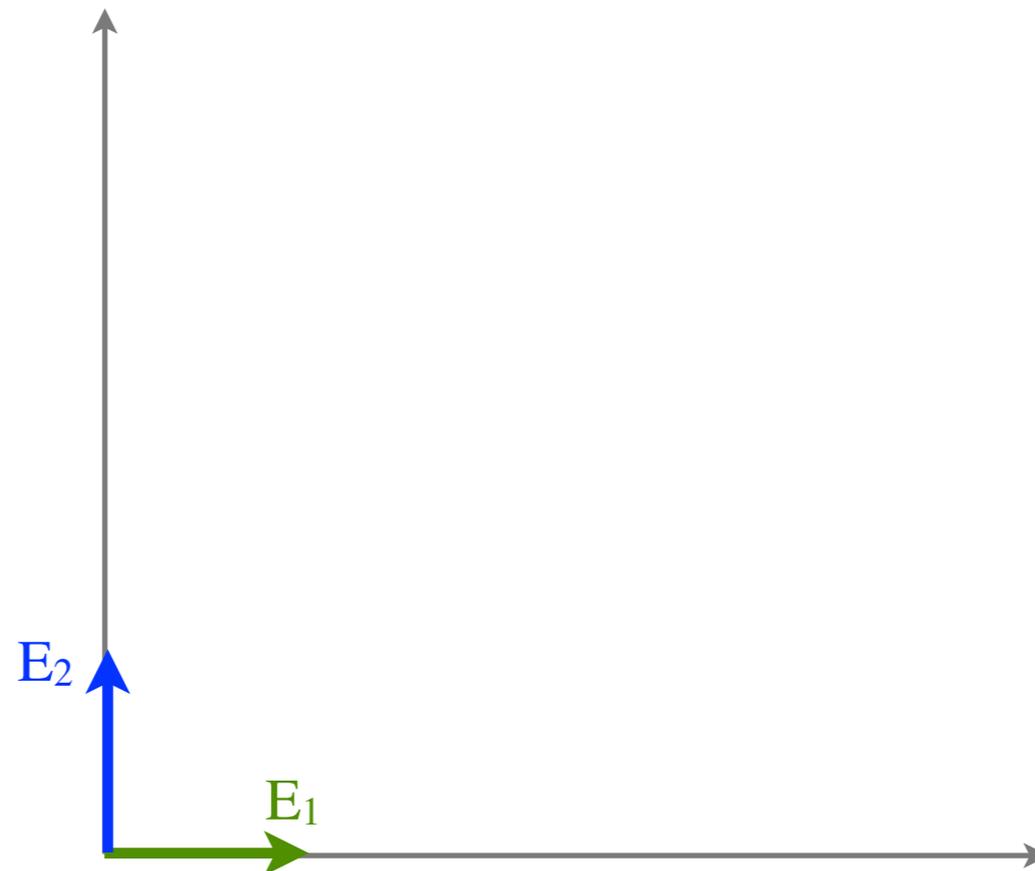


Quadratures, Homodyne Detection and Squeezing ²⁰

- Optical field close to ω_0 can be written in the **quadrature representation**

$$E(t) = E_1(t)\cos\omega_0 t + E_2(t)\sin\omega_0 t \quad E_{1,2}(t): \text{ slowly varying}$$

- Act as modulations when superimposed with single-frequency carrier at ω_0



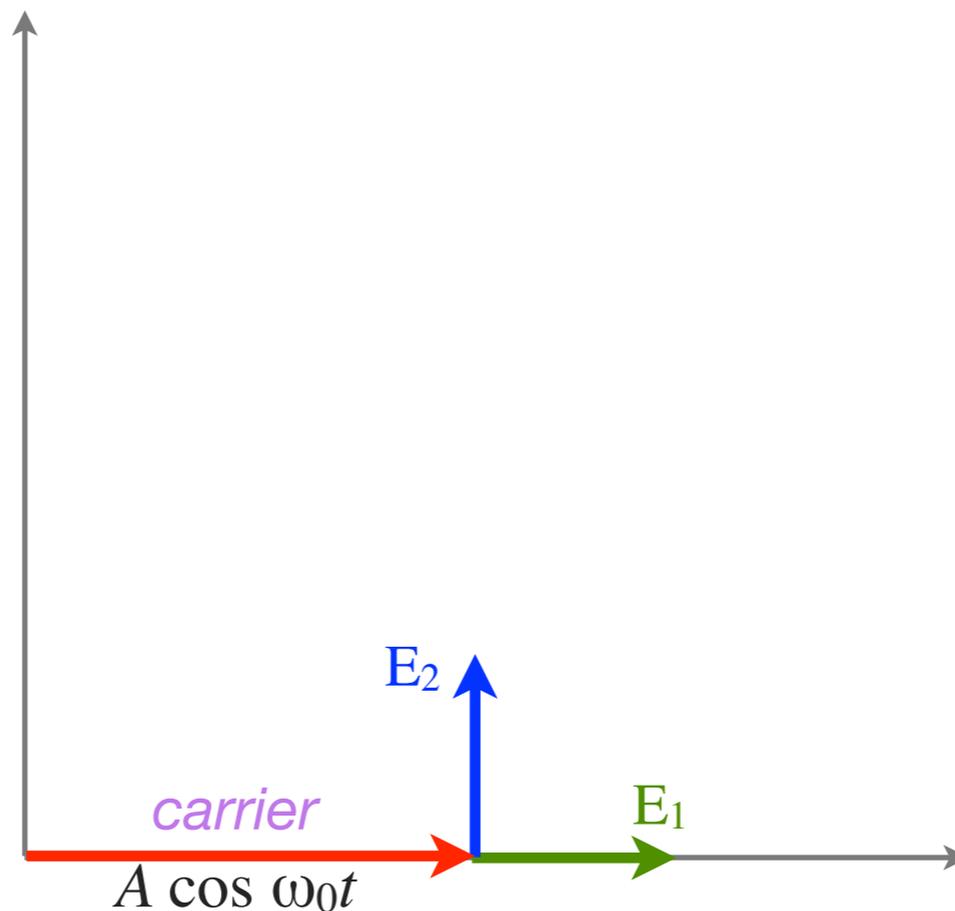
phasor diagram

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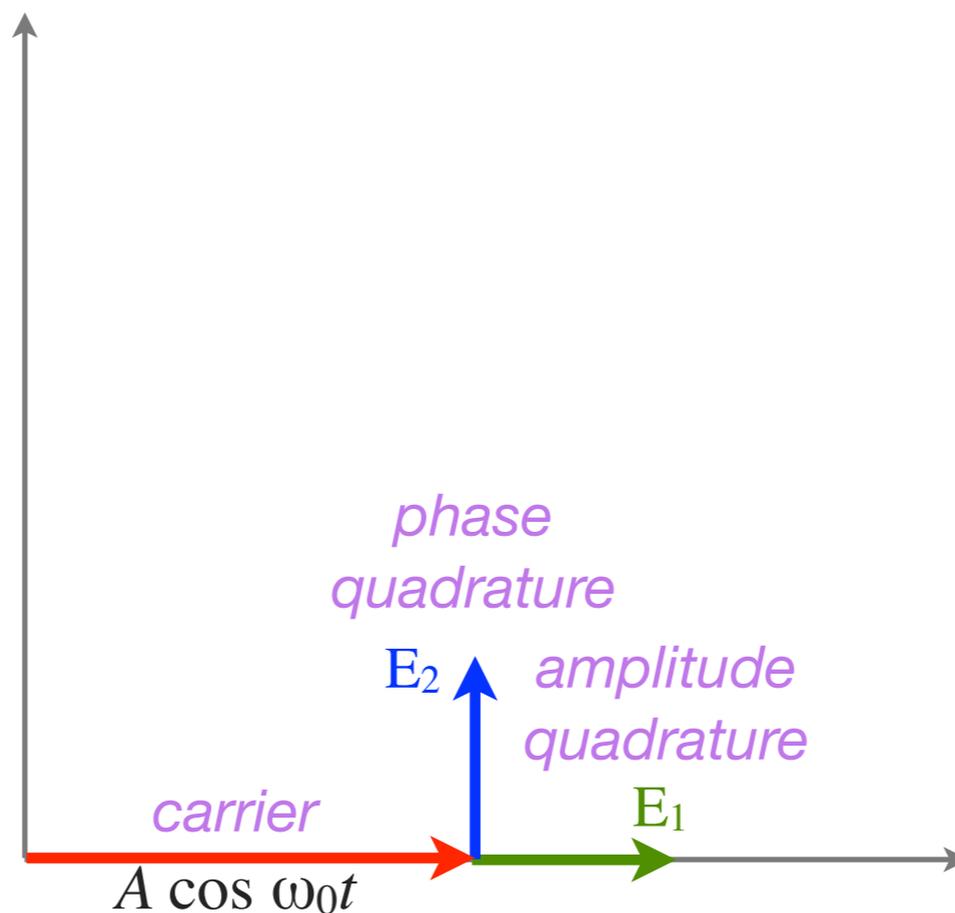
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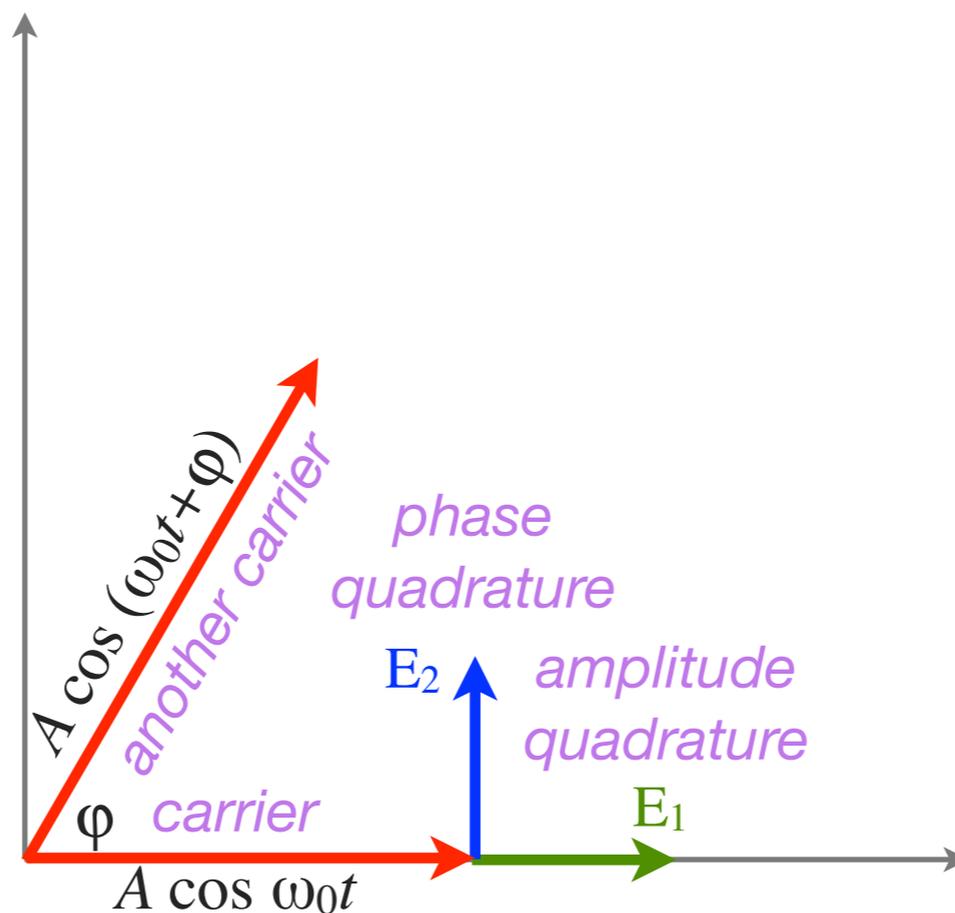
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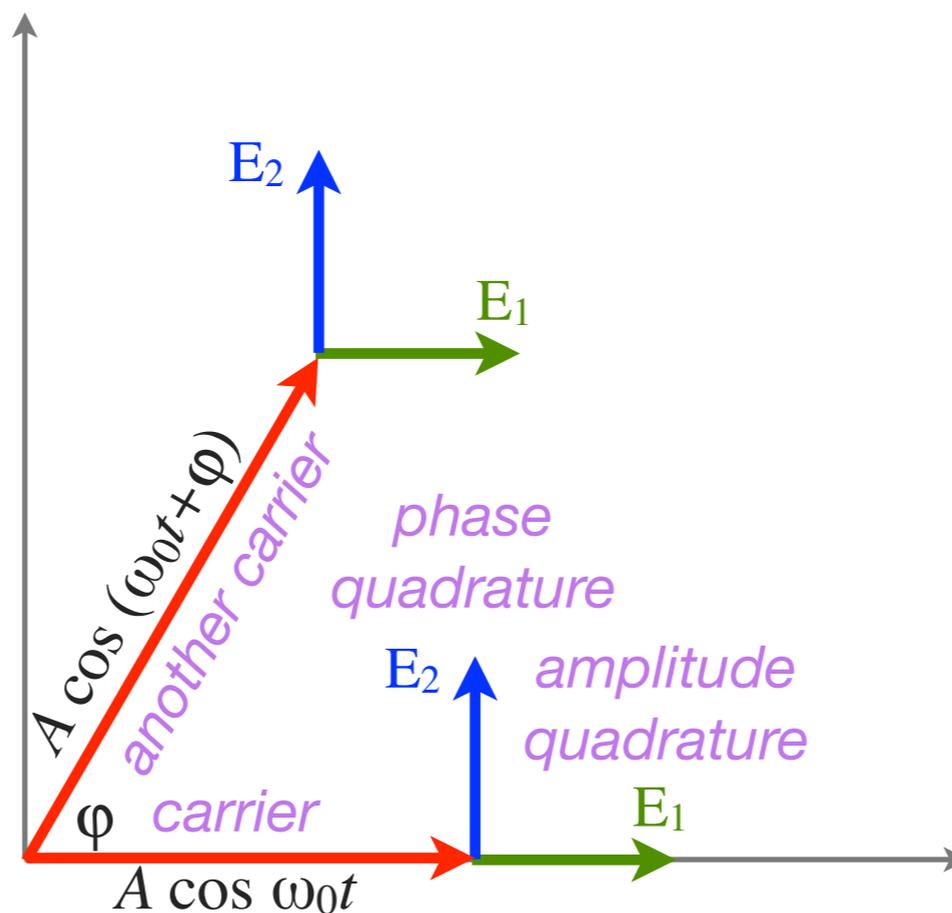
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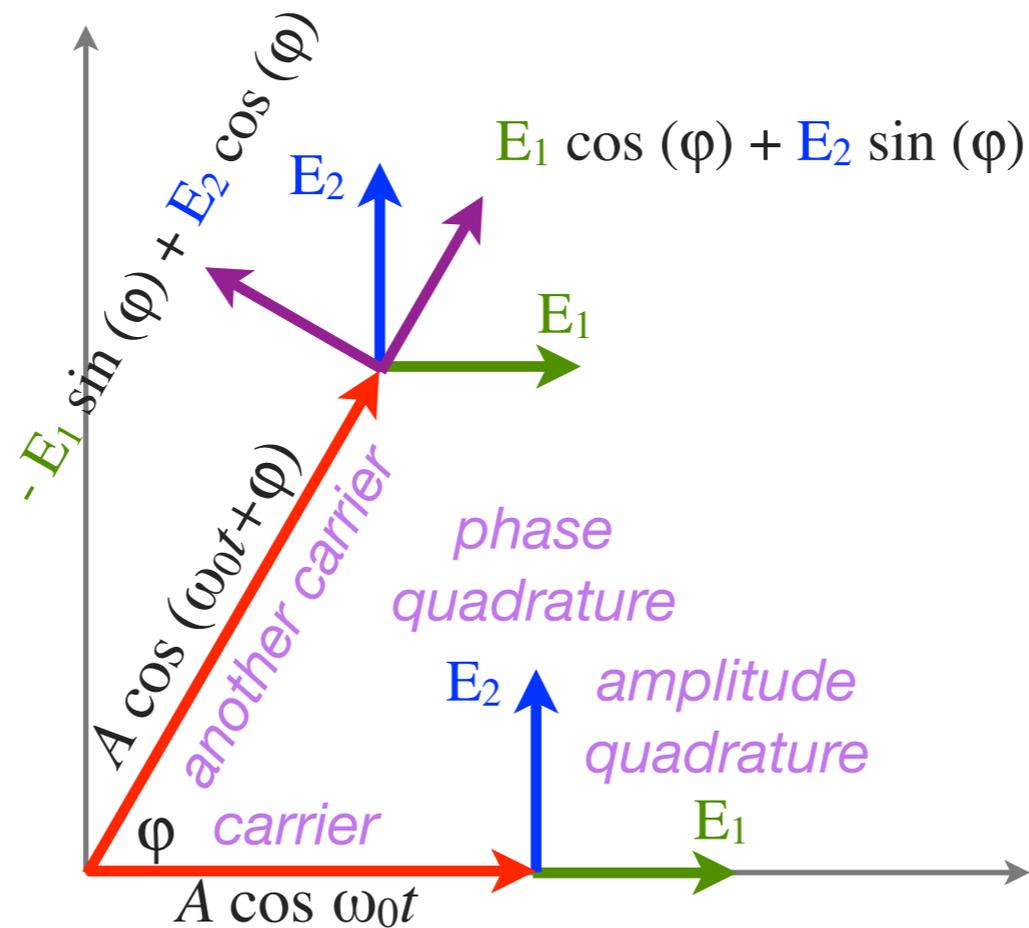
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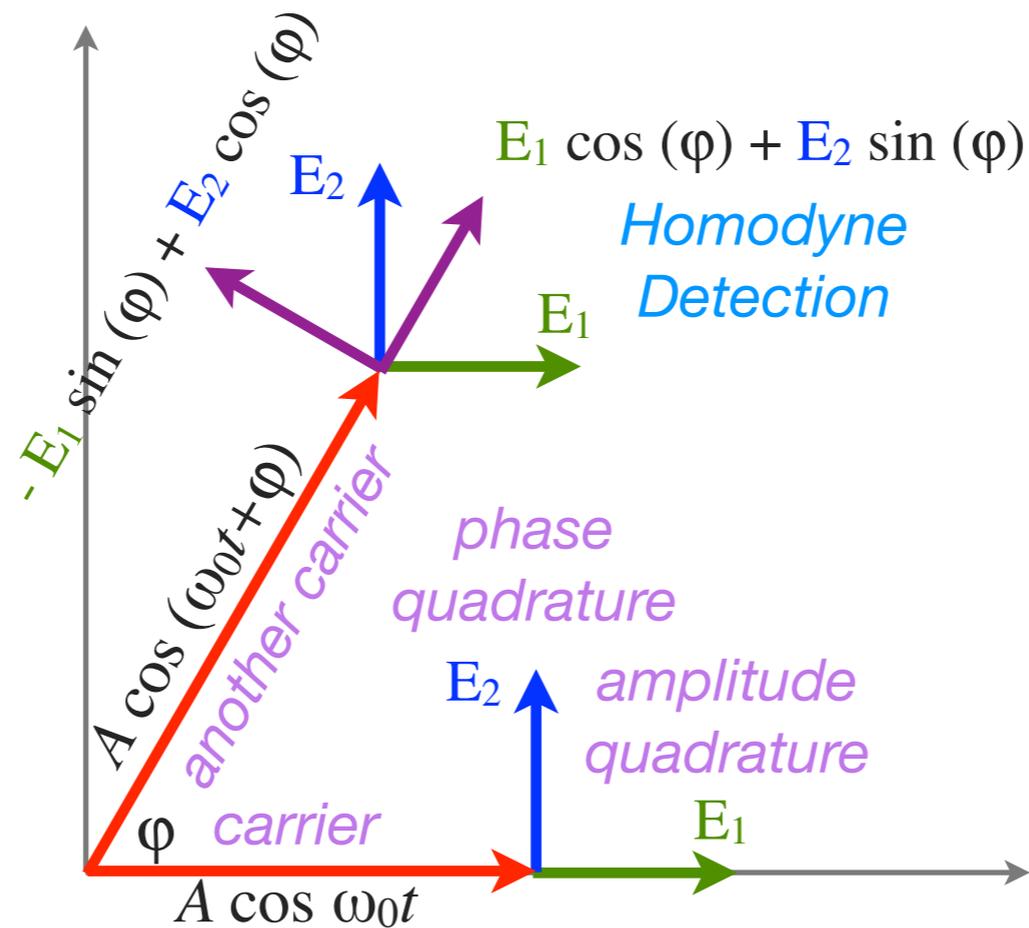


