Weak Values in Metrology

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Weak Values

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How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

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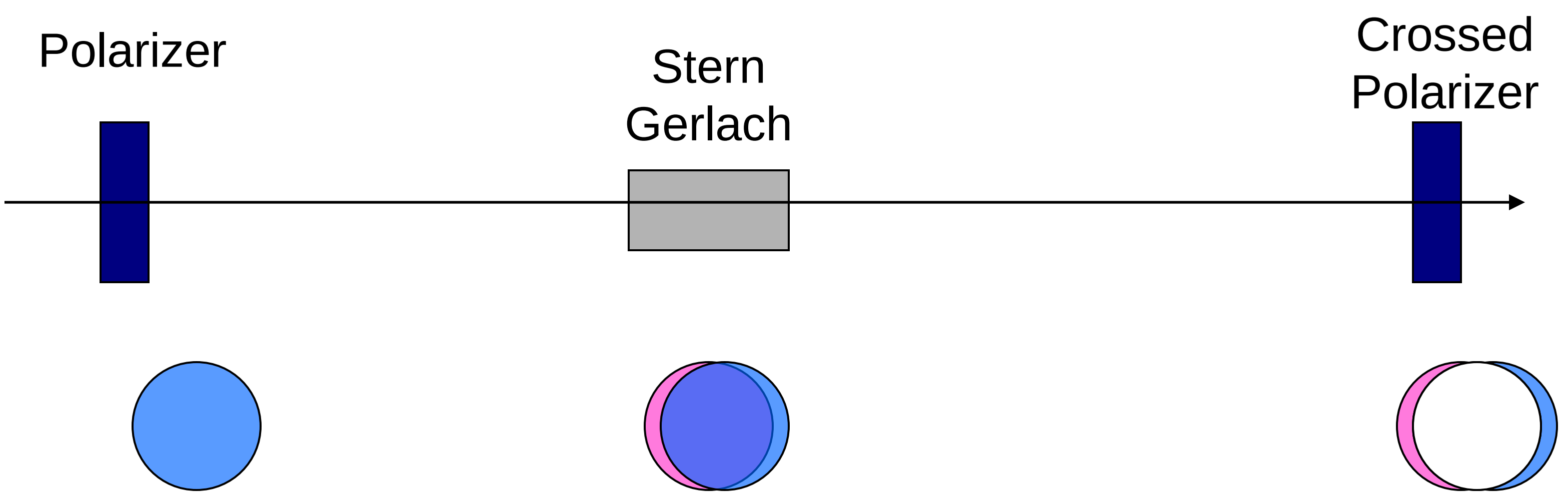
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value.



Weak Values in 3 Steps

- Pre-Selection

  e.g. spatial or polarization mode

- Linking Interaction

  weak perturbation linking 2 d.o.f.

- Post-Selection

  mostly orthogonal to pre-selection

Weak Values Mathematics

$$\langle \widetilde{\text{Path}} | \exp(-ix\mathbf{A}k) \; |\text{Path}\rangle \otimes |\text{Position}\rangle$$

Post-Selection Linking Pre-Selection

Weak Values Mathematics

$$\langle \widetilde{\text{Path}}| \quad 1 - ix\mathbf{A}k \quad |\text{Path}\rangle \otimes |\text{Position}\rangle$$

Post-Selection Linking Pre-Selection

$$\left(\langle \widetilde{\text{Path}}|\text{Path}\rangle - \langle \widetilde{\text{Path}}|ix\mathbf{A}k|\text{Path}\rangle\right) \otimes |\text{Position}\rangle$$

Weak Values Mathematics

$$\langle\widetilde{\text{Path}}| \quad 1 - ix\mathbf{A}k \quad |\text{Path}\rangle \otimes |\text{Position}\rangle$$

