Quantum Limits on Sensing and Imaging

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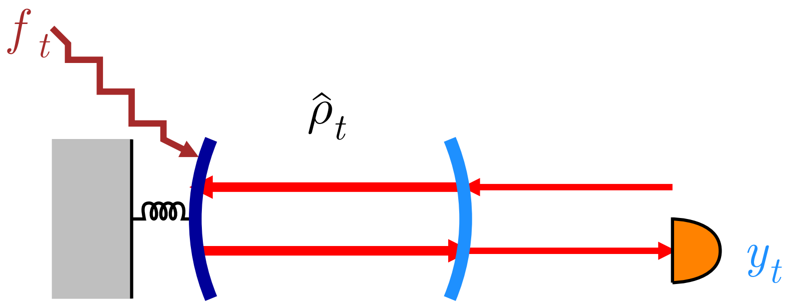
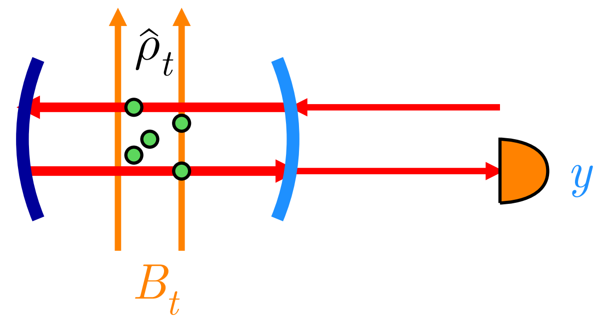
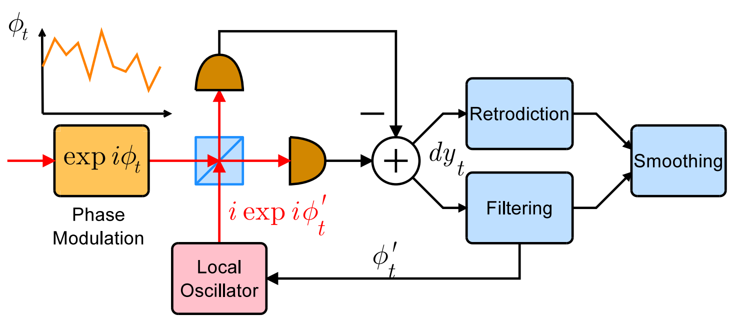
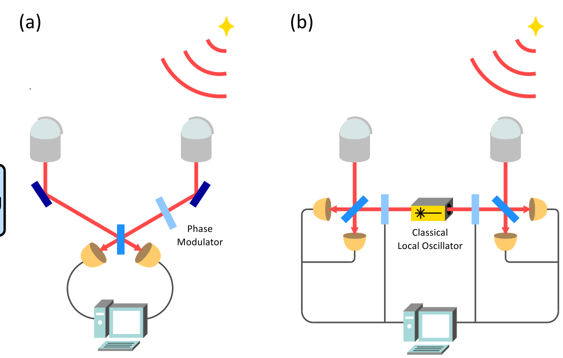
Quantum Waveform Sensing



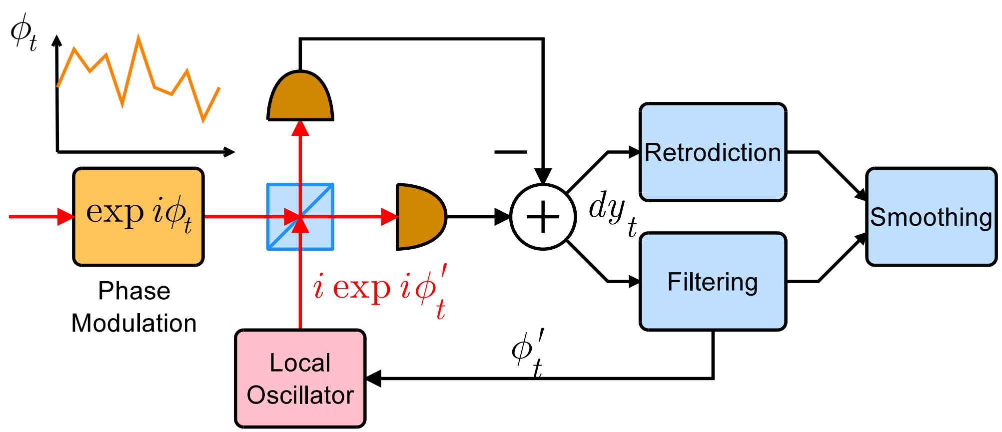






- Estimation/detection of classical waveforms $x(\boldsymbol{r}, t)$ using quantum systems
- Examples: optical interferometry, optical imaging, optomechanical force sensing (gravitational-wave detection), atomic magnetometry, gyroscopy, etc.