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phasor diagram

Quadratures, Homodyne Detection and Squeezing ²¹

- Heisenberg Uncertainty In the Frequency Domain

$$S_{a_1 a_1} S_{a_2 a_2} - |S_{a_1 a_2}|^2 \geq 1$$

Minimum Uncertainty

Gaussian States are:

vacuum state
coherent states
squeezed vacuua
squeezed states

Quadratures, Homodyne Detection and Squeezing ²¹

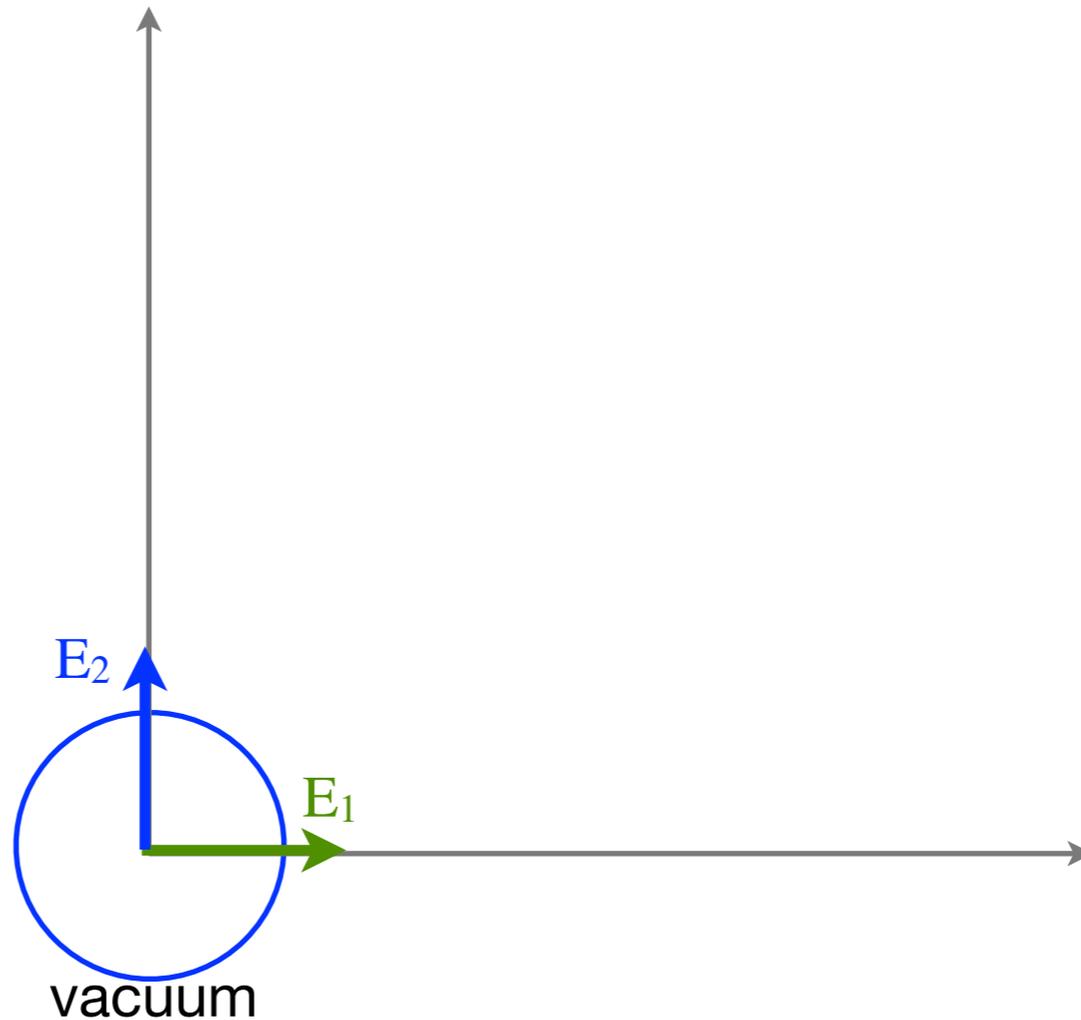
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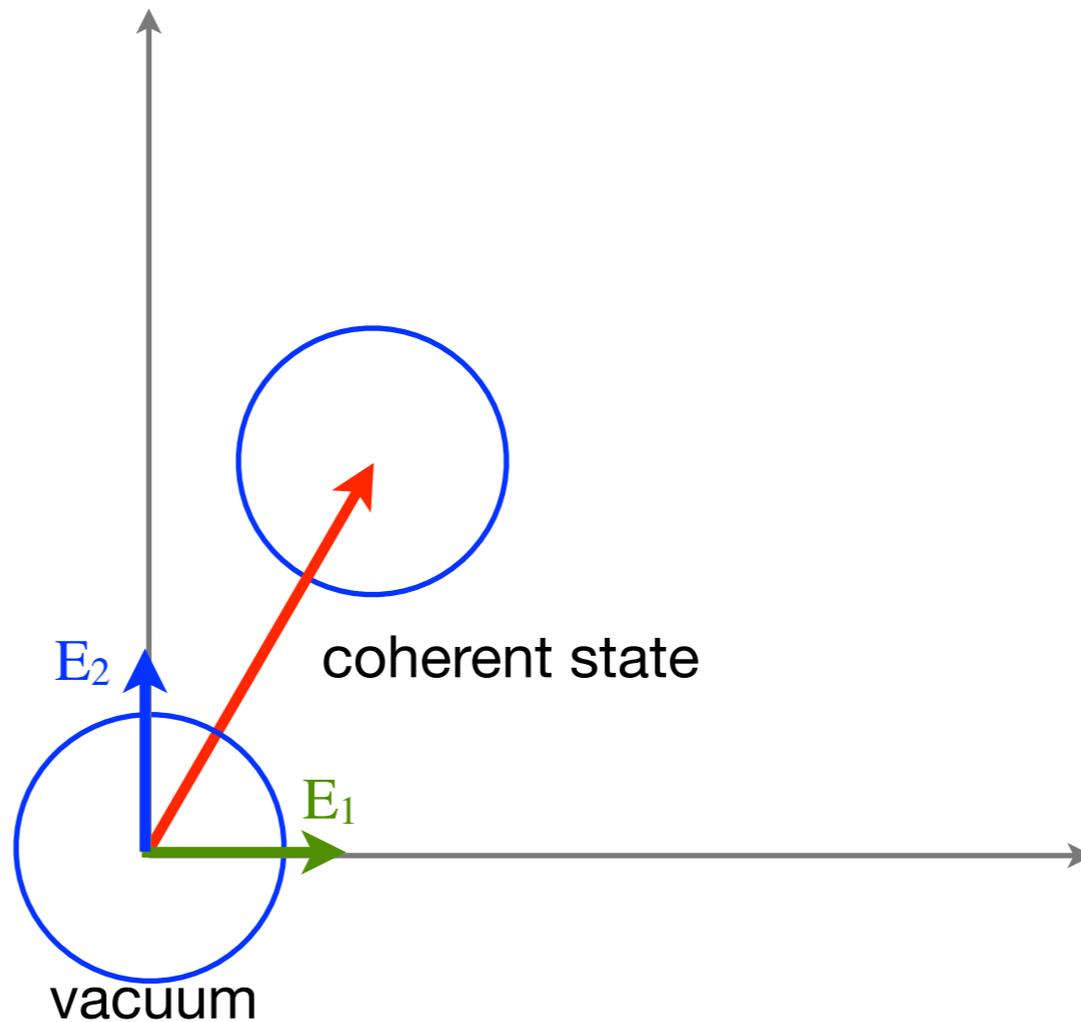
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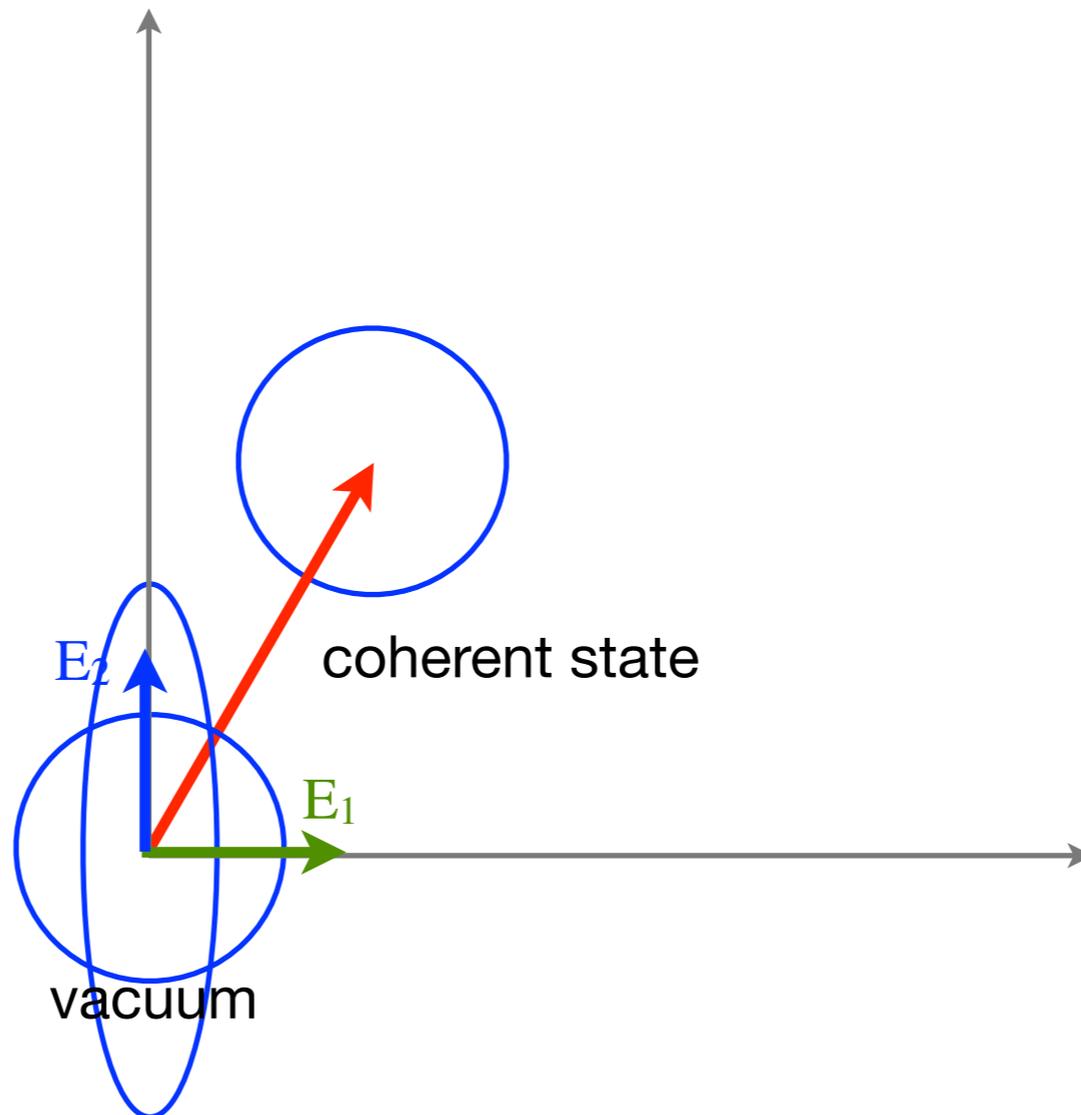
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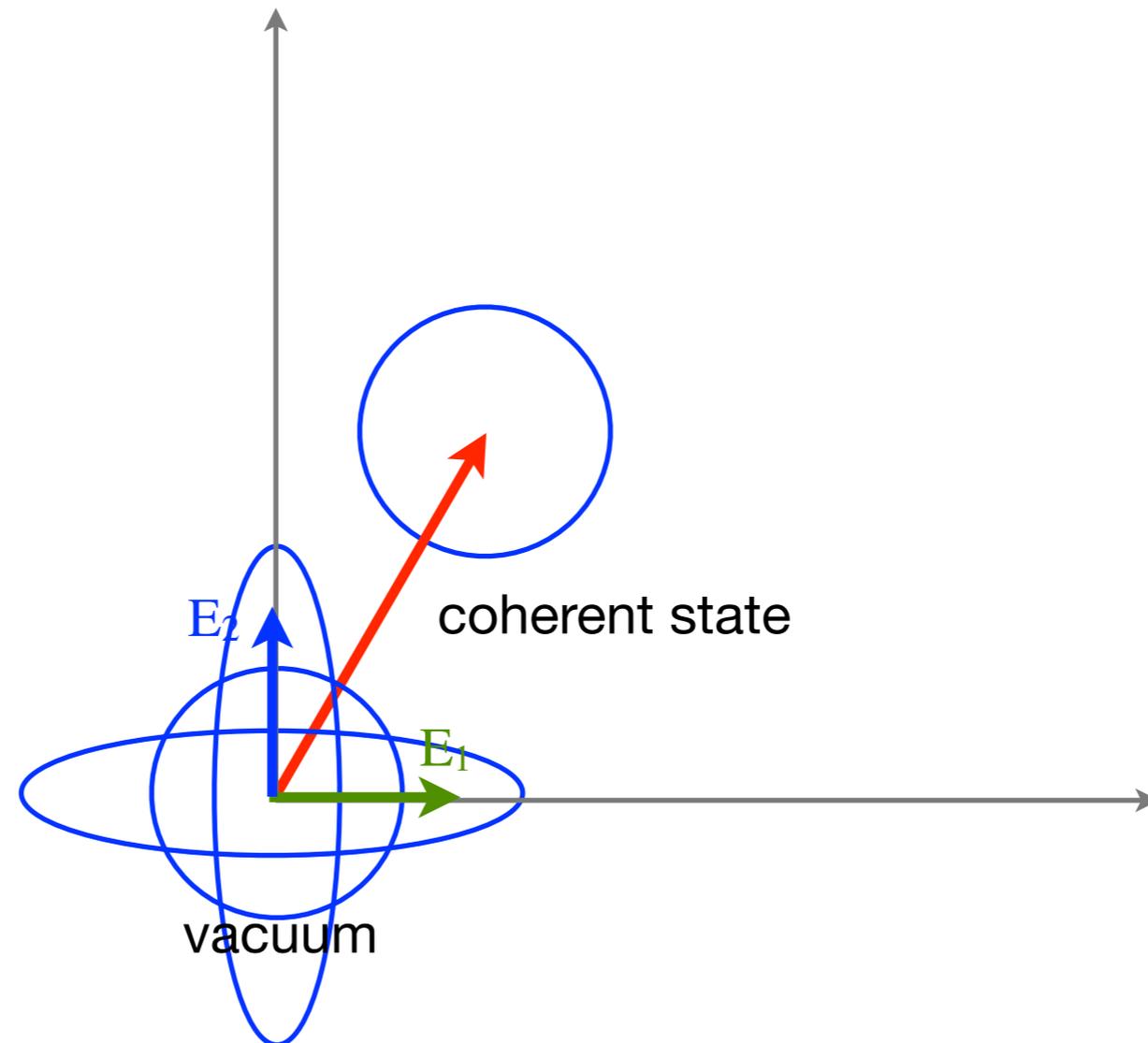