Post-Selection Linking Pre-Selection

$$\langle\widetilde{\text{Path}}|\text{Path}\rangle \times \left(1 - \frac{\langle\widetilde{\text{Path}}|ix\mathbf{A}k|\text{Path}\rangle}{\langle\widetilde{\text{Path}}|\text{Path}\rangle}\right) \otimes |\text{Position}\rangle$$

Weak Values Mathematics

$$\langle\widetilde{\mathrm{Path}}| \quad 1 - ix\mathbf{A}k \quad |\mathrm{Path}\rangle \otimes |\mathrm{Position}\rangle$$

Post-Selection Linking Pre-Selection

$$\langle\widetilde{\mathrm{Path}}|\mathrm{Path}\rangle \times \left(1 - ixk\frac{\langle\widetilde{\mathrm{Path}}|\mathbf{A}|\mathrm{Path}\rangle}{\langle\widetilde{\mathrm{Path}}|\mathrm{Path}\rangle}\right) \otimes |\mathrm{Position}\rangle$$

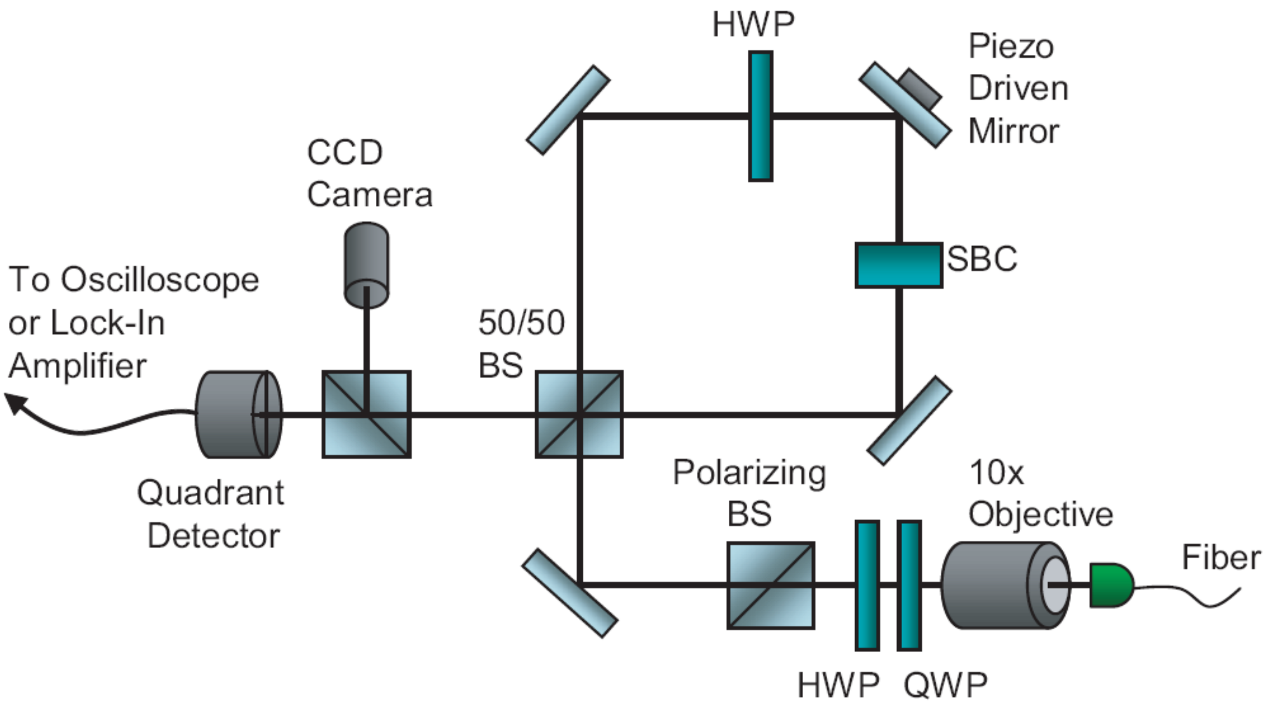
Weak Values Mathematics

$$\langle \widetilde{\text{Path}} | \quad 1 - ix\mathbf{A}k \quad |\text{Path}\rangle \otimes |\text{Position}\rangle$$