Optical Phase and Frequency Estimation



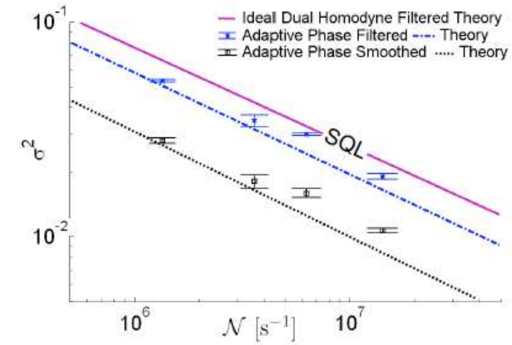


FIG. 3: The variance σ^2 of the adaptive phase estimation for quantum filtering and smoothing as a function of photon number $\mathcal{N}$, compared to the relevant theoretical predictions, and the theoretical predictions for nonadaptive measurements.

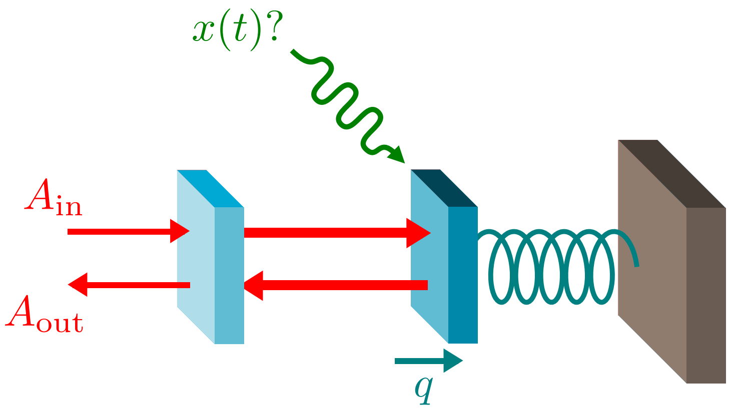
- QCRB [Tsang, Wiseman, and Caves, PRL **106**, 090401 (2011); unpublished]:

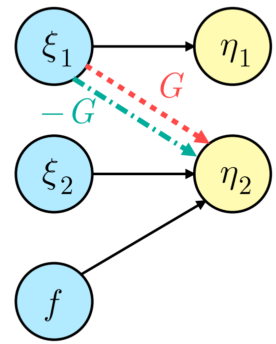
$$\phi(t) = \int_{-\infty}^{\infty} dt' h(t-t')x(t'), \quad \langle \delta x^2 \rangle \geq \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{4|h(\omega)|^2 S_{\Delta \hat{I}}(\omega) + 1/S_x(\omega)},$$

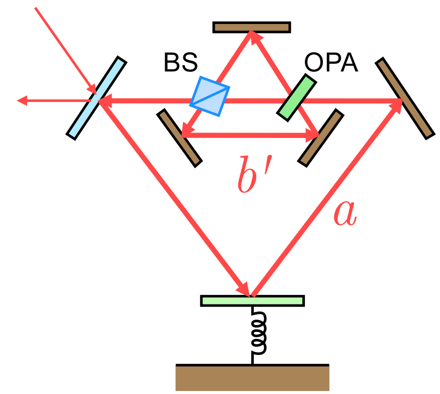
$$\text{e.g. } S_{\Delta \hat{I}}^{\text{coh}}(\omega) = \frac{\bar{P}}{\hbar\omega_0}, \quad S_x^{\text{OU}}(\omega) = \frac{\kappa}{\omega^2 + \epsilon^2}. \tag{1}$$

- Achieved by homodyne phase-locked loop + Smoothing [Personick IEEE TIT **17**, 240 (1971); Tsang, Shapiro, and Lloyd, PRA **78**, 053820 (2008); **79**, 053843 (2009)]
- Wheatley *et al.*, PRL **104**, 093601 (2010).
- Interferometry, ranging, velocimetry, clock synchronization, coherent comm., etc.