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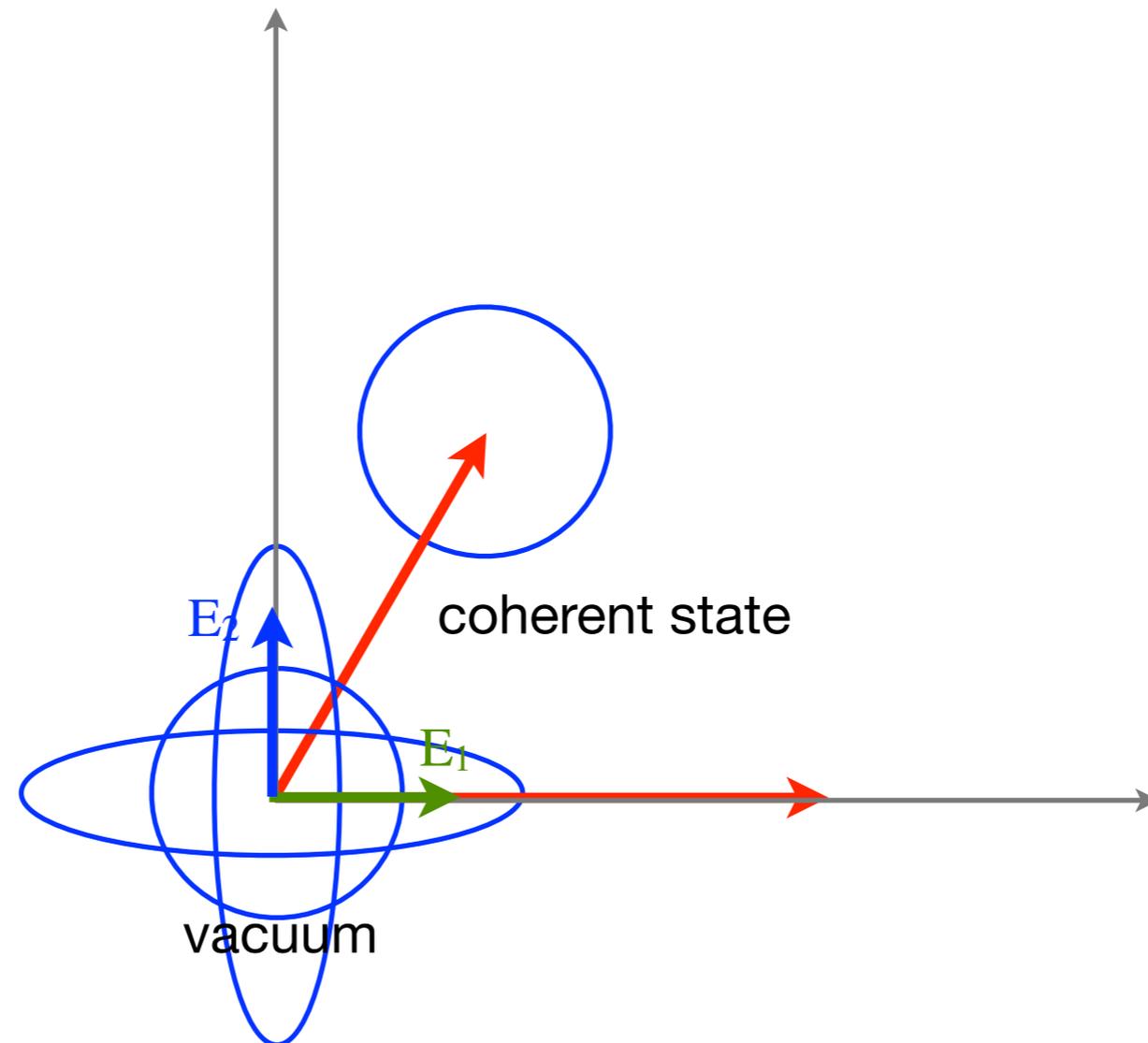
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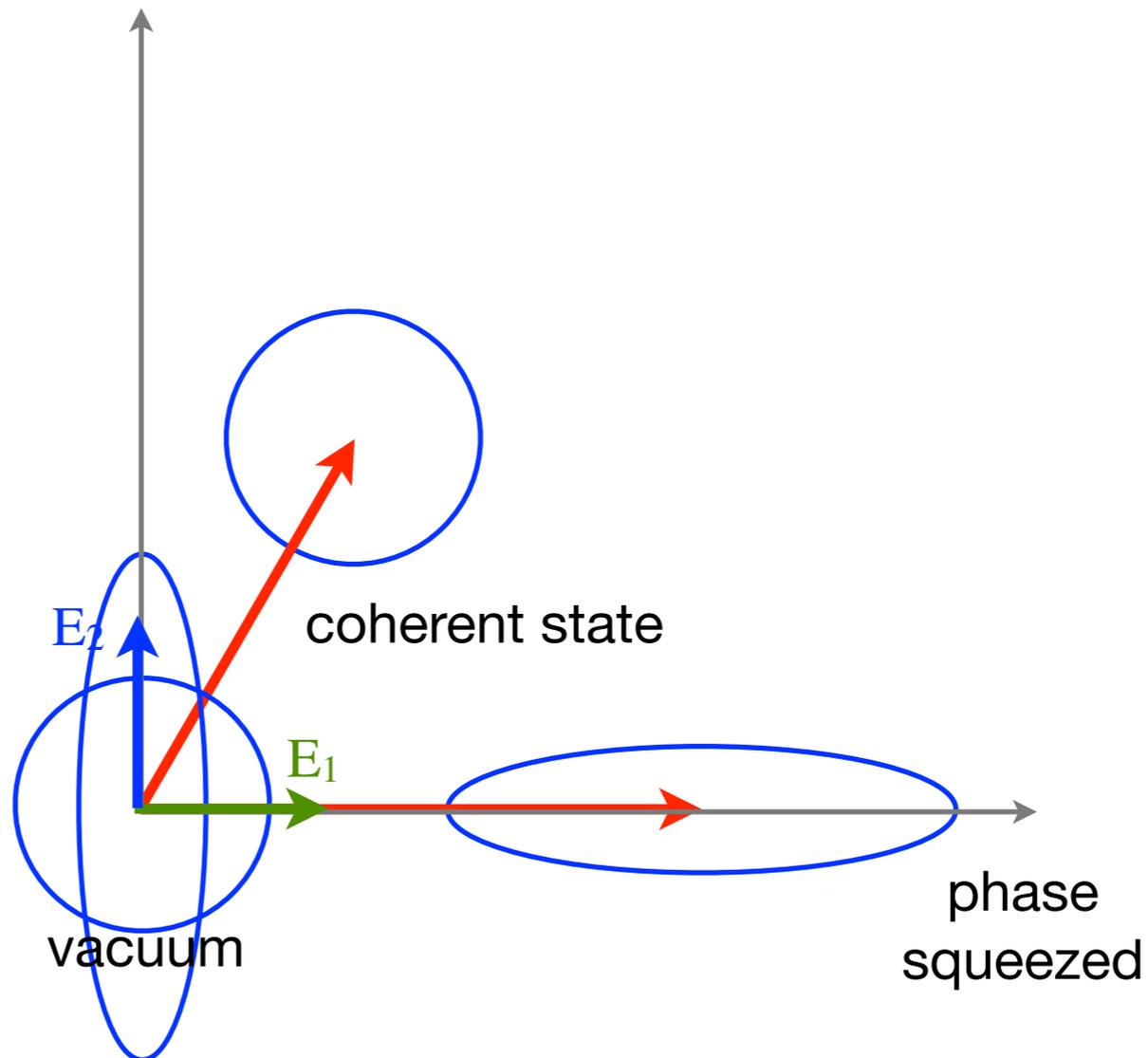
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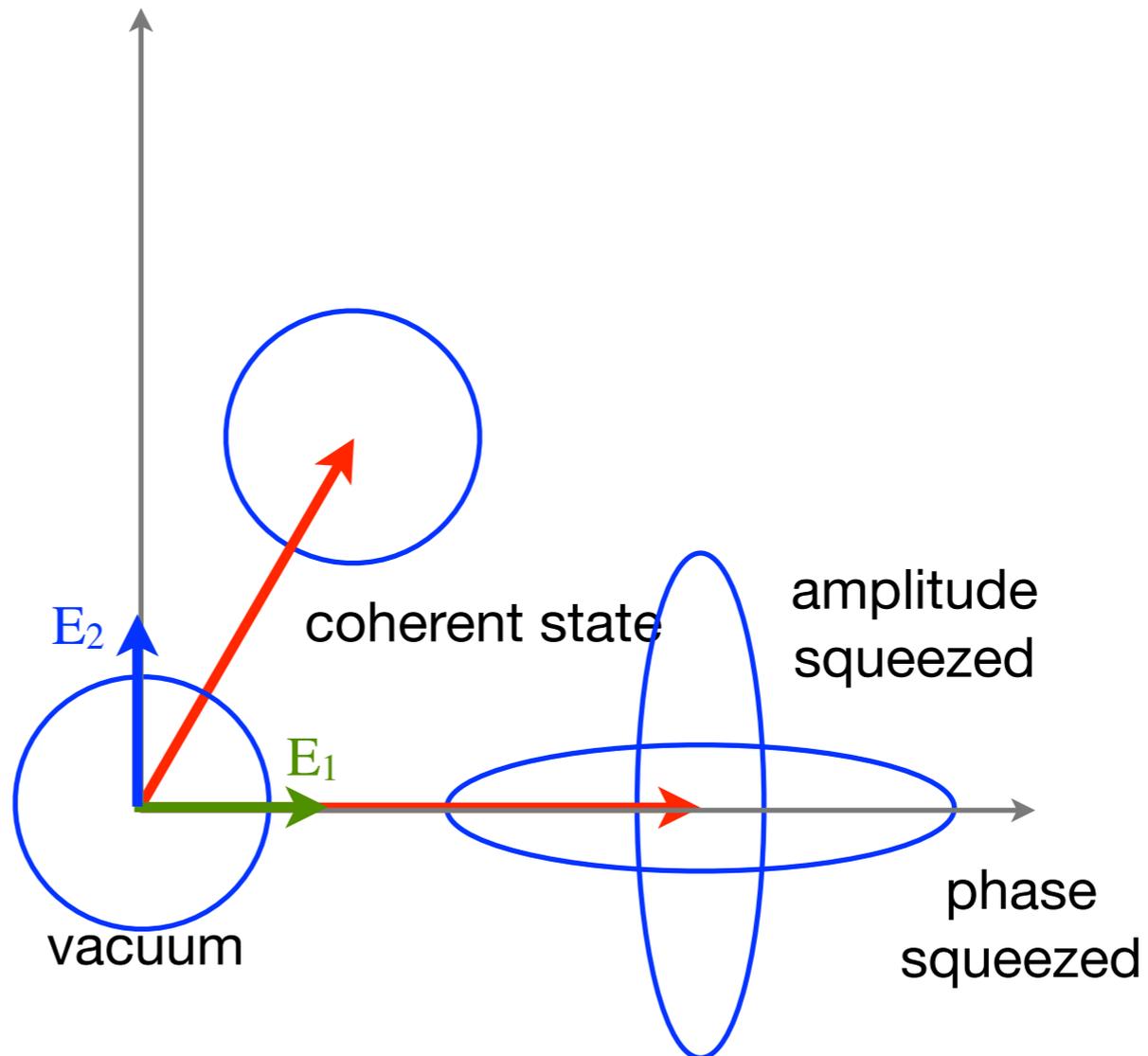
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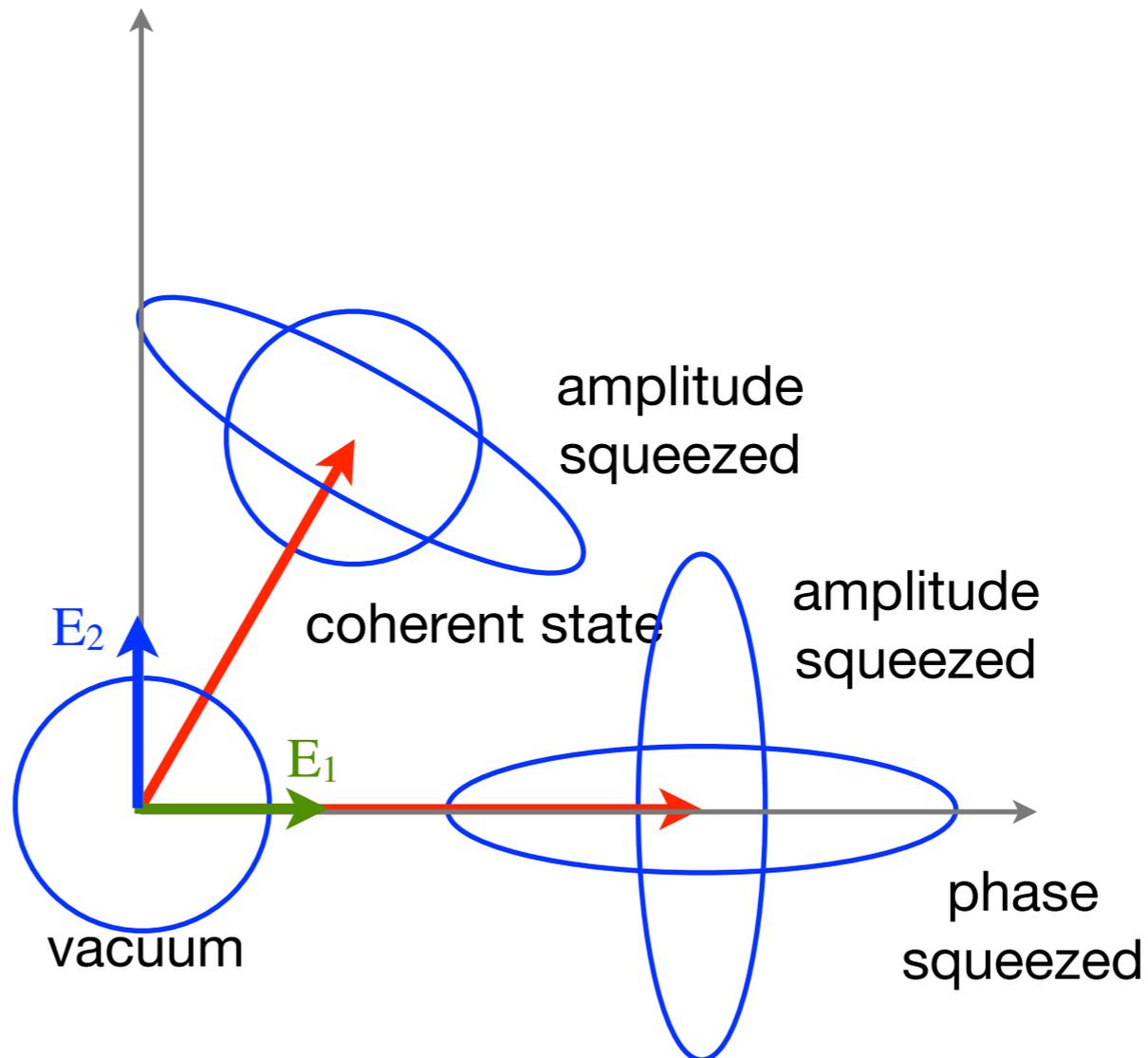
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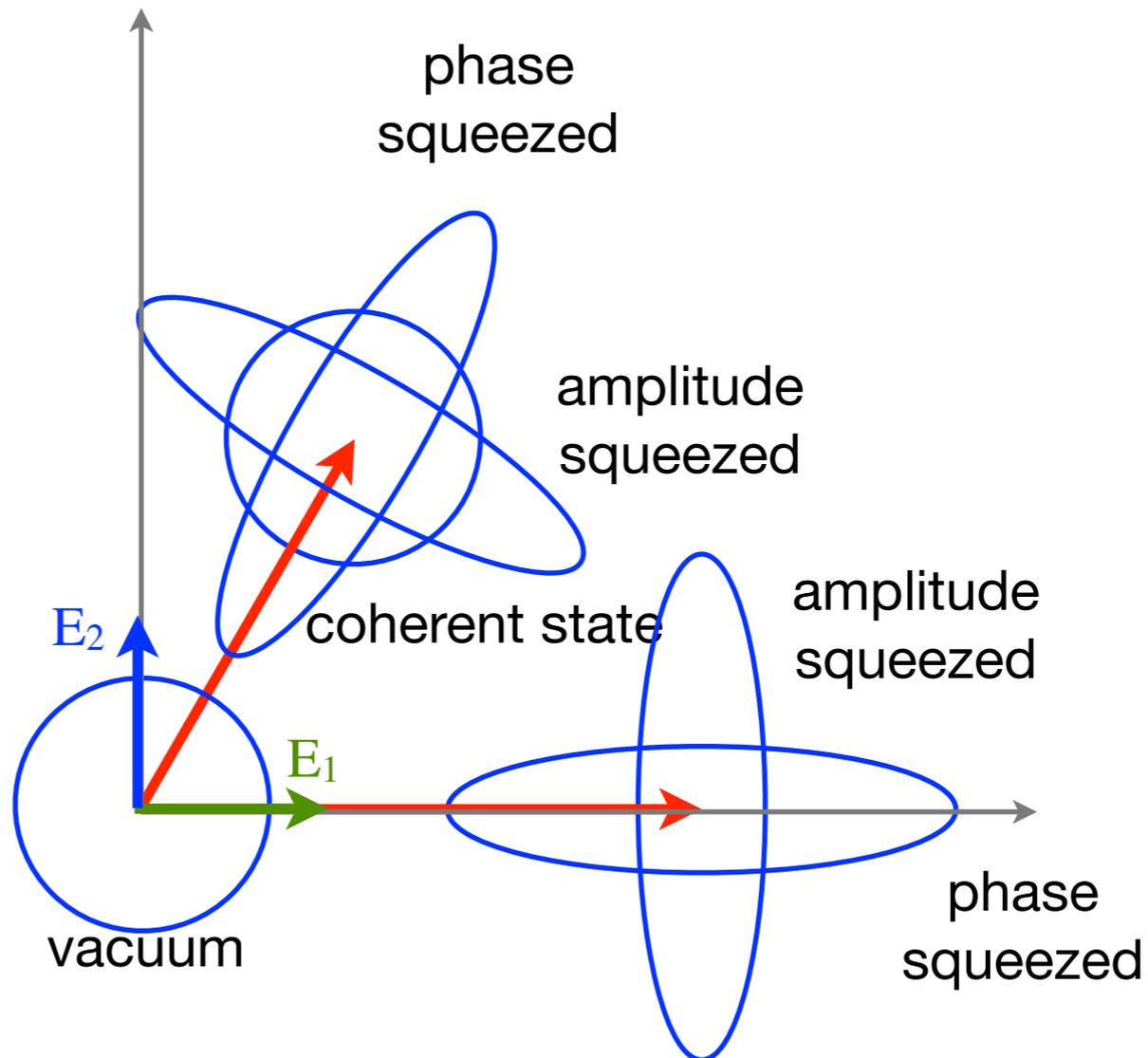
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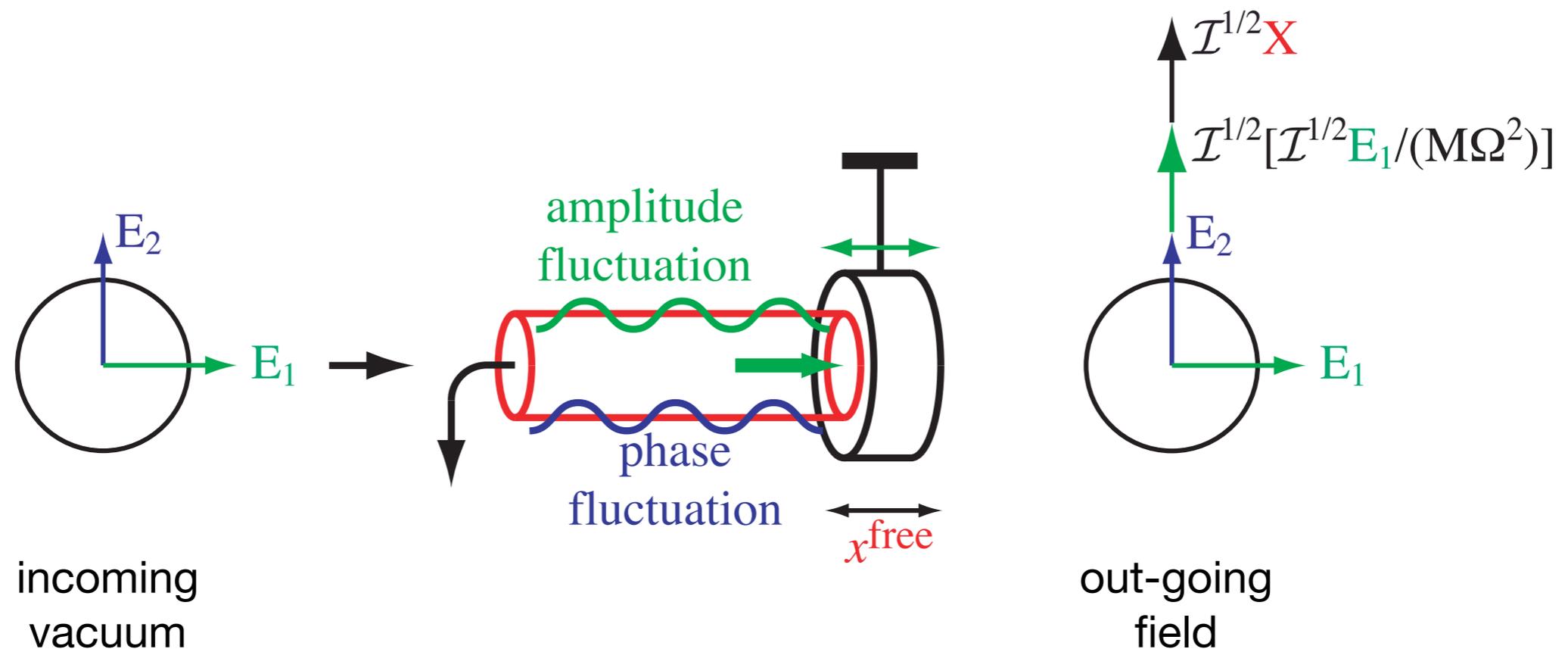
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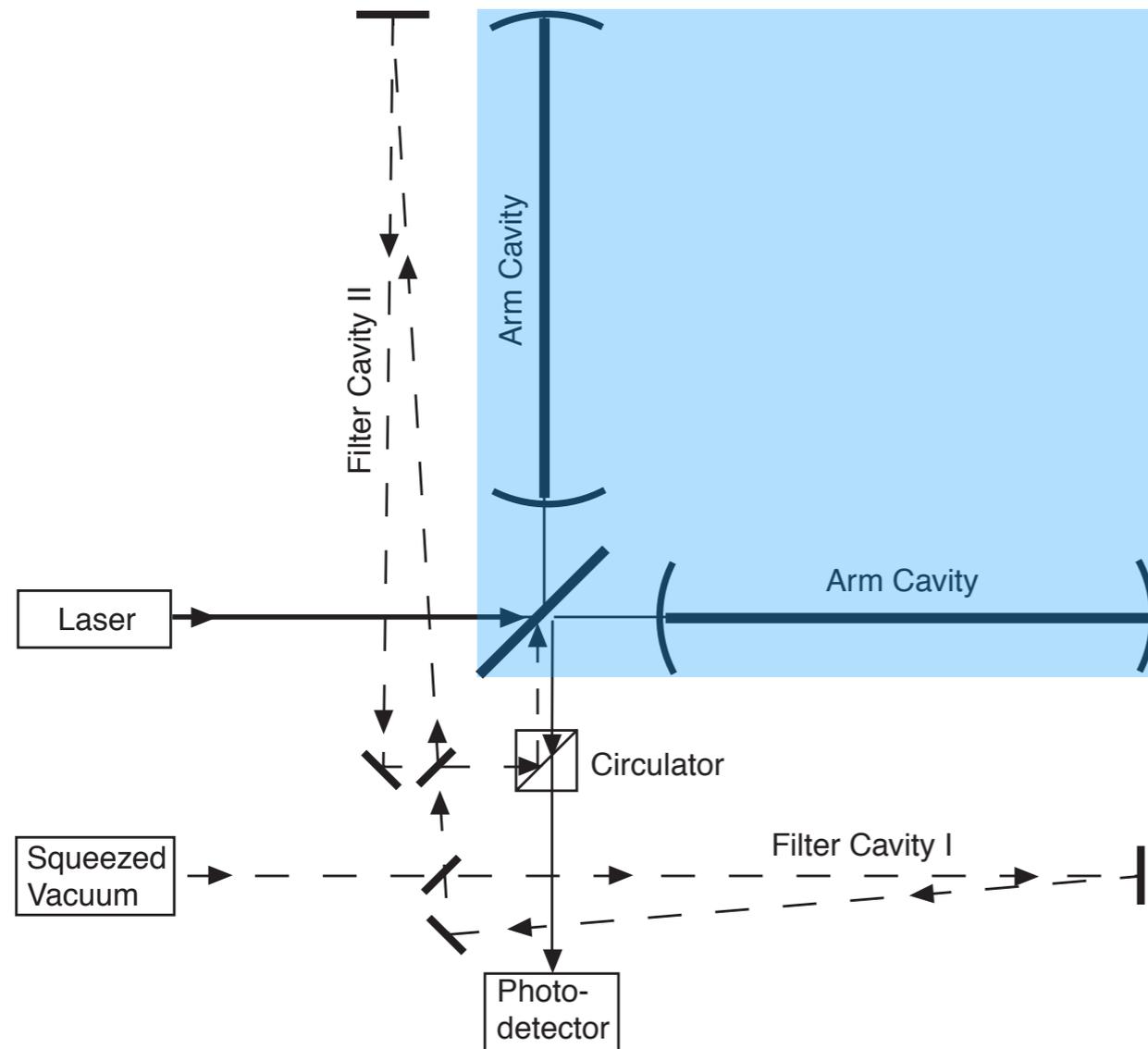