Post-Selection Linking Pre-Selection

$$\langle \widetilde{\text{Path}} | \text{Path} \rangle \times \exp(-ixA_w k) \otimes |\text{Position}\rangle$$

Experimental Setup

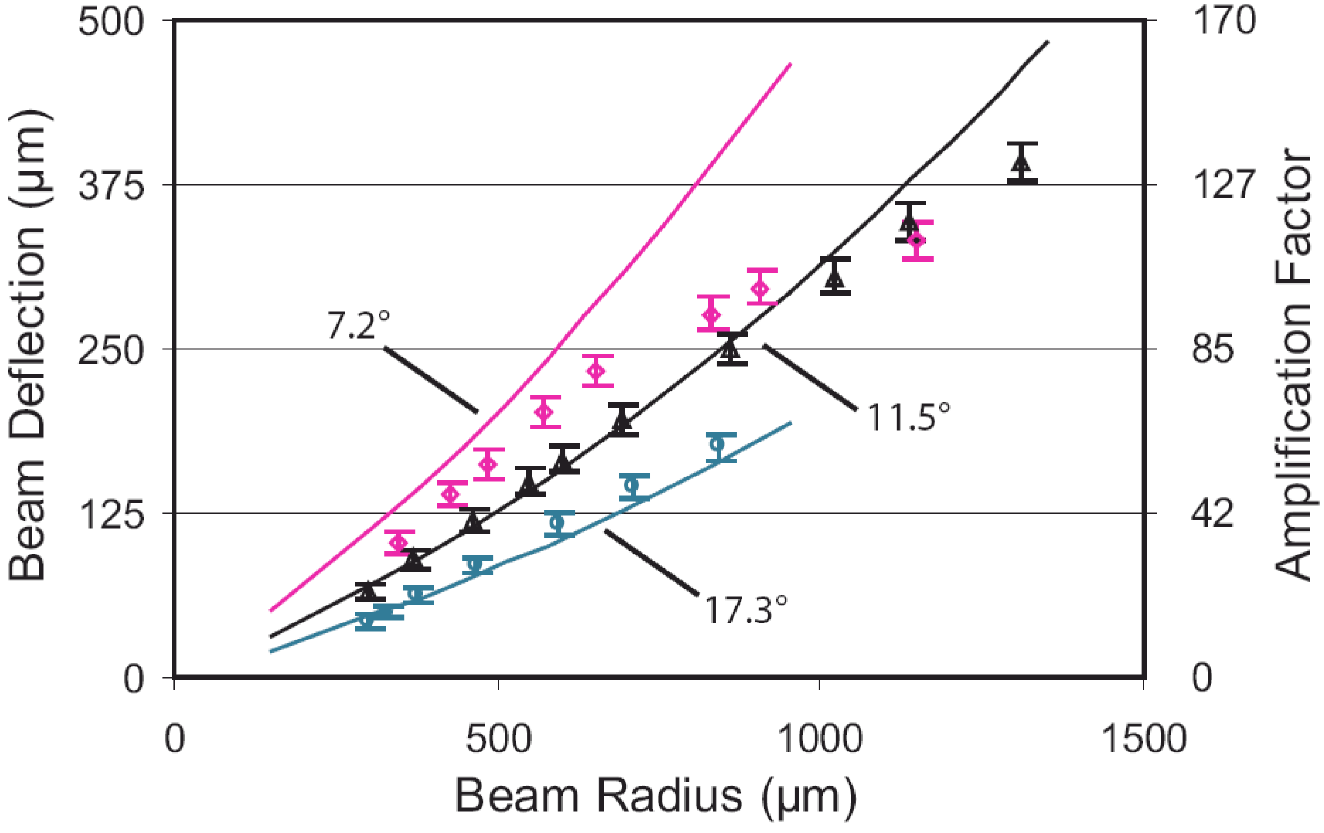


$$A_w = -i\cot(\phi/2) \approx -2i/\phi \qquad \mathcal{A} = \frac{2k_0\sigma^2}{l_{md}}\cot(\phi/2)$$