Optomechanical Force Sensing







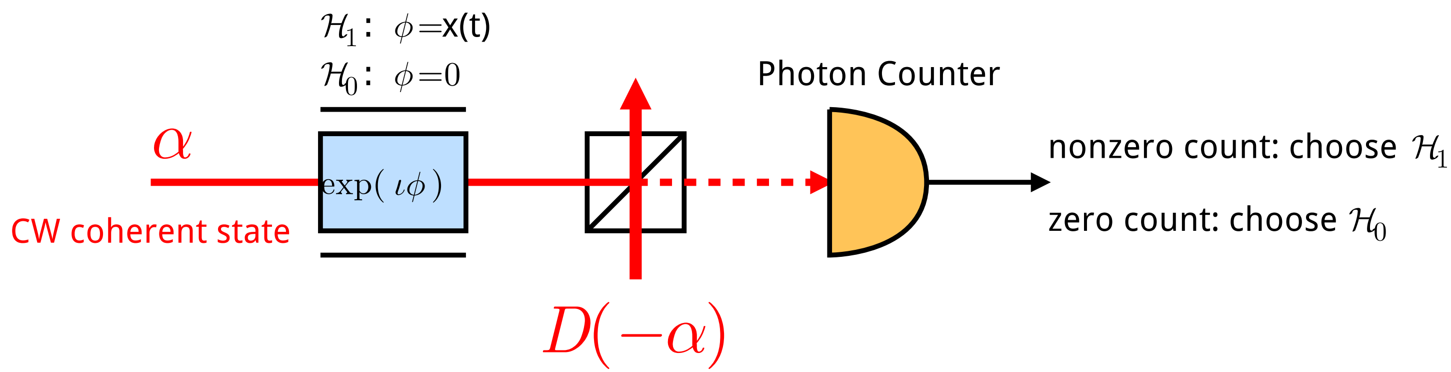
- QCRB [Tsang, Wiseman, and Caves, PRL **106**, 090401 (2011)]:

$$\langle \delta x^2 \rangle \geq \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(4/\hbar^2) S_{\Delta \hat{q}}(\omega) + 1/S_x(\omega)}. \tag{2}$$

- Achieved by Quantum Backaction Noise Cancellation [Kimble *et al.*, PRD **65**, 022002 (2001); Chen, PRD **67**, 122004 (2003); Tsang and Caves, PRL **105**, 123601 (2010)] + Smoothing [Tsang, PRL **102**, 250403 (2009)].

Optical Phase Waveform Detection

- Binary hypothesis testing $\mathcal{H}_0 : \phi(t) = 0$, $\mathcal{H}_1 : \phi(t) = x(t)$
- Tsang, unpublished:

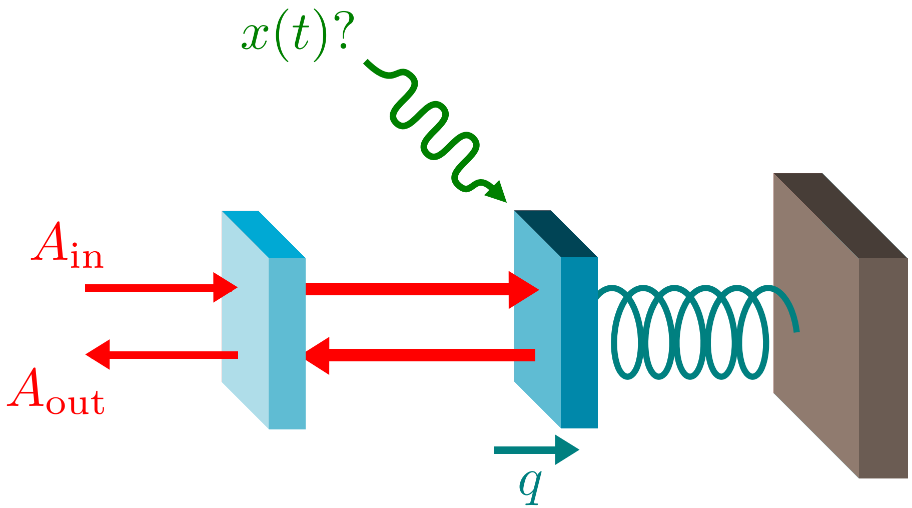


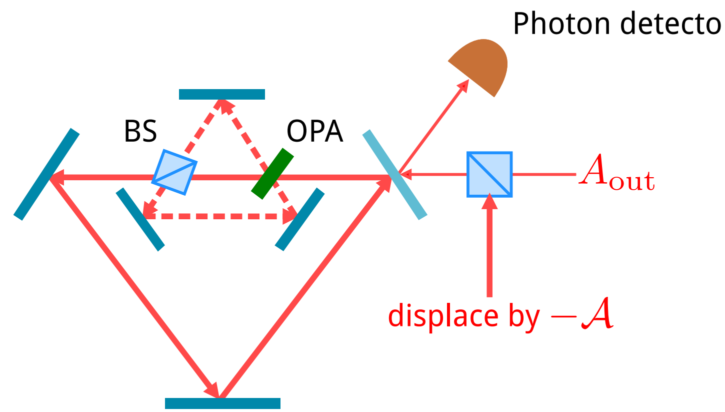
$$P_e \geq \frac{1}{2}\left(1 - \sqrt{1 - 4P_0 P_1 F}\right), \tag{3}$$

$$F = \int DxP[x]\left|\langle\psi|\exp\left[i\int_{t_0}^{T} dt\hat{I}(t)x(t)\right]|\psi\rangle\right|^2. \tag{4}$$

- Kennedy receiver has optimal error exponent for stochastic waveform detection with coherent state.
- Homodyne performance depends on prior waveform statistics

Optomechanical Force Detection





- Tsang and Nair, arXiv:1204.3697 (2012):

$$P_e \ge \frac{1}{2}\left(1 - \sqrt{1 - 4P_0P_1F}\right), \quad F = \int DxP[x]\left|\langle\psi|\mathcal{T}\exp\left[i\int_{t_0}^{T} dt\hat{q}_0(t)x(t)\right]|\psi\rangle\right|^2 \tag{5}$$

- Because the bound is achievable for deterministic $x(t)$ and not limited by backaction noise, SQL can definitely be overcome
- QNC + Kennedy receiver achieves optimal error exponent

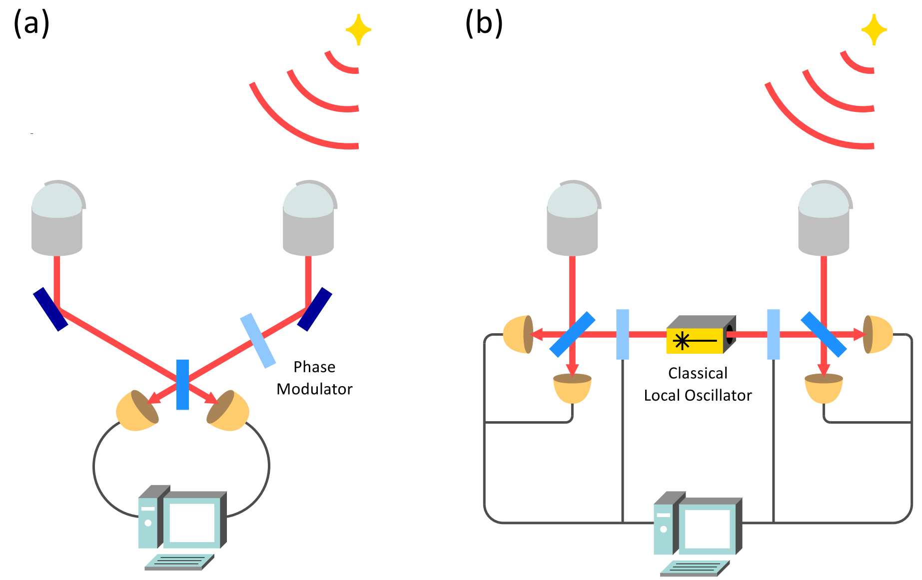
Decoherence

- Significant decoherence/loss rules out any significant quantum enhancement using squeezed state/nonclassical state of light (Durkin *et al.*/Escher *et al.*)
- No-go for nonclassical light in space optics applications
- No result yet about decoherence in waveform estimation/detection, surprise unlikely
- Quantum illumination (Lloyd/Erkmen/Guha/Shapiro/Giovannetti *et al.*): up to 6 dB improvement in error exponent!
  - Producing squeezed state requires strong pump with way more photons
  - can be achieved by coherent state with 6 dB more photons
  - Known receivers can't get to 6 dB
  - Low-photon-number regime only
  - Gaussian noise strengths are assumed to be different under hypotheses for QI to be useful, passive target detection may be better in practice
- Quantum Metrology with POVMs