Amplitude Noise & Phase Noise

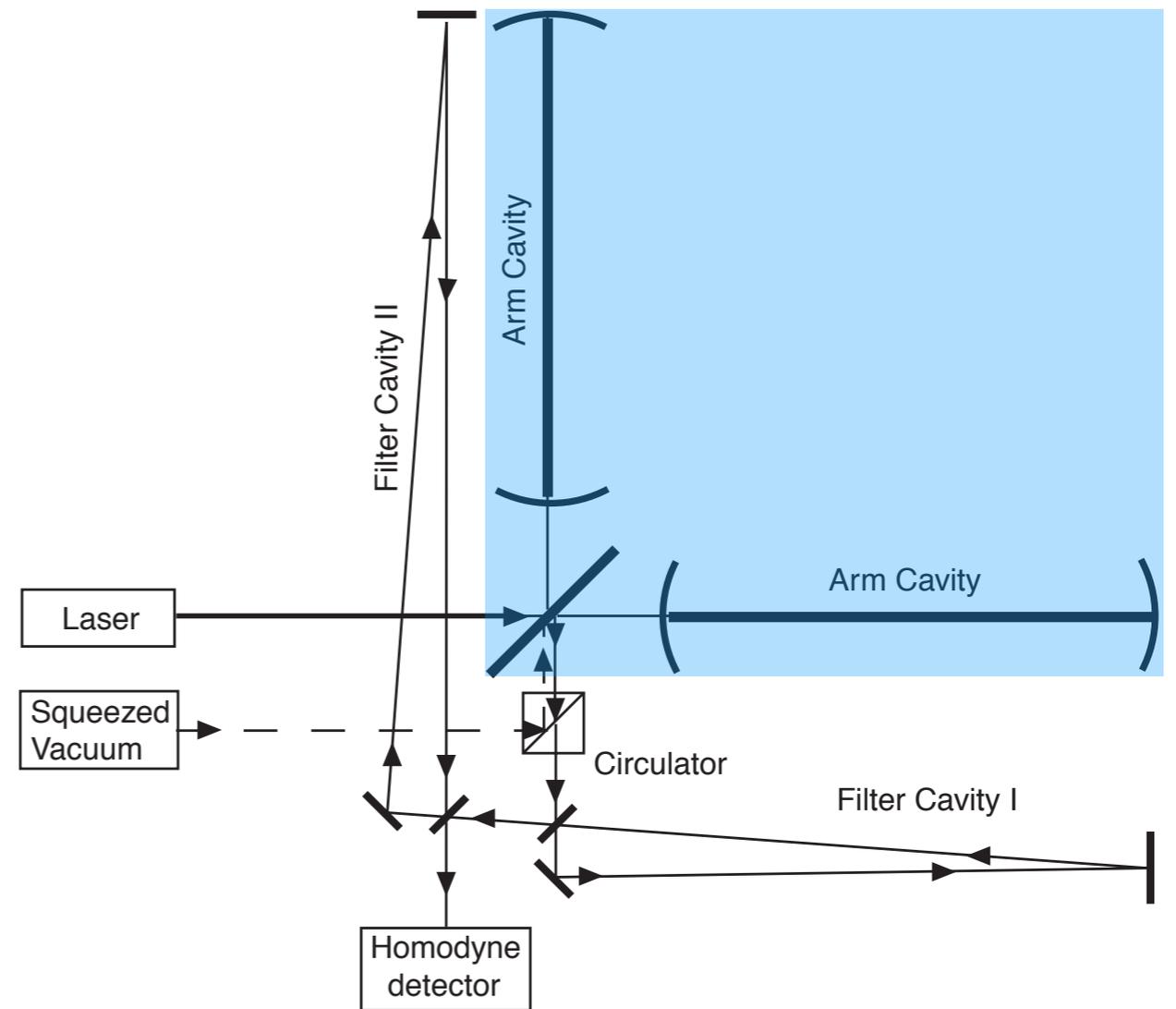


- squeezing **phase noise** will lower shot noise, but increase radiation-pressure noise (good for first-generation detectors, but **doesn't help beating the SQL**)
- squeezing a combination of input **amplitude** and **phase** will help, but only narrow band
- squeezing a **frequency-dependent combination** will help beat the SQL broadband.
- **detecting a combination of output amplitude and phase may even completely remove back-action noise**

Surpassing the SQL in a Michelson interferometer ²³



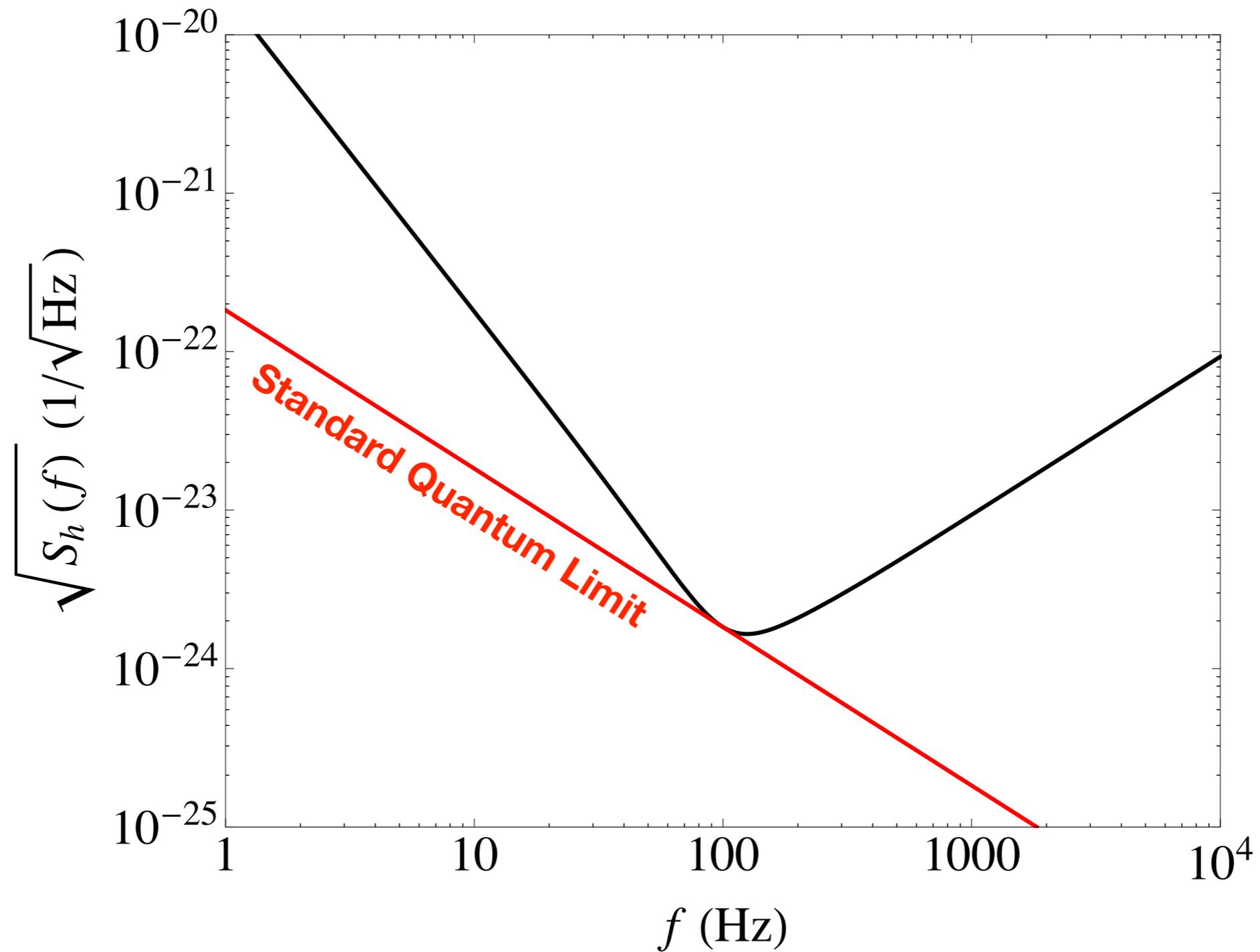
frequency dependent
input squeezed state



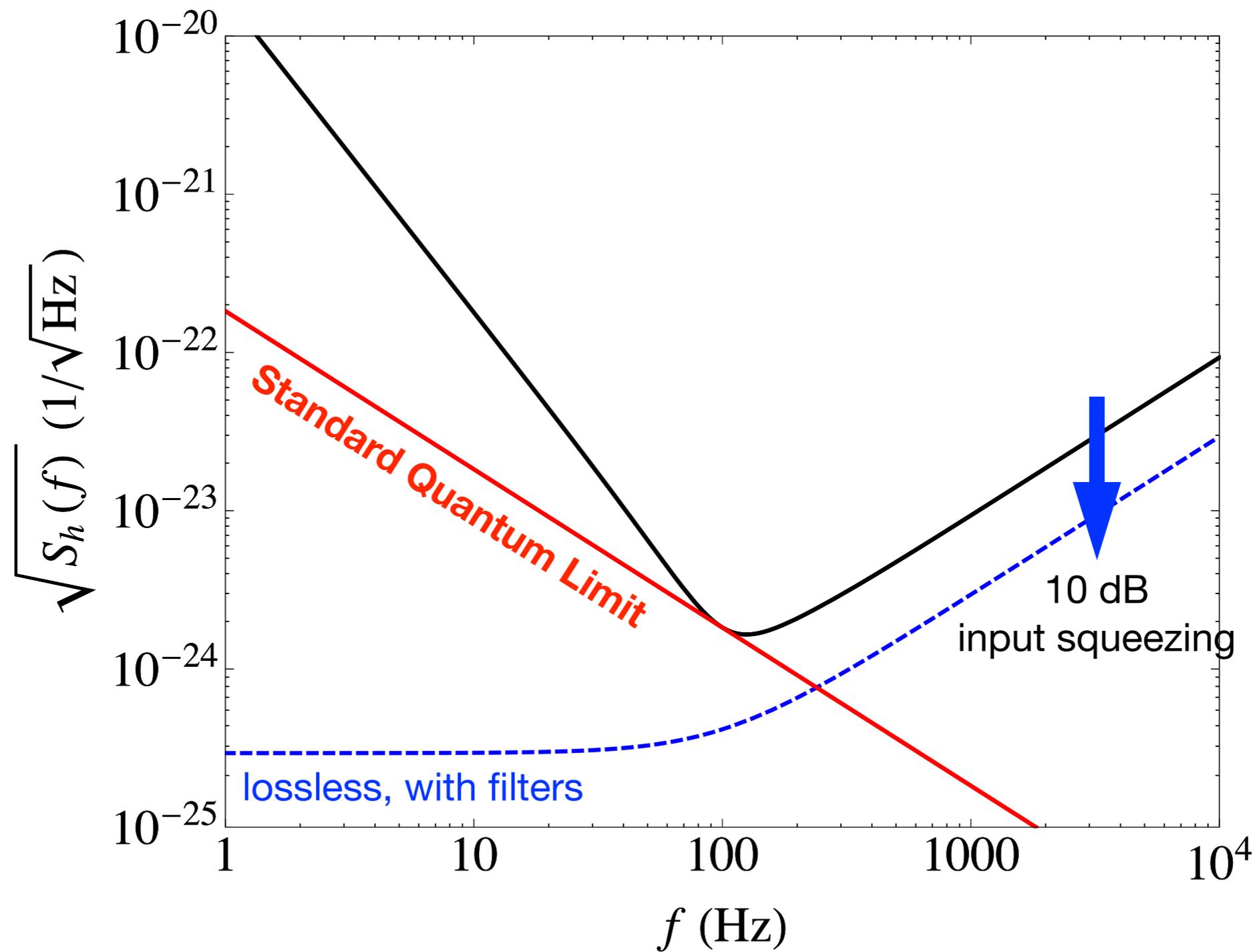
frequency dependent
homodyne detection

[Kimble et al., 2001]

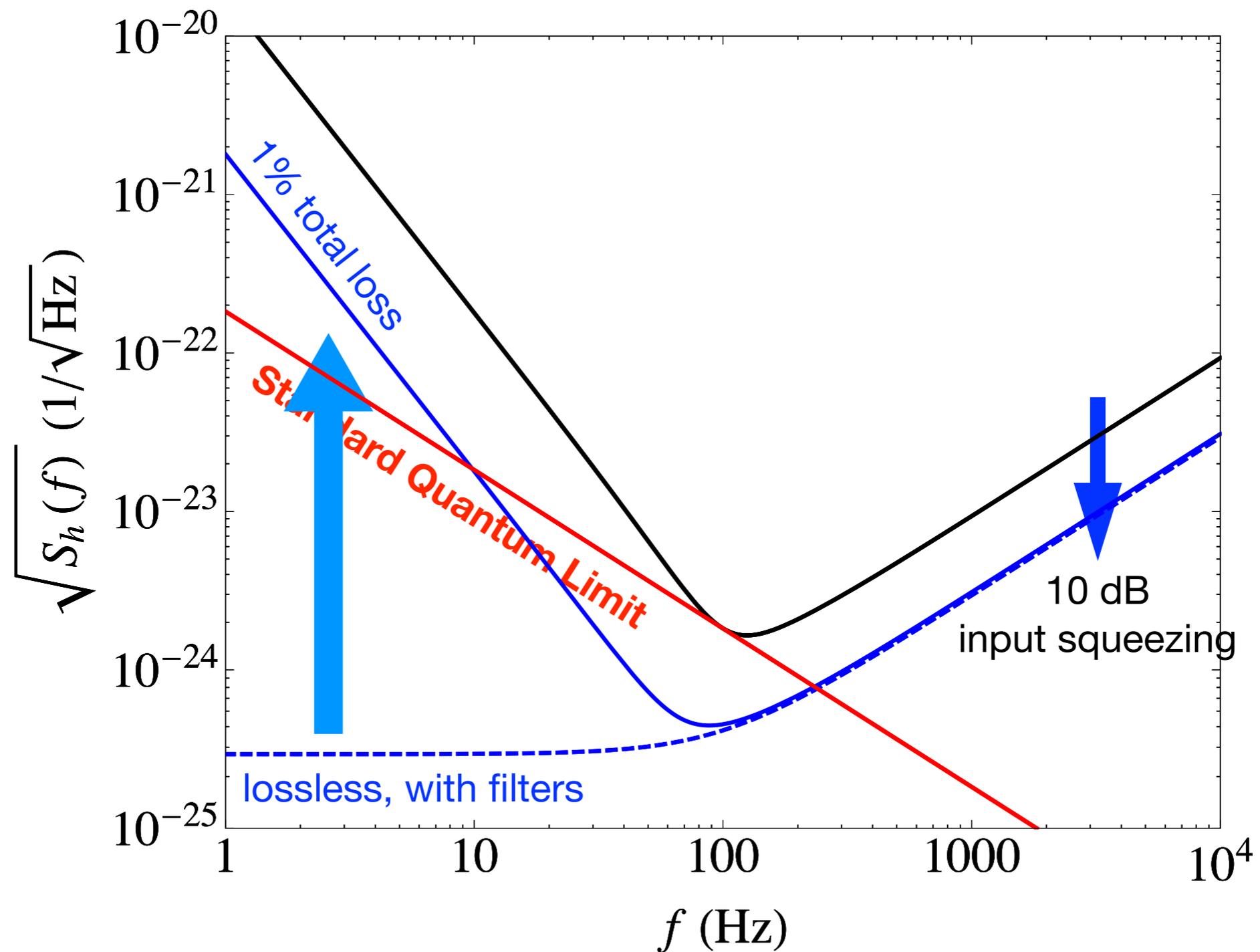
Frequency Dependent Squeezing & Detection