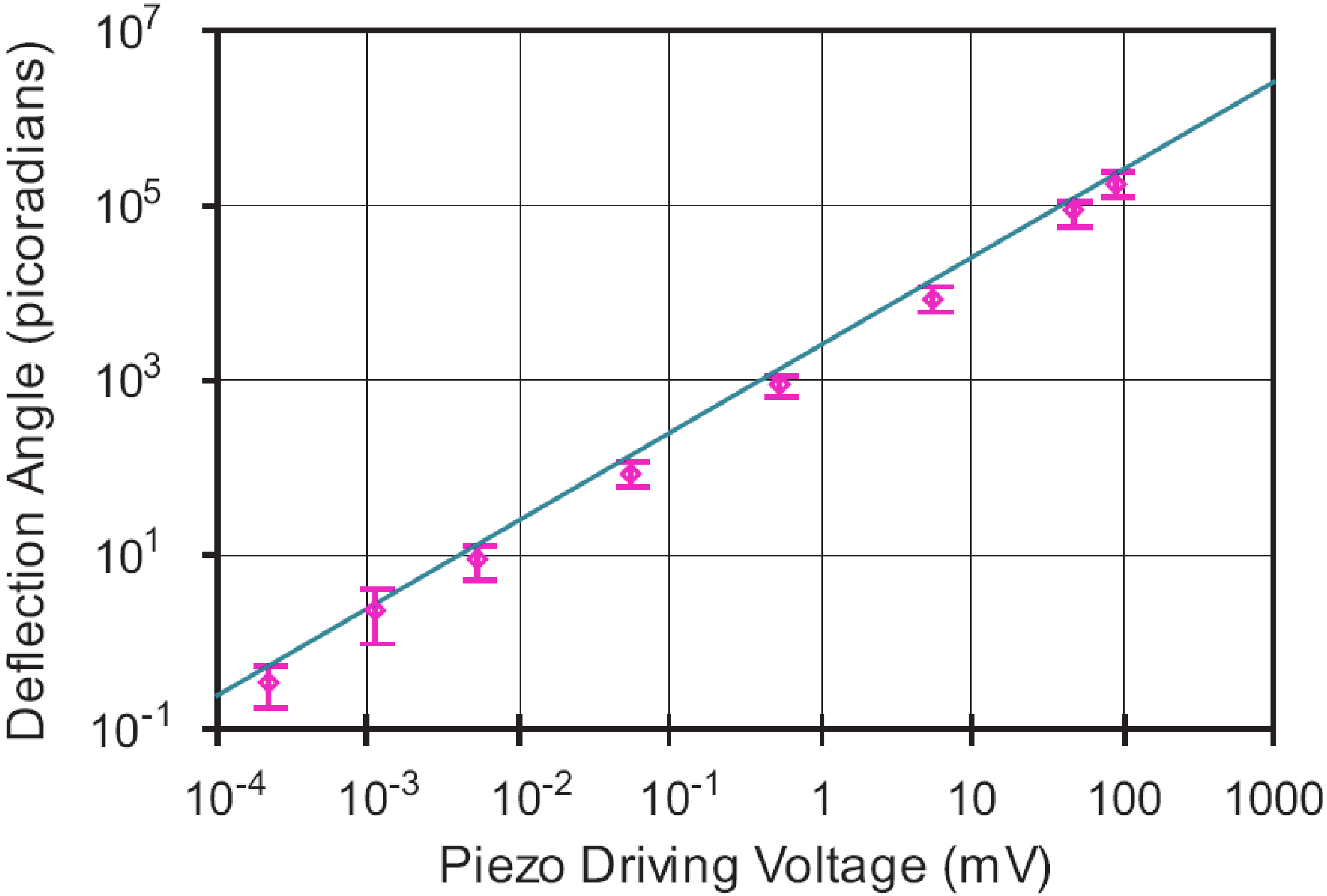
Dixon et al., Phys. Rev. Lett. 102, 173601 (2009)