Quantum Imaging

- Ghost imaging: Shih/Shapiro/Erkmen
- Sub-Rayleigh quantum lithography/imaging: Boto *et al.*, PRL **85**, 2733 (2000); Tsang, PRL **102**, 253601 (2009); Giovannetti *et al.*, PRA **79**, 013827 (2009)
- Experiments: D'Angelo *et al.*, PRL **87**, 013602 (2001); Guerrieri *et al.*, PRL **105**, 163602 (2010); Shin *et al.*, PRL **107**, 083603 (2011)
- Classical computational sub-Rayleigh imaging: STORM/PALM [Zhuang, Nature Photon. **3**, 365 (2009)]; STED (Hell), etc.
- Computational imaging for astronomy [Fienup]
- Not much rigorous work in quantum imaging that uses estimation/detection theory

Quantum Camera Design

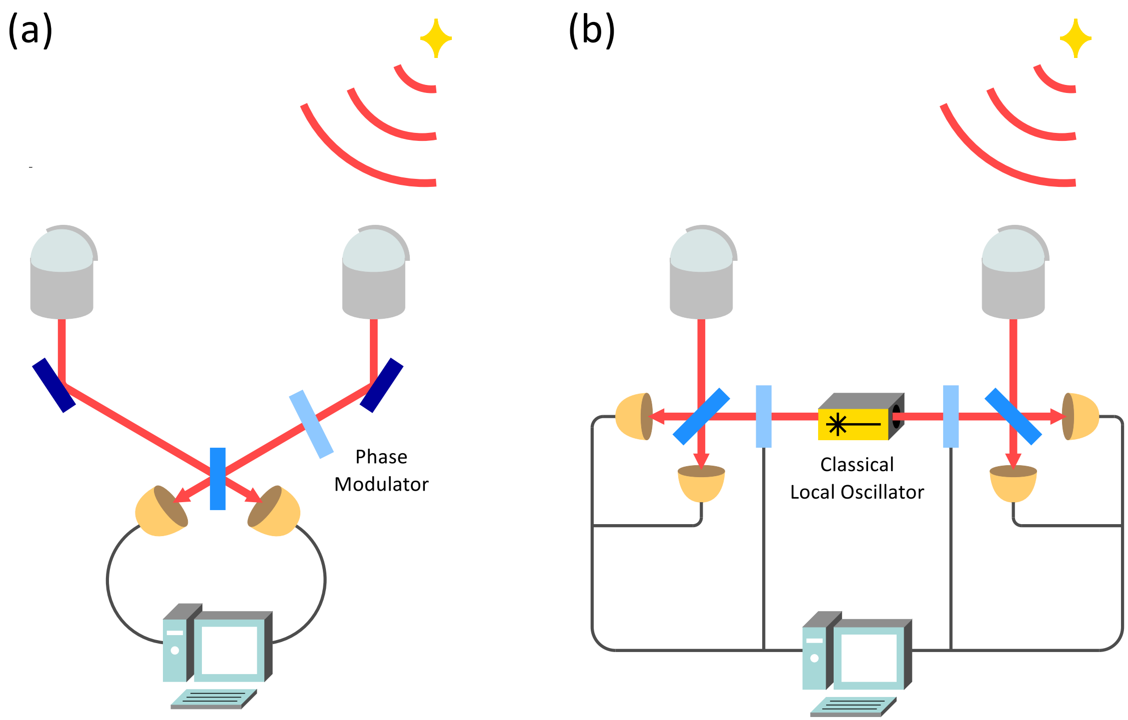


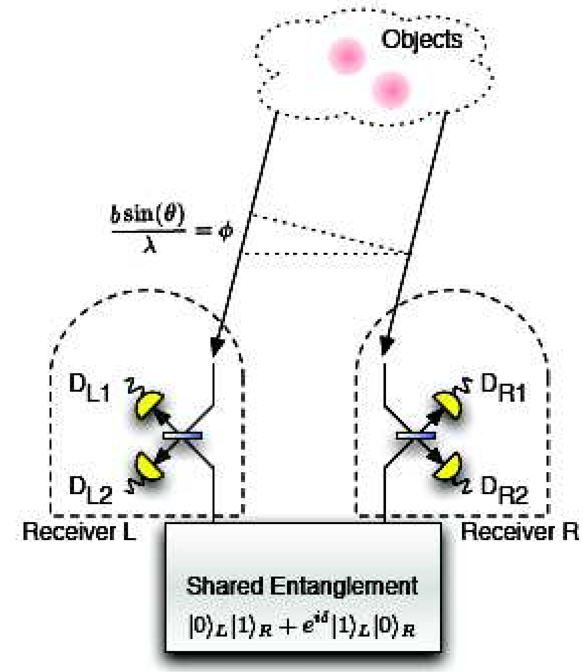
- start with multi-spatial-mode $\rho_{x(\boldsymbol{r})}$ (e.g., multimode thermal or coherent state)
- Record with imaging system/CCD/interferometer/digital holography (model by POVM $\boldsymbol{E}[y(\boldsymbol{r})]$)

$$P[y(\boldsymbol{r})|x(\boldsymbol{r}')] = \mathrm{tr}\left\{E[y(\boldsymbol{r})]\Phi_{\mathrm{aperture}}\Phi_{\mathrm{diffract}}\rho_{x(\boldsymbol{r}')}\right\} \tag{6}$$