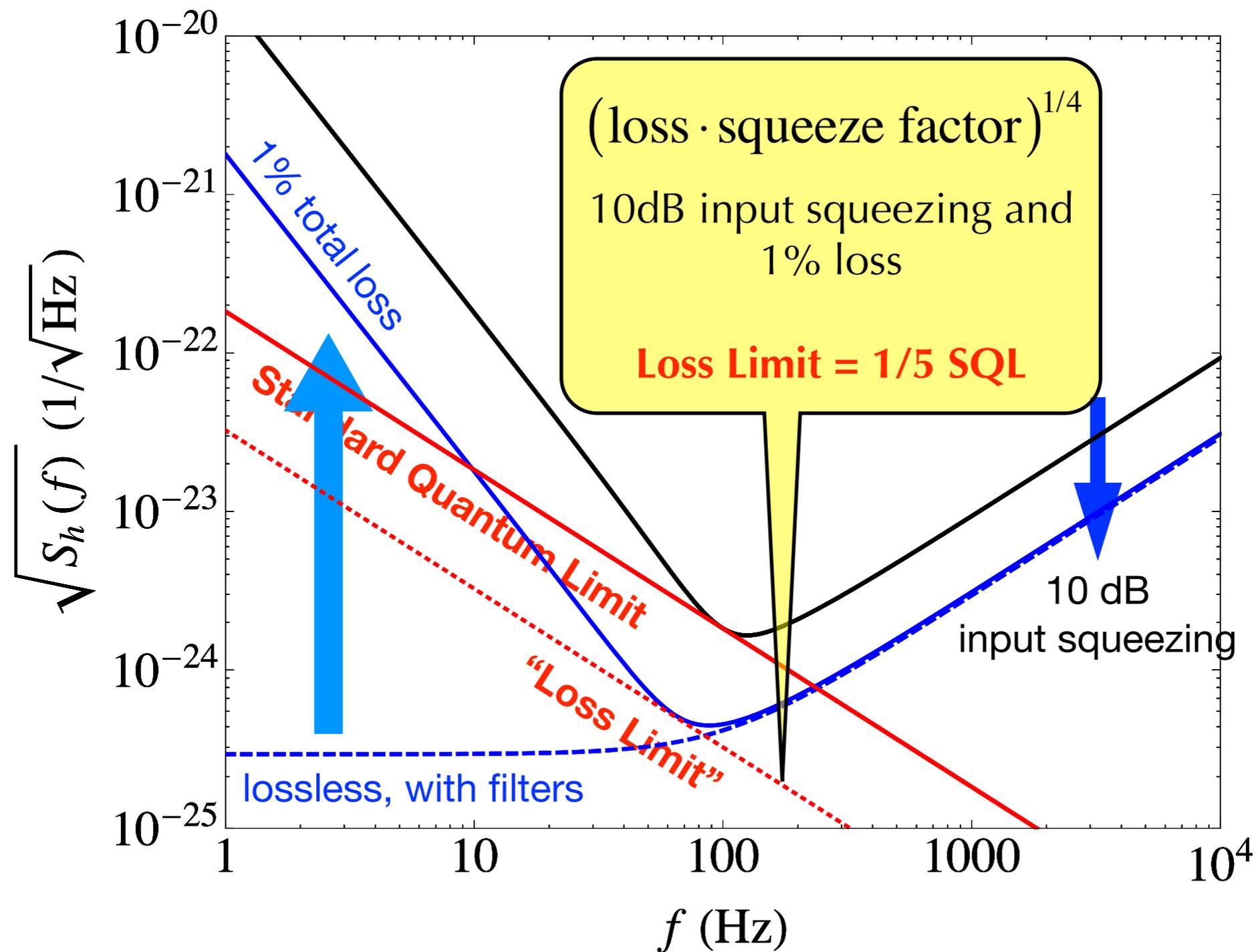
Frequency Dependent Squeezing & Detection



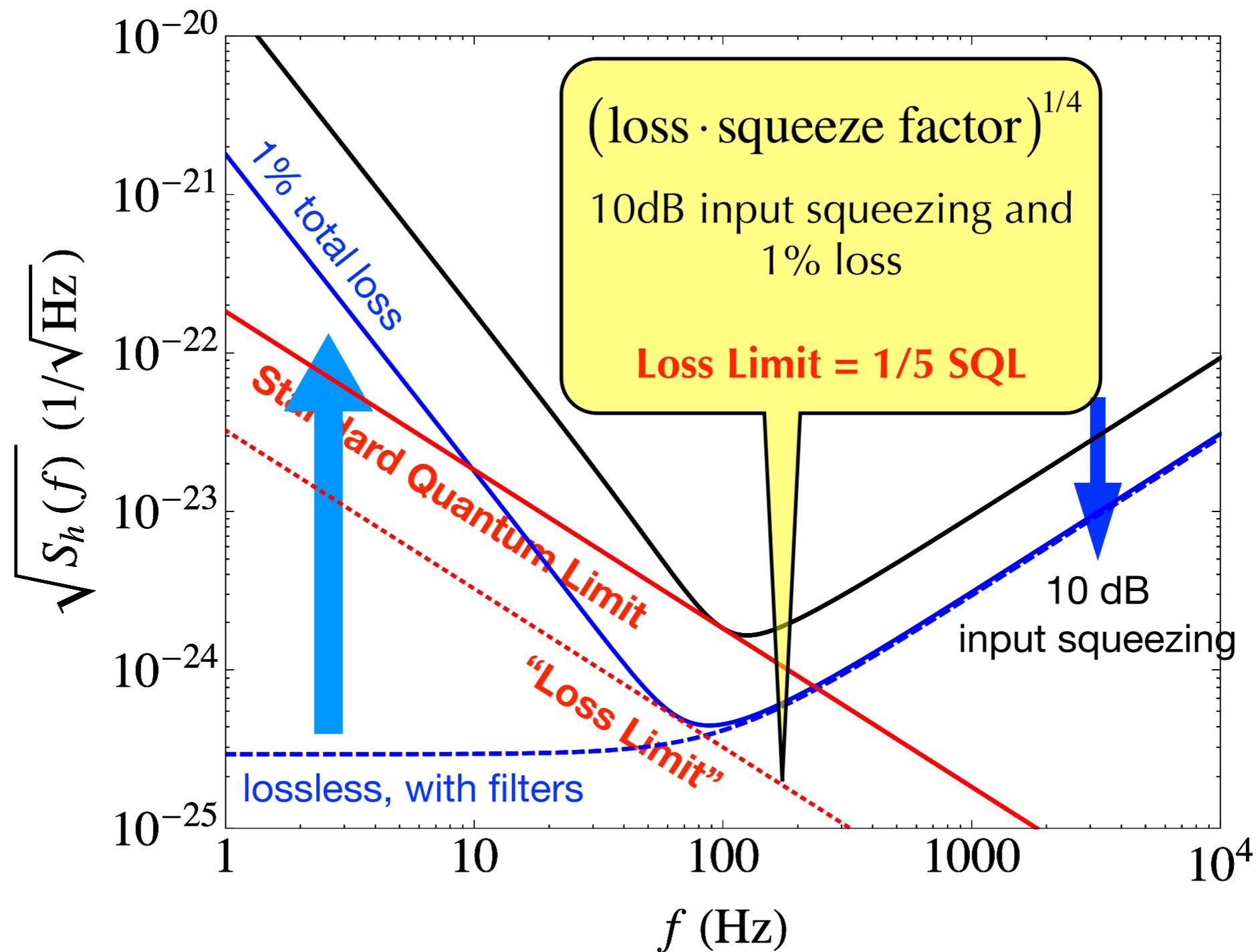
Frequency Dependent Squeezing & Detection



Frequency Dependent Squeezing & Detection



Frequency Dependent Squeezing & Detection



Quantum Enhancement of Sensitivity Requires Low Loss!

Generation of Squeezed Vacuum

Nonlinear Optics

$$H_I = \chi E^3$$



quantize

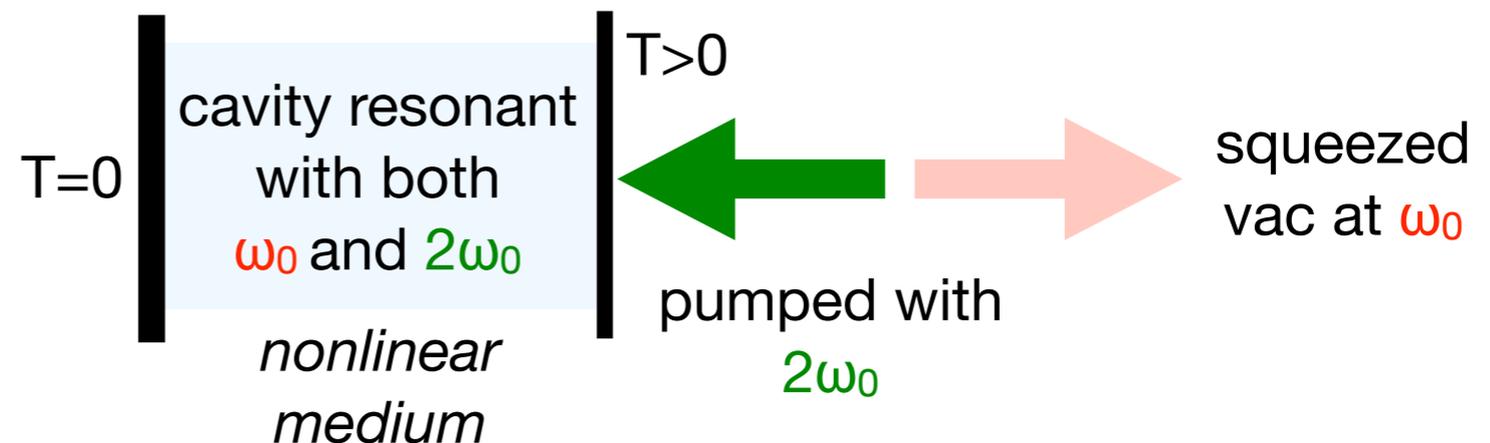
$$H = \dots + a_{2\omega_0}^\dagger a_{\omega_0+\Omega} a_{\omega_0-\Omega} + a_{2\omega_0} a_{\omega_0+\Omega}^\dagger a_{\omega_0-\Omega}^\dagger$$



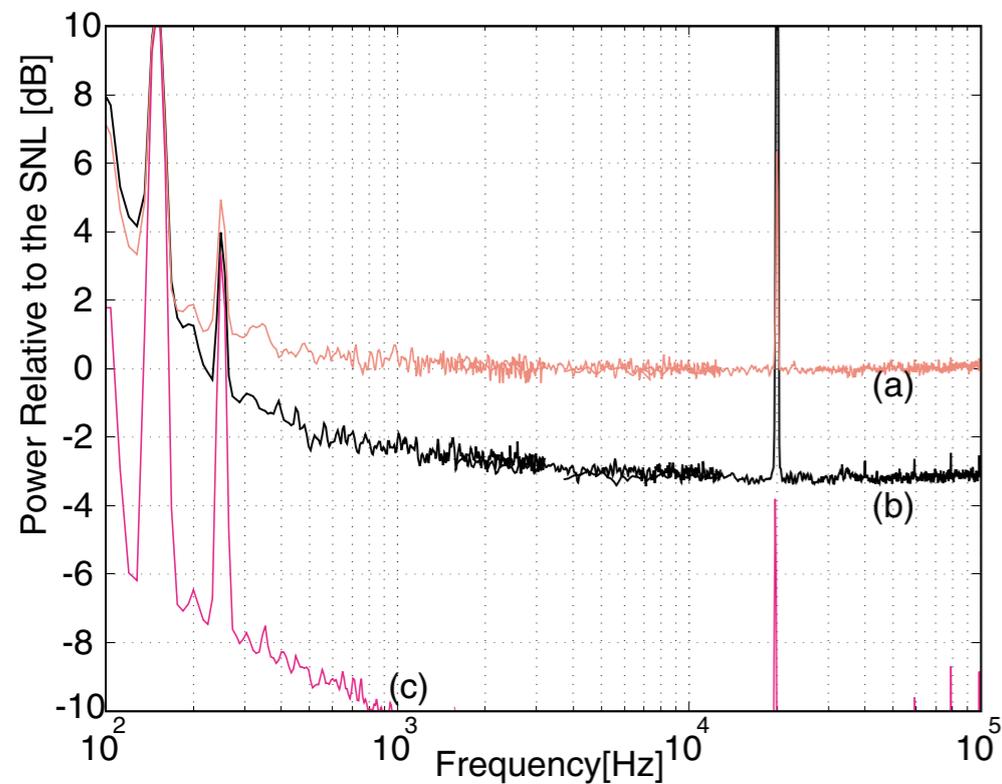
when non-linear medium pumped with $2\omega_0$
and phase-matching condition satisfied

$$H = \dots + \int \frac{d\Omega}{2\pi} \left[A_{2\omega_0}^* a_{\omega_0+\Omega} a_{\omega_0-\Omega} + A_{2\omega_0} a_{\omega_0+\Omega}^\dagger a_{\omega_0-\Omega}^\dagger \right]$$

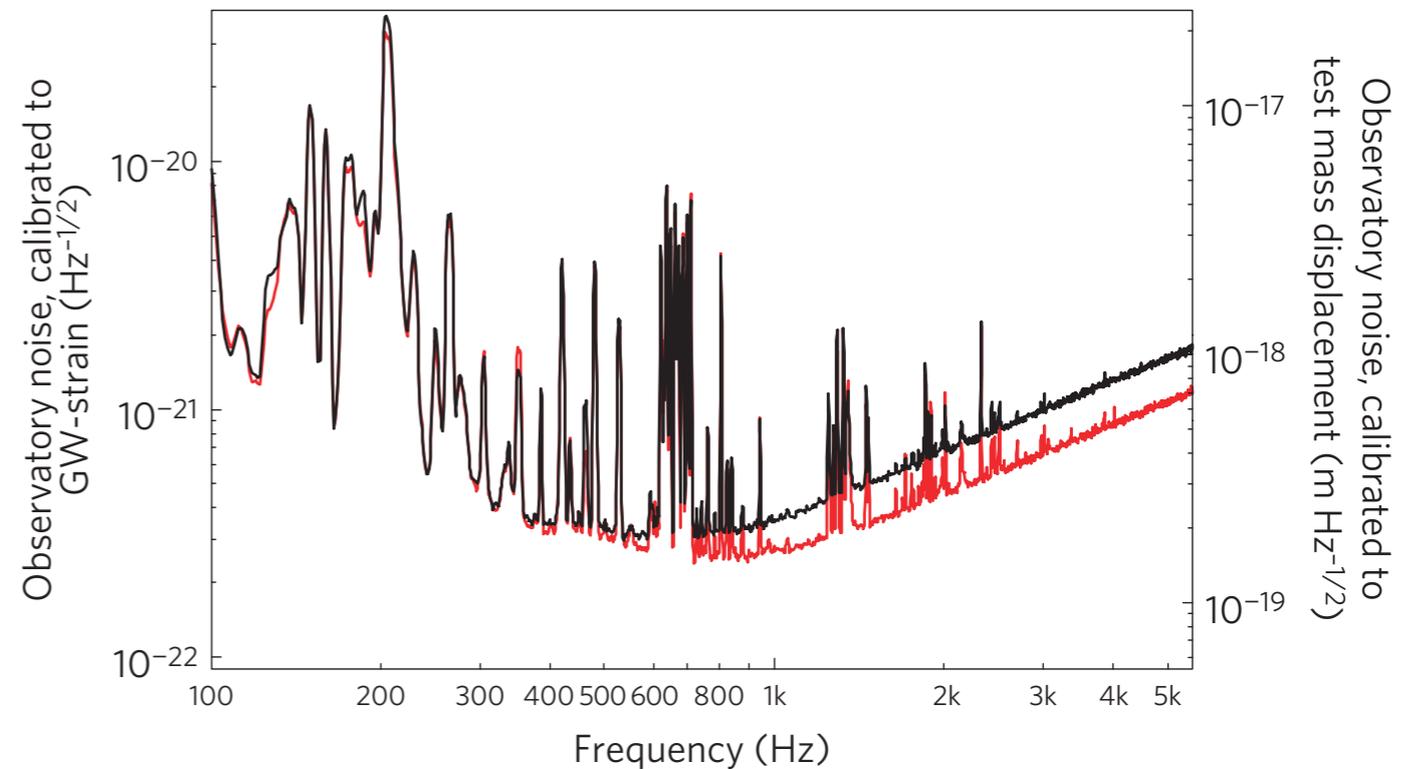
this term becomes effective and generates squeezing



Squeezing for GW Detectors



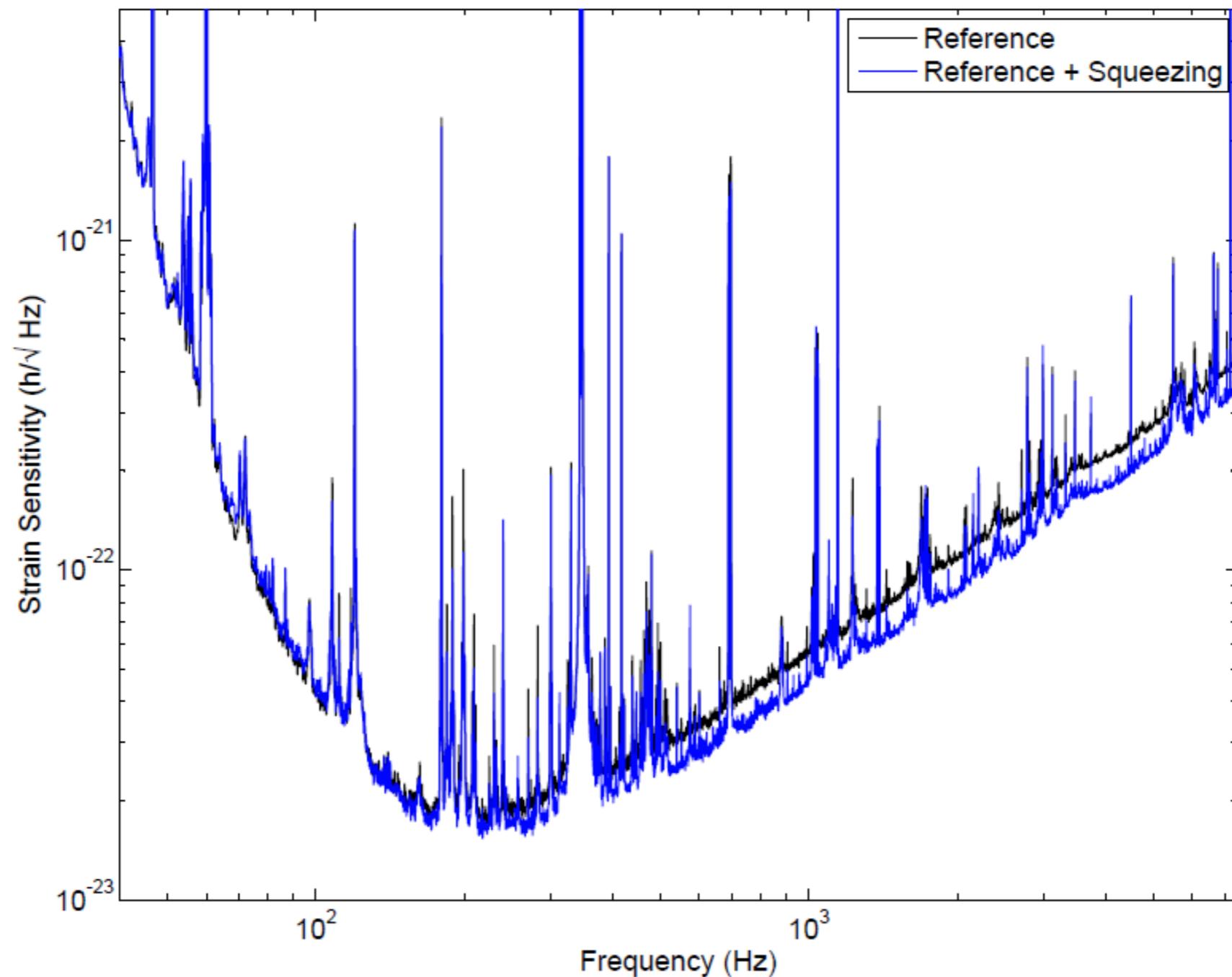
Low-Frequency Squeezing
K. McKenzie et al., 2004



[GEO Squeezing result,
LIGO Scientific Collaboration, 2011]

- First demonstration of squeezing in the GW band (sub kHz) [K. McKenzie et al., 2004]
- Squeezing injection at the Caltech 40 m prototype lab [K. Goda et al., 2008]
- 3.5 dB Squeezing at GEO 600 detector [LSC, 2011; H. Vahlbruch, 2010]
- 2+dB squeezing of LIGO Hanford, achieving best-ever sensitivity to GWs at 200+Hz.