Characterization

$$\mathcal{A} = \frac{2k_0\sigma^2}{l_{md}}\cot(\phi/2)$$



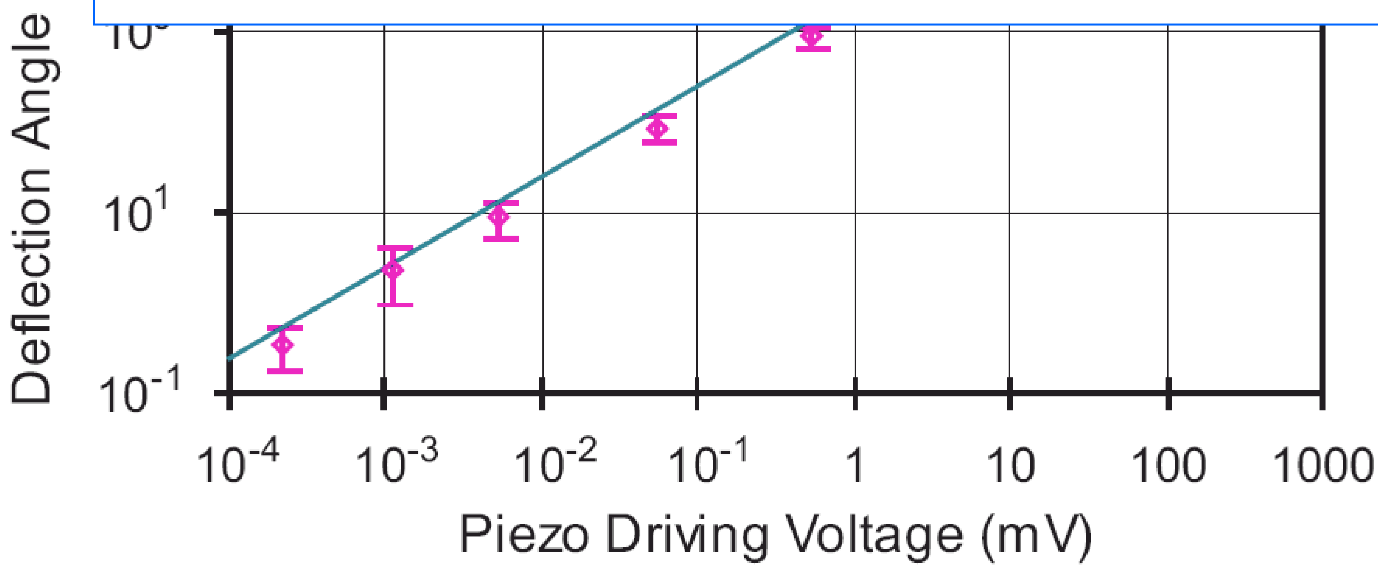
Deflection Measurement



Deflection Measurement

“measured the angular deflection of a light beam with the precision of literally a hair’s breadth at the distance of the Moon.”