- Quantum bounds: multiparamter QCRB, etc.
- Do conventional imaging systems saturate these bounds?
- How to implement optimal POVM?

Stellar Interferometry





Gottesman, Jennewein, and Croke, e-print arXiv:1107.2939.

- Estimation of coherence:

$$\Gamma_{ab} = \langle b^\dagger a \rangle, \qquad g^{(1)} = \frac{\langle b^\dagger a \rangle}{\sqrt{\langle b^\dagger b \rangle \langle a^\dagger a \rangle}} \quad \text{(normalized)}. \tag{7}$$

Old-School Quantum Optics

- P representation:

$$\rho = \int d^2\alpha d^2\beta \Phi(\alpha,\beta)|\alpha,\beta\rangle\langle\alpha,\beta|. \tag{8}$$

- $\Phi(\alpha,\beta)$ is a two-mode zero-mean Gaussian for thermal light, i.e. no entanglement
- weak thermal light $\epsilon \equiv \langle a^\dagger a\rangle = \langle b^\dagger b\rangle \ll 1$ in photon-number basis:

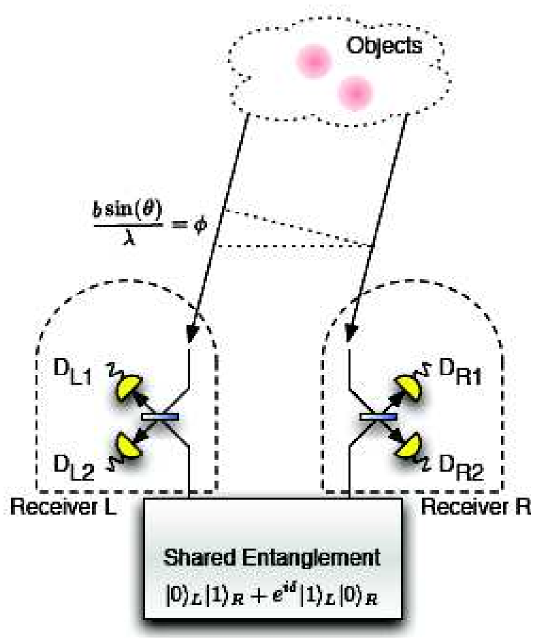
$$\rho = (1-\epsilon)|0,0\rangle\langle 0,0| + \frac{\epsilon}{2}\left[|0,1\rangle\langle 1,0| + |1,0\rangle\langle 1,0| + g^*|0,1\rangle\langle 1,0| + g|1,0\rangle\langle 0,1|\right] + O(\epsilon^2), \tag{9}$$