Squeezing Status & Prospects

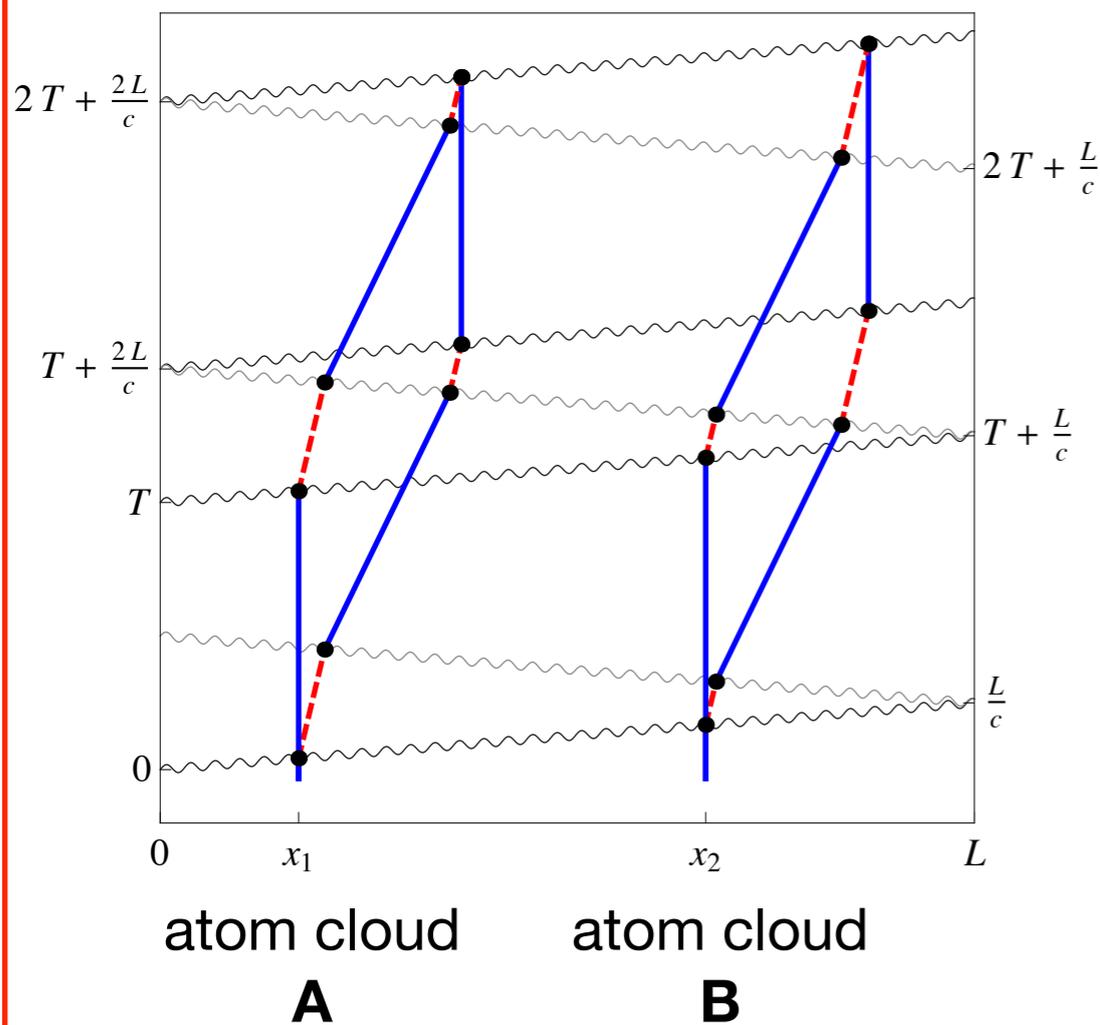


	initial LIGO	Advanced LIGO	aim of future LIGO
total loss	55-60%	20%	<2%
detected	2+dB	6dB	10-15dB

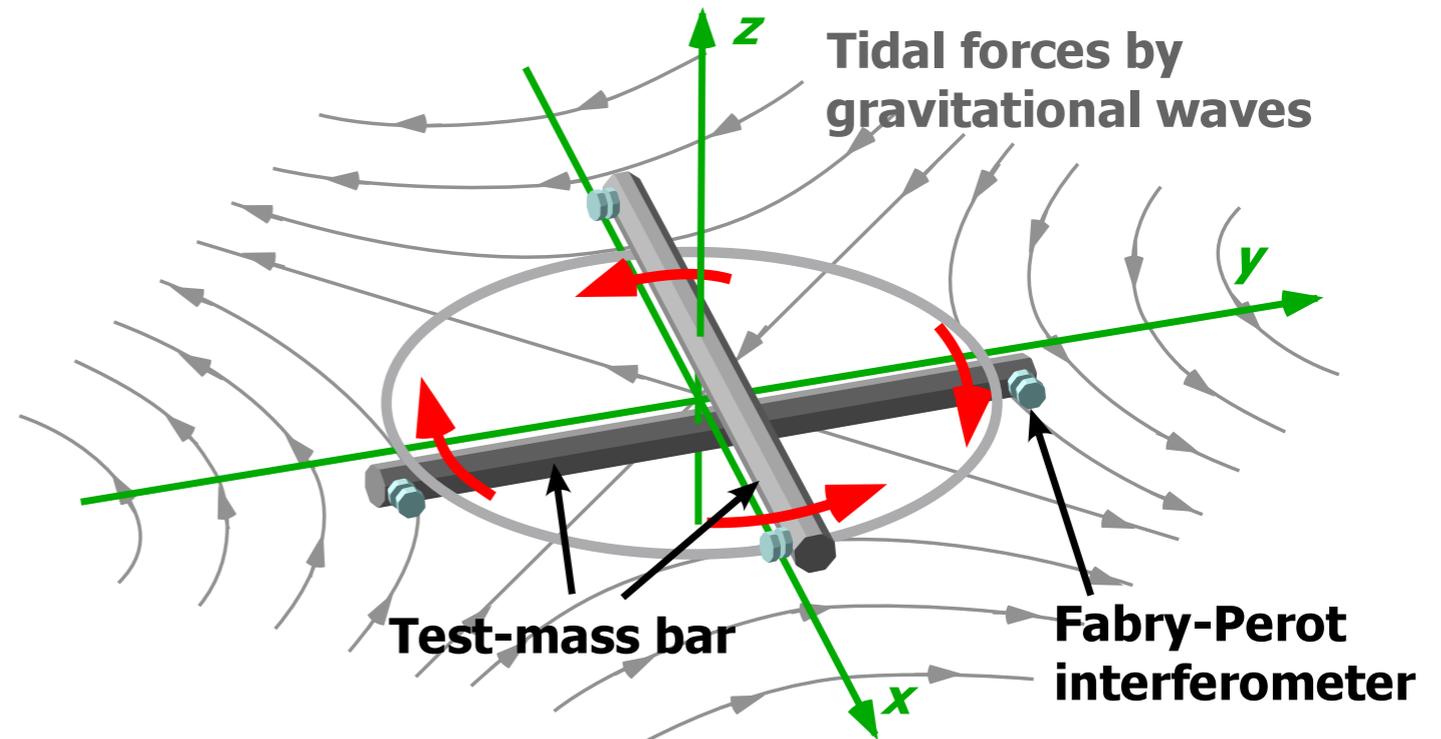
[slide & numbers from Sheila Dwyer, GW Adv Detector Workshop, 2012]

Low-frequency barrier on the earth?

- Suspension Thermal Noise
 - a pendulum's thermal noise seems a strong limitation
 - other methods are being considered
 - magnetic levitation
 - juggling mirrors?
 - atom interferometers
 - TOBA?



[Dimopoulos, Graham, et al.]

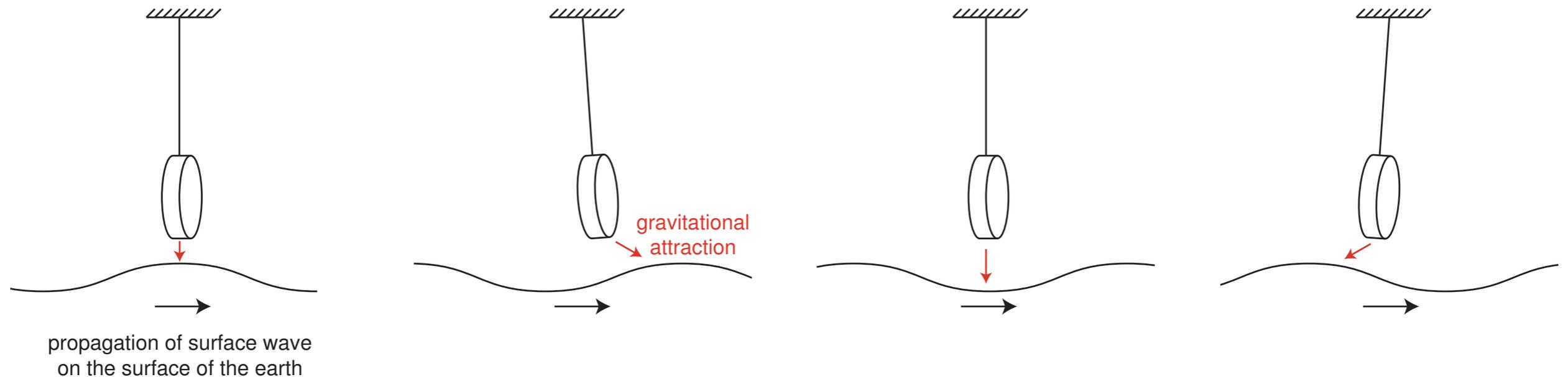


access down to 0.1 Hz
experimental prototypes built in Kyoto & Tokyo

[Ando et al., 2010]

Gravity Gradient Noise

- Seismic motion driving fluctuations in newtonian gravity field



[figure from Pitkin et al. 2011]

- Can be suppressed by monitoring ground motion and subtracting the predicted effect.
- For LIGO (between 10 Hz and 20 Hz)
 - 5x suppression required to not affect Advanced LIGO
 - 30x suppression required to not affect 3rd generation designs [\[J. Driggers, 2012\]](#)
- Moving detector underground may suppress level and allow better subtraction.

Space-Based GW Detection

- Space-based GW
 - interferometers with long arms (compared with GW wavelength)
 - Laser Interferometer Space Antenna (LISA)
 - *quantum enhancements of a LISA-like mission?*
 - Other space missions

Plane Gravitational Wave

- Coordinates can be chosen such that a plane wave along z direction can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$h_{ij}^{\text{TT}}(t, x, y, z) = \begin{bmatrix} h_+(t-z) & h_\times(t-z) & 0 \\ h_\times(t-z) & -h_+(t-z) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad i, j = x, y, z$$

This is called the TT gauge because h is transverse, and traceless.

$h_{+, \times}$ are the two polarizations of the plane GW

Influence of GW on Light and Matter

- Propagation of Light in the “Transverse Traceless” gauge

the scalar wave equation $g^{ab} \nabla_a \nabla_b \Phi = 0 \Leftrightarrow \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0$

$$\Phi = A \exp(ik_\mu x^\mu + i\delta\phi) = A \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{x} + i\delta\phi)$$

flat-space solution plus *additional phase due to GW*

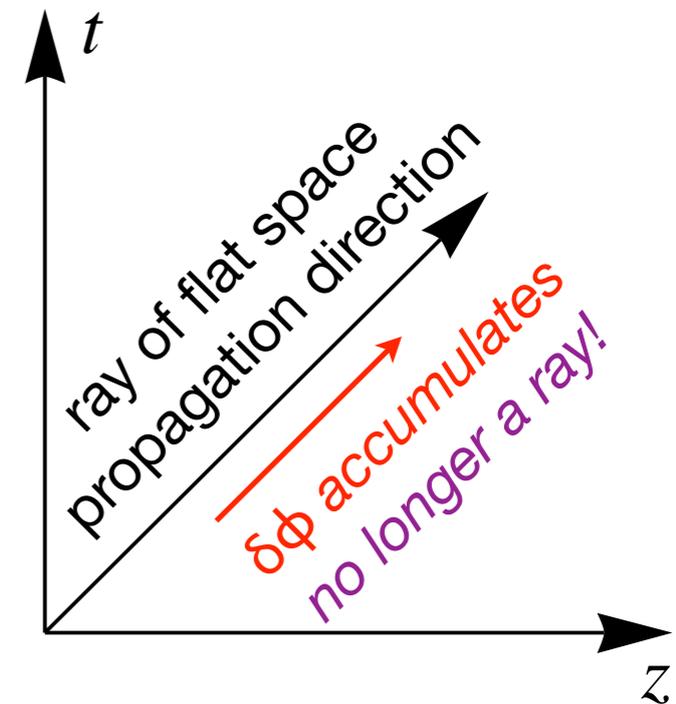
$$k^\mu = (\omega, \mathbf{k}) = \omega(1, \hat{\mathbf{k}})$$

4-wavevector ang freq 3-wavevector propagation direction

$\delta\phi$ slowly varying $\rightarrow k^\mu \partial_\mu \delta\phi = \frac{h_{\mu\nu}^{TT} k^\mu k^\nu}{2}$

additional phase accumulates *along rays* as wave propagates

$$\delta\phi(t_0 + L, \mathbf{x}_0 + \hat{\mathbf{k}}L) = \frac{\omega}{2c} \int_0^L \hat{k}^i \hat{k}^j h_{ij}(t_0 + \xi, \mathbf{x}_0 + \hat{\mathbf{k}}\xi) d\xi$$



Light propagation is modified by GW

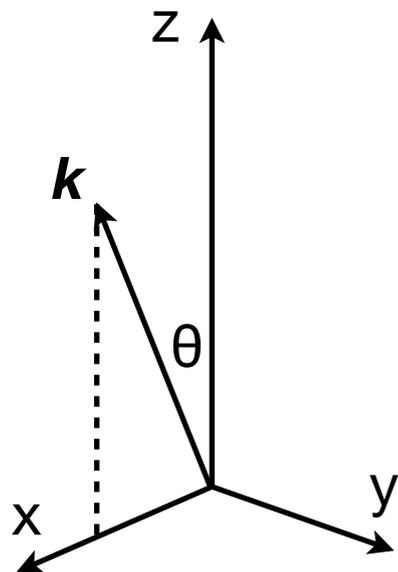
Response of Laser Interferometers: TT Gauge

- For larger separation (\sim reduced wavelength): **oscillatory nature matters**

$$h_p = H_p e^{-i\Omega(t-z)}, \quad p = +, \times \quad \dots \text{ plane wave with propagation direction } \mathbf{N}$$

$$\delta\phi(t_0 + L, \mathbf{x}_0 + \hat{\mathbf{k}}L) = \frac{\omega_0}{c} \frac{LH_p e^{ij} \hat{k}^i \hat{k}^j}{2} e^{-i\Omega(t-z)} \frac{e^{-i\Omega L(1-\hat{k}_z)} - 1}{-i\Omega L(1-\hat{k}_z)}$$

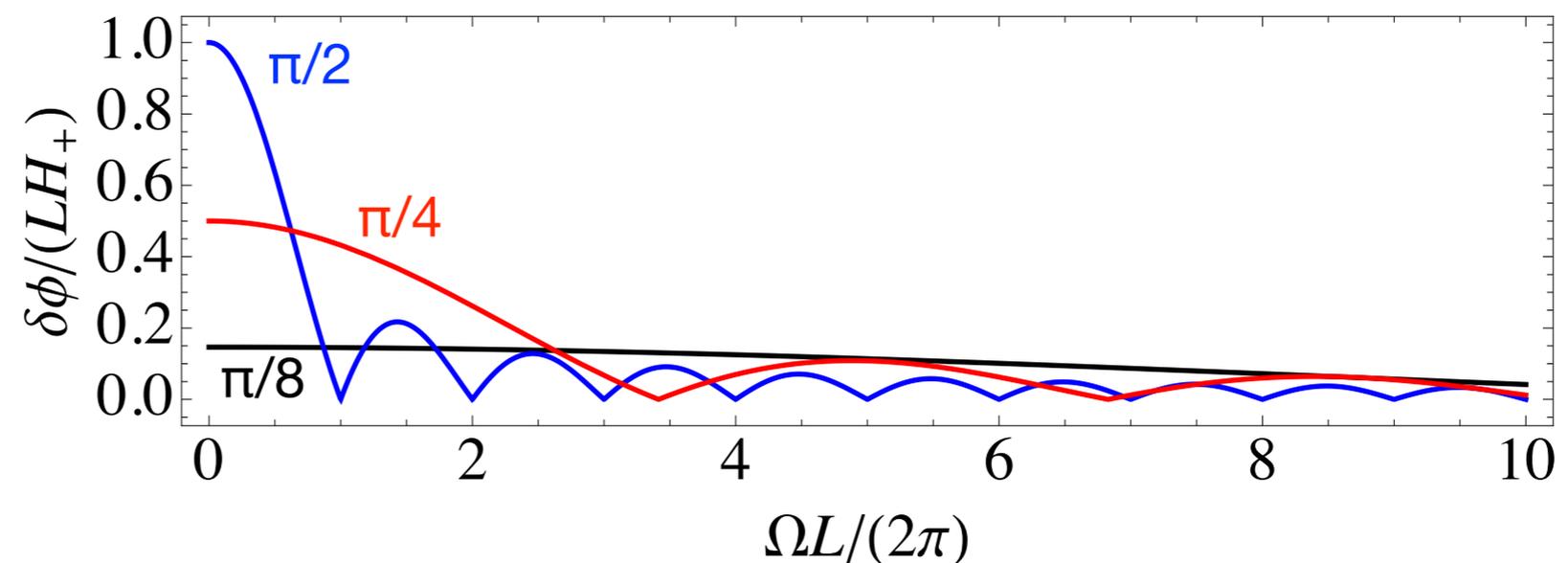
GW along z



+ polarized

same as before
this favors
 \mathbf{k} orthogonal to \mathbf{N}
(transverse wave)
proportional to L