A. M. Steinberg in News and Views, *Nature* **463**, 890-891



Signal to Noise Ratio

$$\mathcal{R} = \sqrt{\frac{2}{\pi}}\frac{\sqrt{N}d}{\sigma}$$

Weak Values

$\alpha\,\mathcal{R}$

$$\boxed{\alpha = 2k_0\sigma^2\cos(\phi/2)/l_{md}}$$

Focusing Lens

$\alpha_f\,\mathcal{R}$

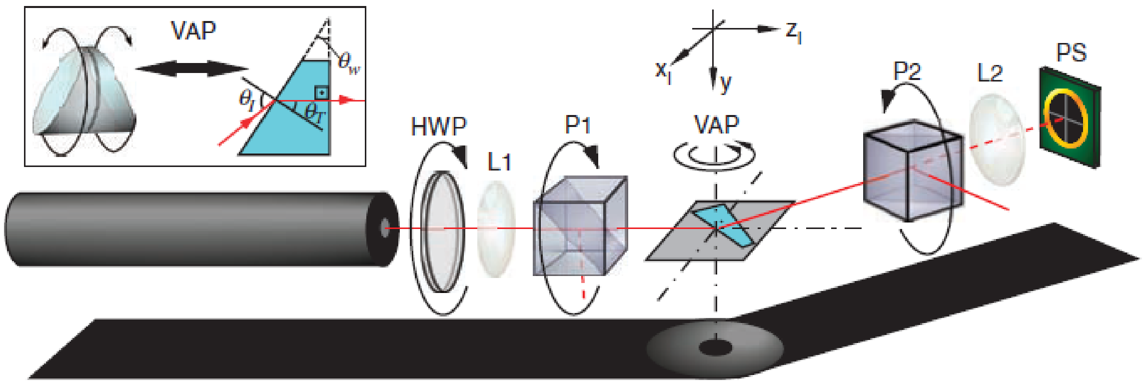
$$\boxed{\alpha_f = 2k_0\sigma^2/l_{md}}$$

Starling, Dixon, Jordan, and Howell, Phys. Rev. A 80, 041803(R) (2009)

More Capabilities

Observation of the Spin Hall Effect of Light via Weak Measurements

Onur Hosten* and Paul Kwiat



Hosten and Kwiat, Science 319, 787 (2008)

More Capabilities

Observation of the Spin Hall Effect of Lig

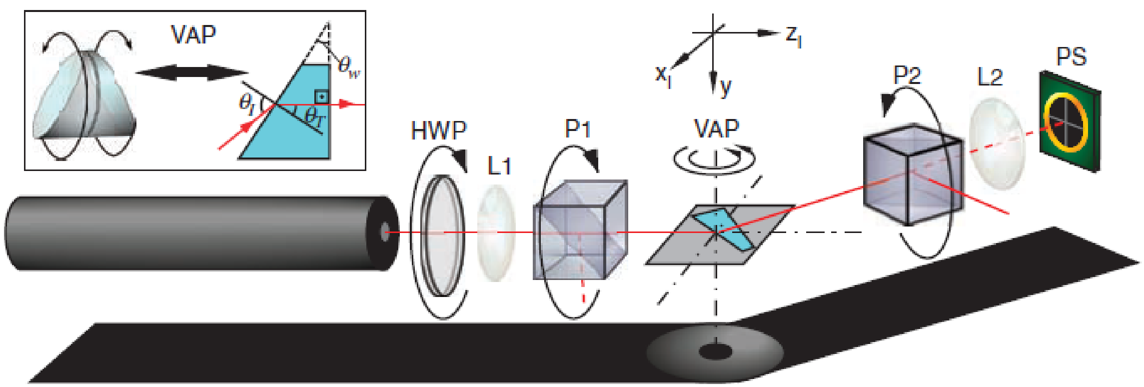
Onur Hoster

Amplifying Single-Photon Nonlinearity Using Weak Measurement

Amir Feizpour,[1,*] Xingxing Xing,[1] and Aephraim M. Steinberg[1]

[1] *Centre for Quantum Information and Quantum Control, and Institute for Optical Sciences, Department of Physics, University of Toronto, 60 St George Street, Toronto, Ontario, Canada M5S 1A7*

We show that weak measurement can be used to "amplify" optical nonlinearities at the single-photon level, such that the effect of one properly post-selected photon on a classical beam may



Hosten and Kwiat, Science 319, 787 (2008)   Feizpour et al. Phys. Rev. Lett. 107, 133603 (2011)