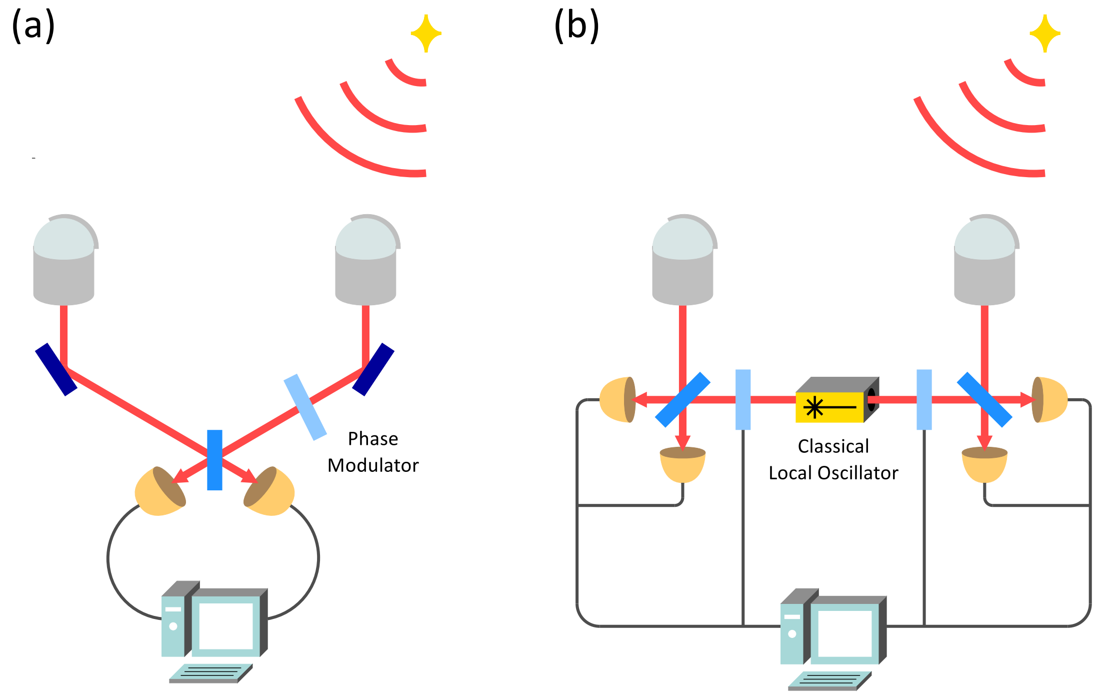
$$P(y|g) = \operatorname{tr}\left[E(y)\rho\right]. \tag{10}$$

- Classical Fisher information for $g = g_1 + ig_2$:

$$F_{jk} = \left\langle \frac{\partial}{\partial g_j}\ln P \frac{\partial}{\partial g_k}\ln P \right\rangle, \qquad \Sigma \geq \frac{1}{M}F^{-1} \tag{11}$$

Bound for Local Measurements

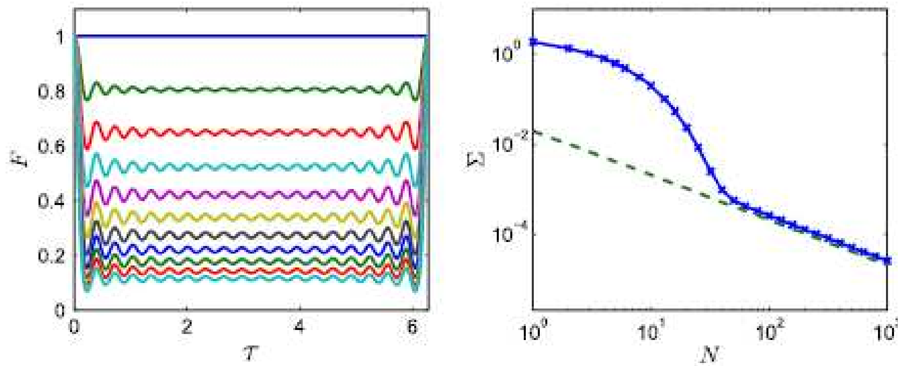




- Nonlocal measurements (direct detection, shared-entanglement): $||F|| \sim \epsilon$.
- A necessary condition for local (LOCC) measurement is the PPT condition applied to the POVM [Terhal *et al.*, PRL 86, 5807 (2001)]. Then $||F|| \leq \epsilon^2 + O(\epsilon^3)$.
- Generalizable to repeated LOCC measurements
- Quantum nonlocality in measurement of nature, even if the state has no entanglement.
- M. Tsang, PRL **107**, 270402 (2011).

Misc.

- Quantum Ziv-Zakai bounds [Tsang, PRL **108**, 230401 (2012)]



- Continuous quantum hypothesis testing (for tests of physics using continuous quantum measurements) [Tsang, PRL **108**, 170502 (2012)]
- Cavity quantum microwave photonics [Tsang, PRA **81**, 063837 (2010); **84**, 043845 (2011)]