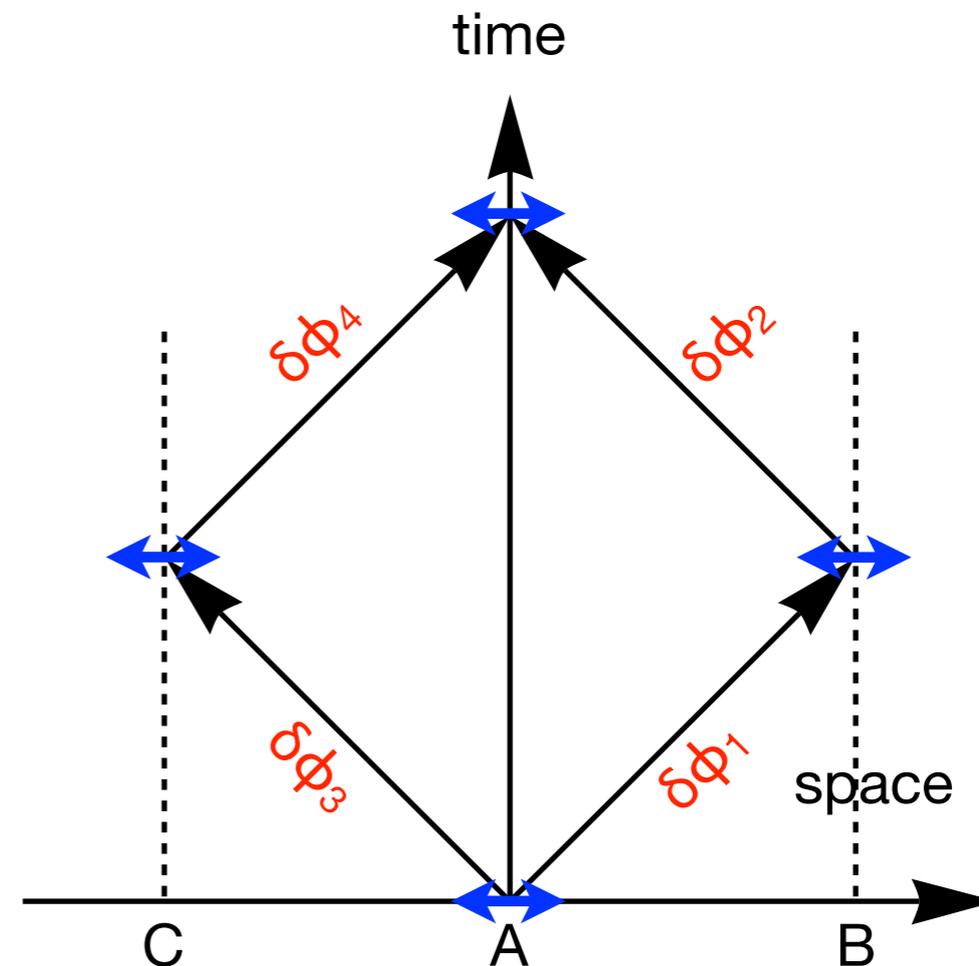
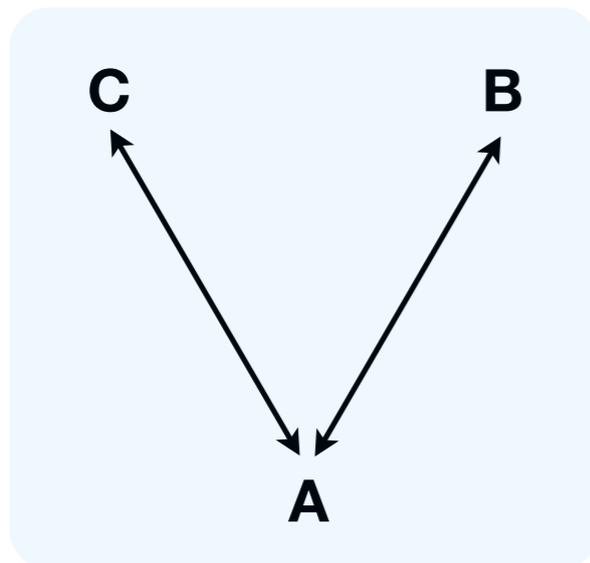
additional phase factor
due to propagation effect
this favors \mathbf{k} along \mathbf{N}



suppression of phase shift from simple Lh

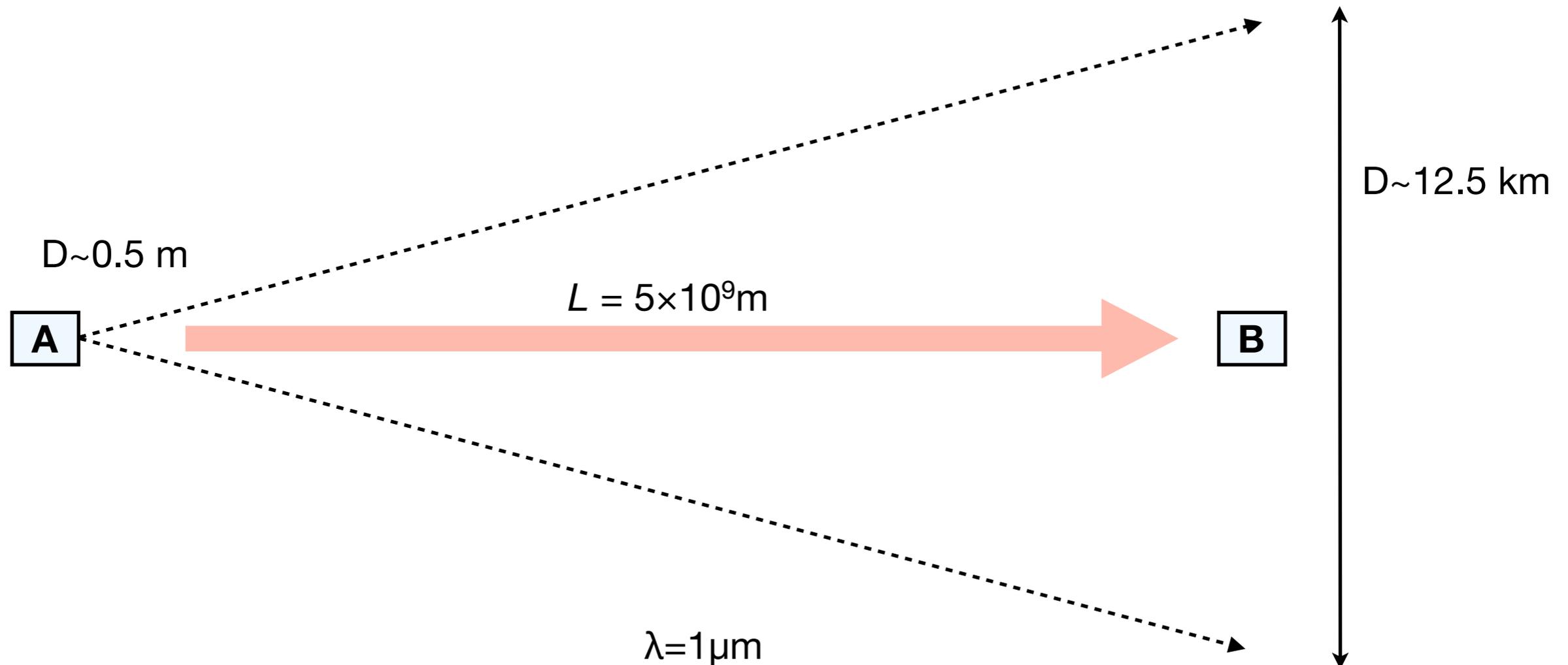
Response of Masses and Building an Interferometer³⁴

- In TT gauge: low-speed motion of test masses not affected by GW!
- But test masses won't stay at fixed locations; they will be moving under noisy forces!
- Simplest interferometer
 - **A**, **B**, and **C** freely fall + noisy motion
 - **A** sends light to **B** and **C**
 - **B** and **C** reflect light back to **A**
 - **A** compares phase between light from **B** and light from **C**.
- This gives signal of $(\delta\phi_1 + \delta\phi_2) - (\delta\phi_3 + \delta\phi_4)$
- Plus local displacement noises (driven by force noise) & shot noise



Arm Length?

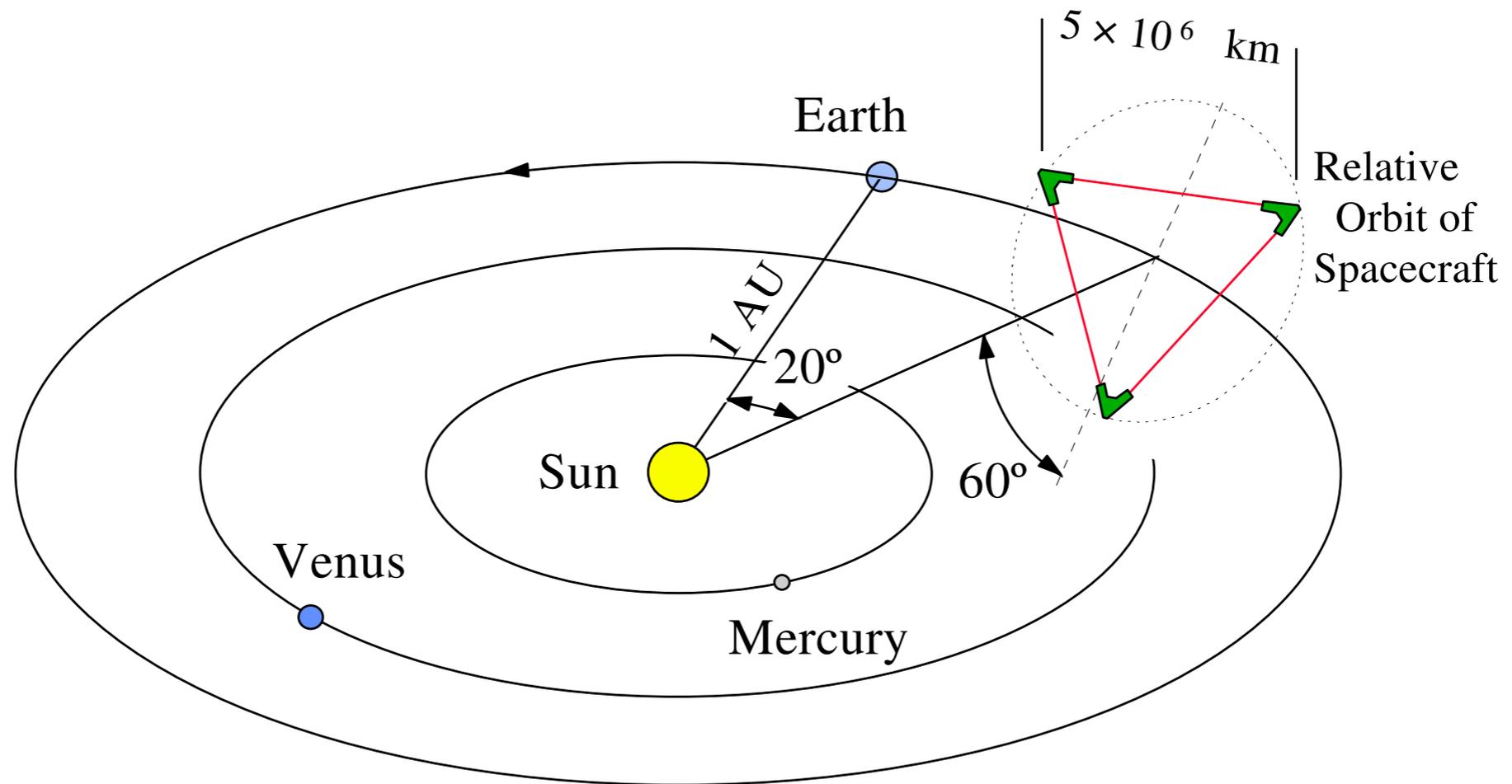
- What if frequency is $f = 10\text{mHz}$.
- Reduced wavelength is $\lambda/(2\pi) \sim 5 \times 10^9\text{m} \sim 5 \times 10^6\text{km}$
- This is the most optimal arm length to reduce effect of local force noise



very small amount of light ($\sim 0.2\%$) is received by B

to collect most of the light, the mirror diameter has to be $> (\lambda L)^{1/2} \sim 71\text{ m}$
or, reduce to $L < D^2/\lambda \sim 250\text{ km}$

Laser Interferometer Space Antenna

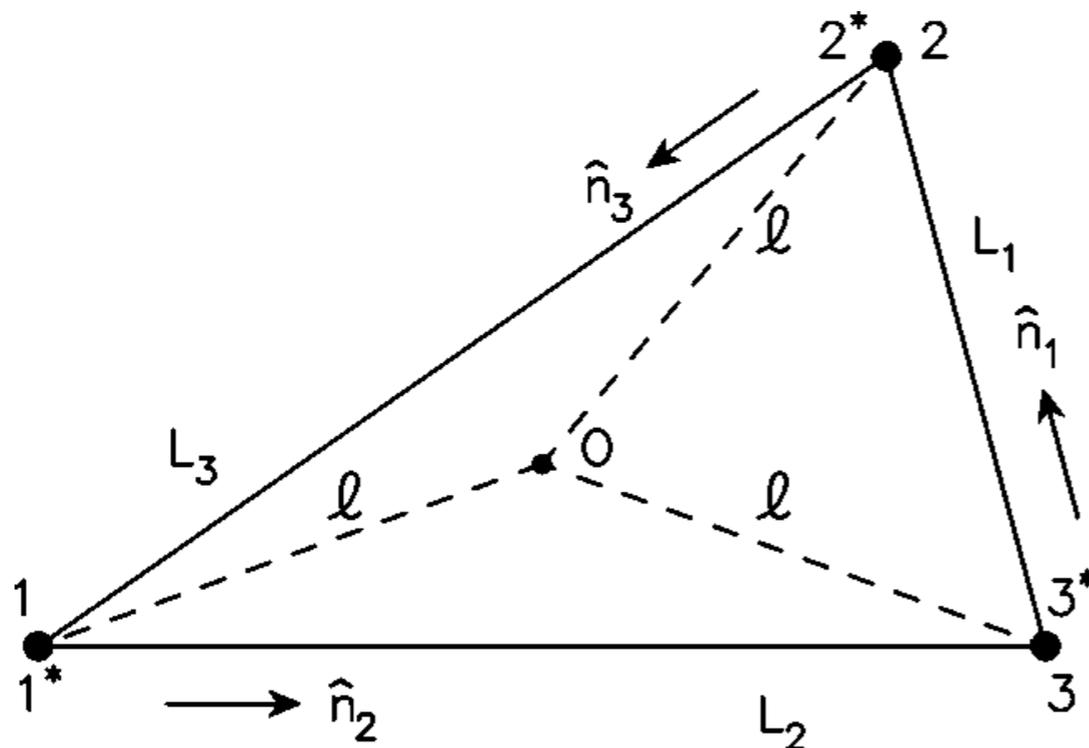


$$L = 5 \times 10^6 \text{ km} = 5 \times 10^9 \text{ m}$$

Equilateral Triangle, tilted at 60 degrees

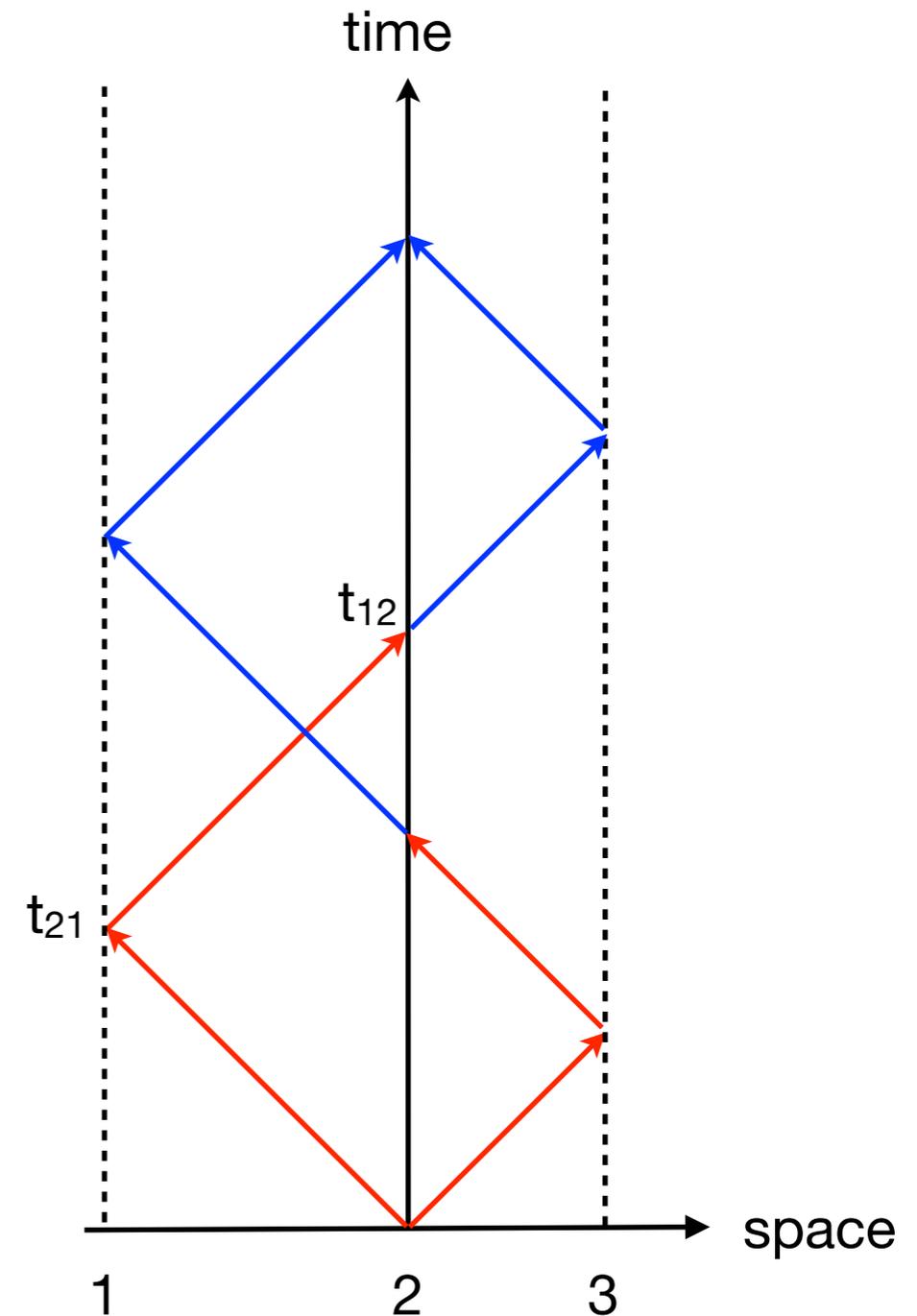
LISA's Time-Delay Interferometry (TDI)

- LIGO-like interferometry does not work for LISA, because
 - light is too weak
 - arm lengths are not equal enough
- Armstrong, Estabrook & Tinto's **Time-Delay Interferometry**
 - light not bounced back by mirrors, but detected
 - interferometry signal synthesized, with length difference accounted for



naive view: test masses compare each other's clock by sending & receiving light pulses

6 links between the 3 spacecraft, each with 1 clock
6 channels - 3 clock noises = 3 noise-free channels