More Capabilities

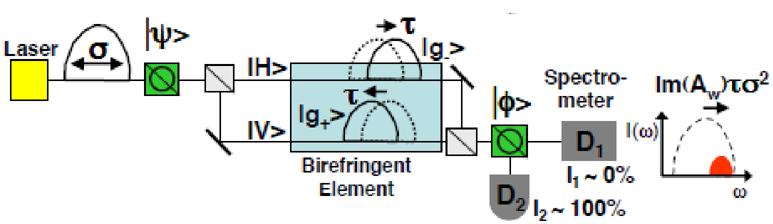
Measuring small longitudinal phase shifts: weak measurements or standard interferometry?

Nicolas Brunner[1] and Christoph Simon[2]

[1]H.H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, United Kingdom
[2]Institute for Quantum Information Science and Department of Physics and Astronomy, University of Calgary, Calgary T2N 1N4, Alberta, Canada

(Dated: July 12, 2010)

Recently, weak measurements were used to measure small effects that are transverse to the propagation direction of a light beam. Here we address the question whether weak measurements are also useful for measuring



Brunner and Simon, Phys. Rev. Lett. 105, 010405 (2010)

More Capabilities

Measuring small longitudinal phase shifts: weak measurements or standard interferometry?

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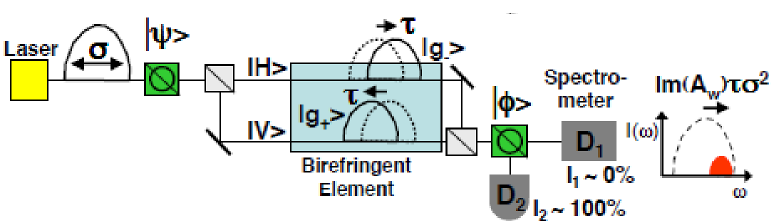
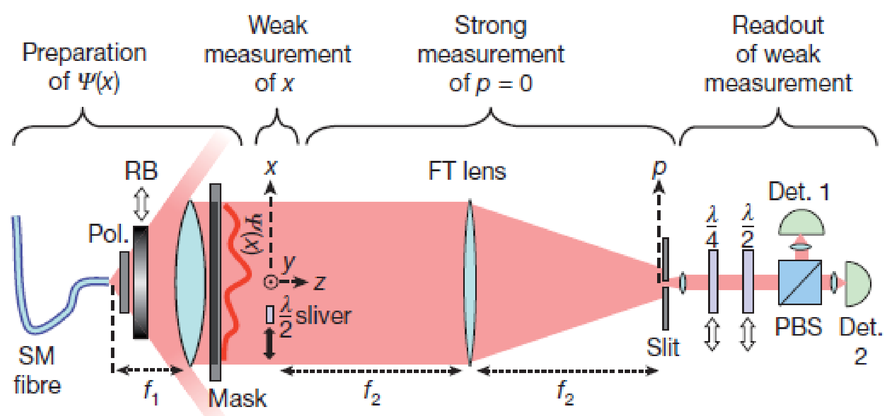
(Dated: July 12, 2010)

Recen
tion of a

Direct measurement of the quantum wavefunction

Jeff S. Lundeen[1], Brandon Sutherland[1], Aabid Patel[1], Corey Stewart[1] & Charles Bamber[1]





Brunner and Simon, Phys. Rev. Lett. 105, 010405 (2010)

Lundeen et al., Nature 474, 188–191 (2011)

Possible Benefits for Space

- Requires lower power on detector
  - Easier to meet requirements
  - Lower detector noise
- No need to focus
  - Can’t focus? Don’t want to focus?
- ~90% of light remains
  - Recycle the light
  - Measure Something Else

Possible Difficulties for Space

- Requires small perturbations
  - Only tracking, not searching
- Need to control the linking interaction
  - Only works on objects we have access to