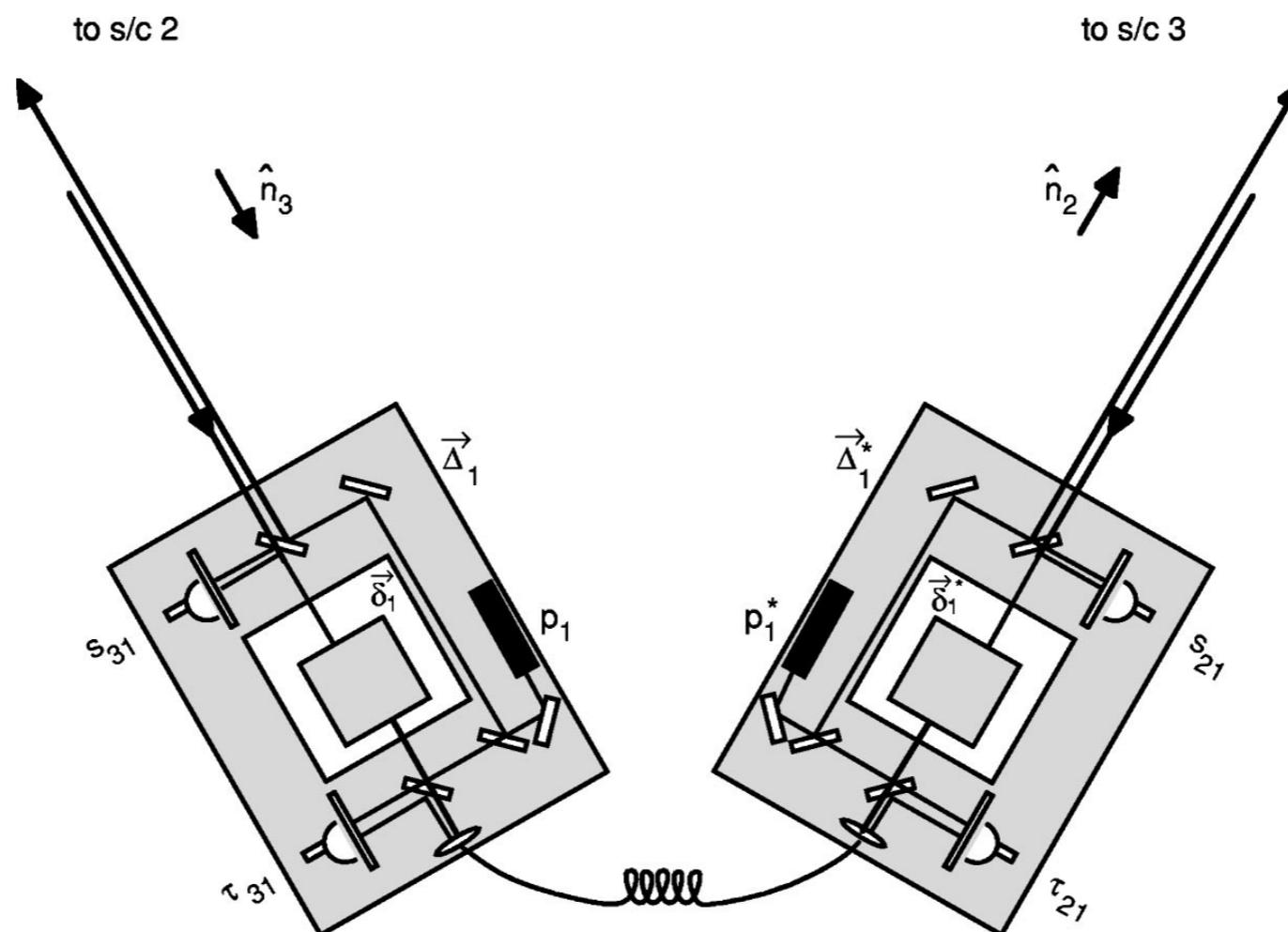


$t_{21}(L_{12}) + t_{12}(2L_{12})$: cancels noise of 1
 $t_{23}(L_{23}) + t_{32}(2L_{23})$: cancels noise of 3

subtracting the two doesn't cancel clock noise of 2!!

but we can complete the loop!!

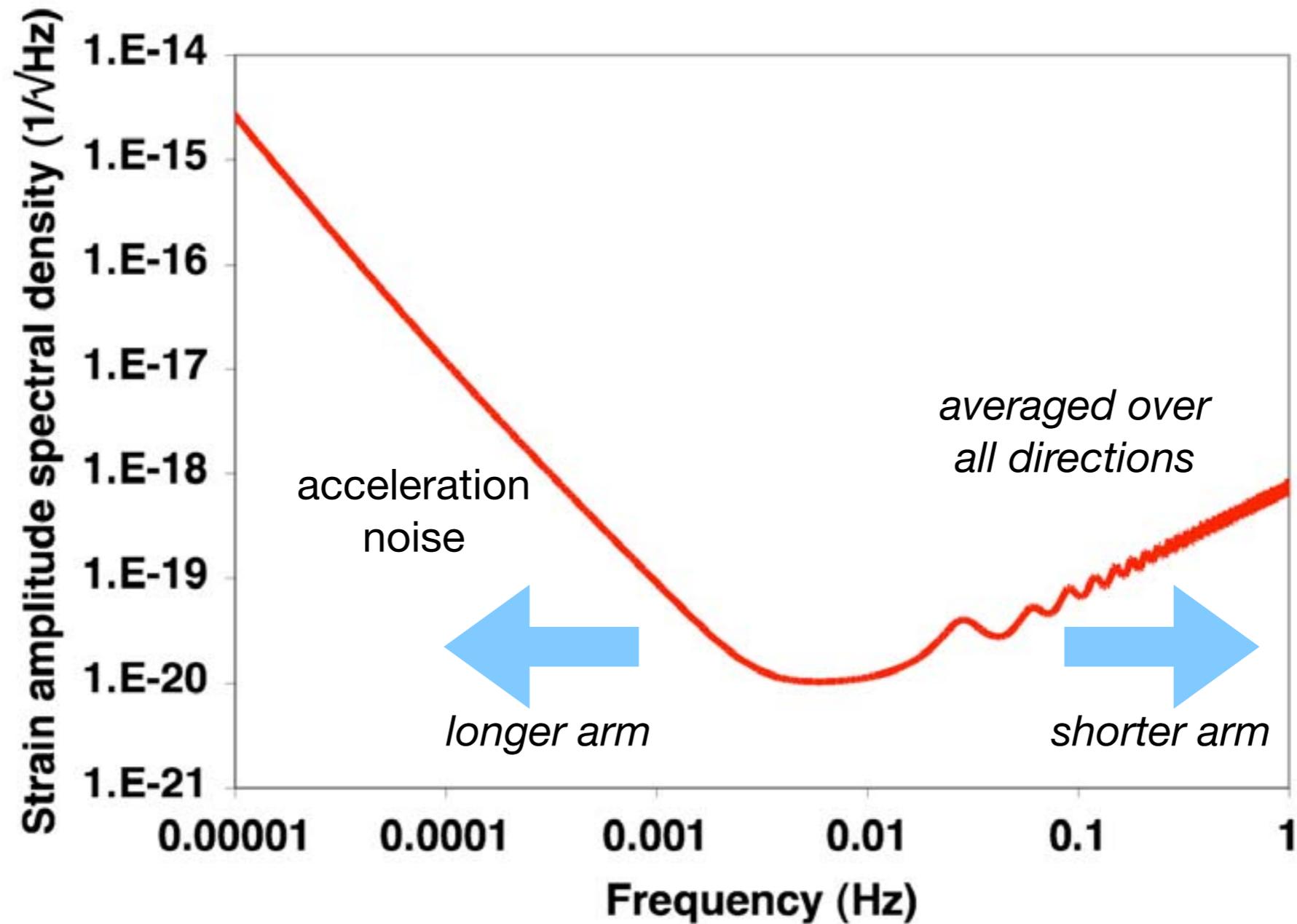
The Real Time-Delay Interferometry



Tinto, Estabrook & Armstrong (2002)

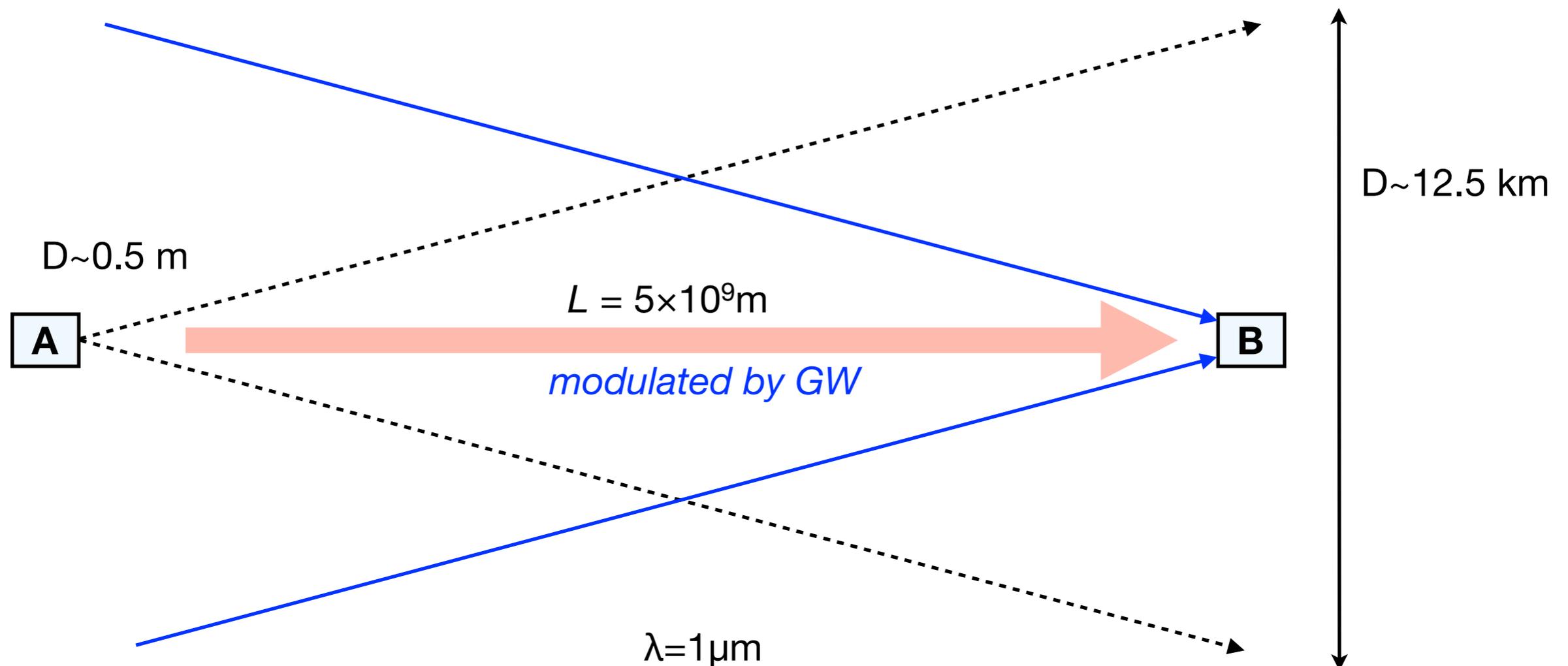
- Two Lasers & Two Test masses on board each spacecraft
 - 6 additional links
 - 3 additional channels of laser noise
 - 3 additional test-mass degrees of freedom
 - these are arranged to also cancel

LISA Noise Spectrum



[LISA Science Requirement Document]

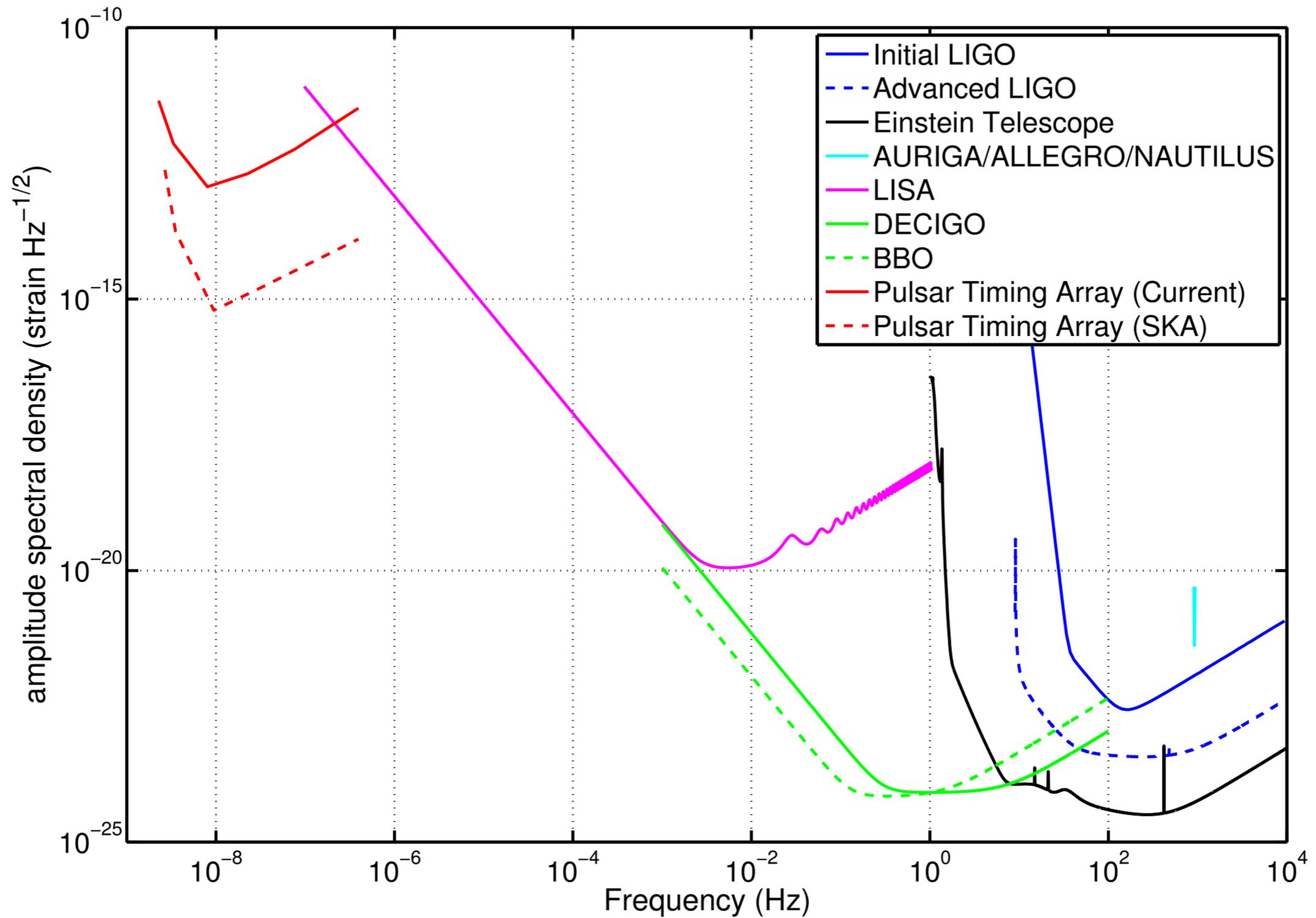
Squeezing?



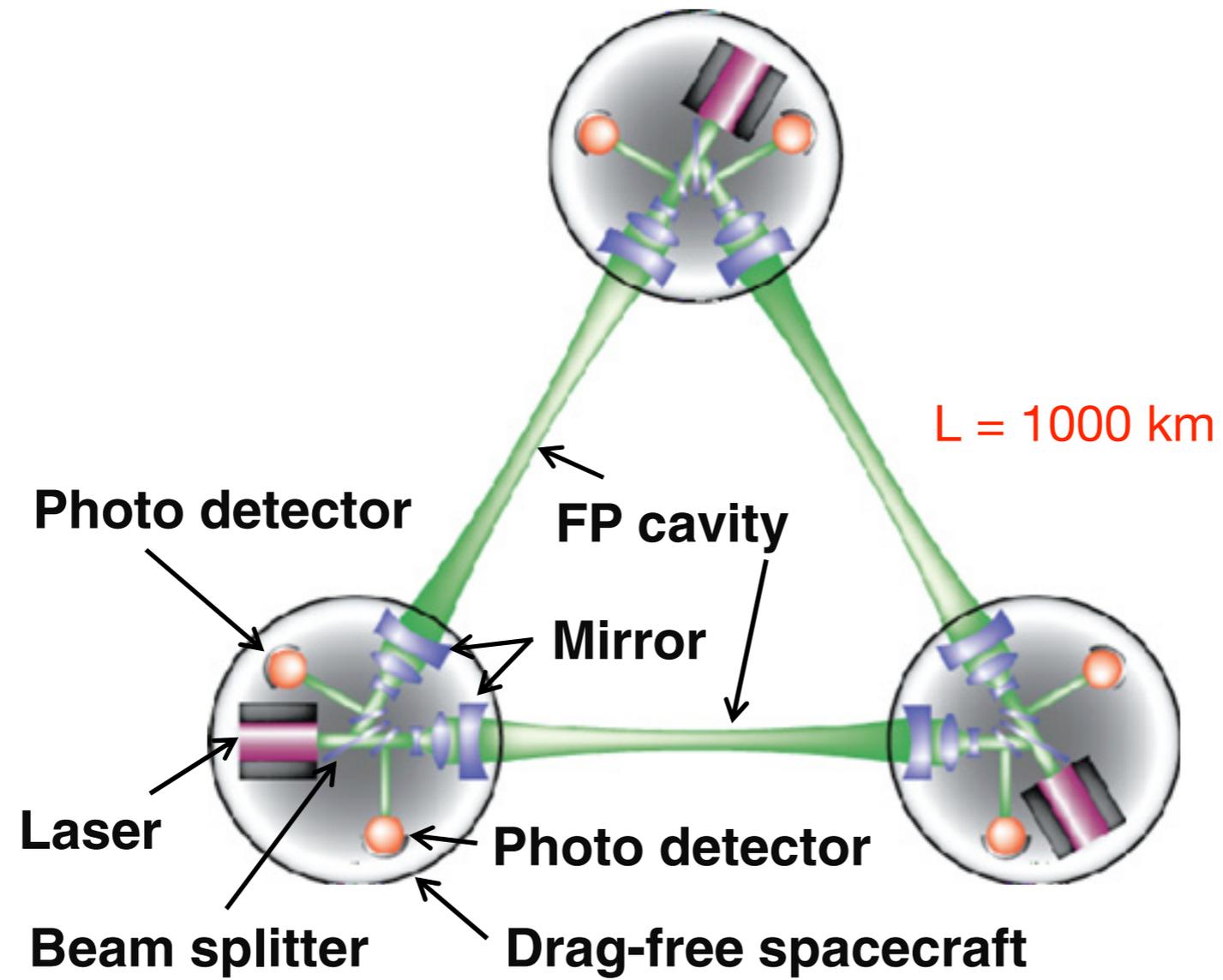
- Signal mode: very wide Gaussian cut by **B**'s aperture, **flat-top mode**
- Local oscillator at **B** must match this mode (mixing in any other mode will only lose)
- Can we squeeze this mode (or approximately this mode)?
 - let's propagate it backwards ...
 - it's not possible to squeeze this mode, unless we have larger apertures!!

*Being limited by Aperture Size & Acceleration Noise,
LISA cannot be improved quantum mechanically ...*

Beyond LISA: BBO & DECIGO



DECIGO



Summary

- Laser Interferometry can be used to detect gravitational waves.
- Squeezing already improves sensitivity of ground-based interferometry.
- Space-based GW detection goes after low-frequency sources