

*Workshop on "Quantum Communication, Sensing and Measurement in Space",  
Keck Institute for Space Studies, California Institute of Technology, Pasadena, CA, June 25-29, 2012*

# **Solar System Tests of Relativistic Gravity:**

## *History, Recent Progress and Possible Future Directions*

Slava G. Turyshev

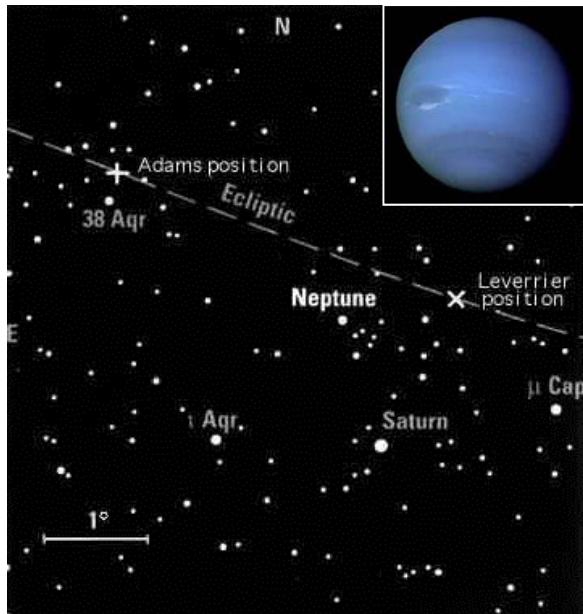
*Jet Propulsion Laboratory, California Institute of Technology  
4800 Oak Grove Drive, Pasadena, CA 91009 USA*

## Outline: Solar System Tests of Gravity

### The talk will cover:

- Theoretical Landscape in the 20th Century:
  - (brief...) History of the tests of general relativity
  - Frameworks used: the PPN formalism and Robertson-Mansouri-Sexl
  - Recent progress in the tests of general relativity
- Beginning of the 21st Century...:
  - Motivations for high-precision tests of gravity
  - What to expect in the near future? and some proposed experiments
- Main objective:
  - Remind where we came from and what lessons we learned
- Themes for discussion:
  - Are the solar system tests still useful?
  - Is there a discovery potential? Or what is the importance of new improved limits?
  - What tests are most valuable?

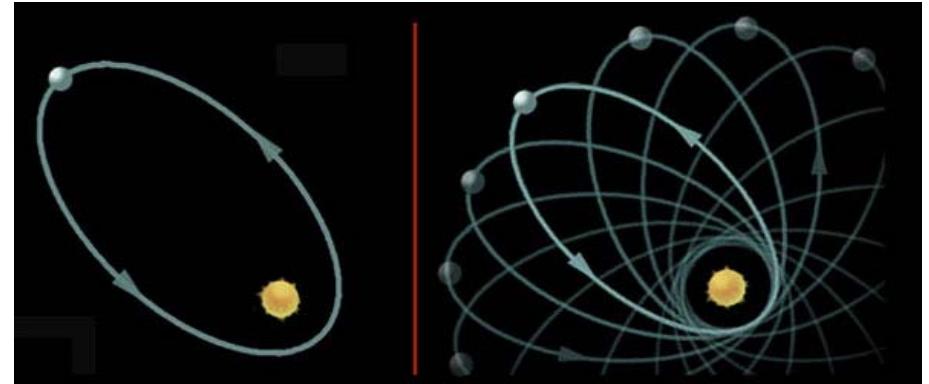
## Triumph of Mathematical Astronomy in 19<sup>th</sup> Century



Discovery of Neptune: 1845

Urbain LeVerrier  
(1811-1877)

- 1845: the search for Planet-X:
  - Anomaly in the Uranus' orbit → Neptune
  - Anomalous motion of Mercury → Vulcan



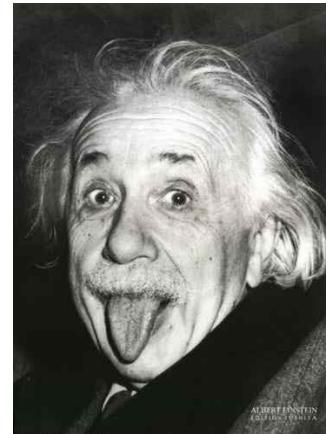
Newtonian Gravity

General Relativity

Sir Isaac Newton  
(1643-1727)

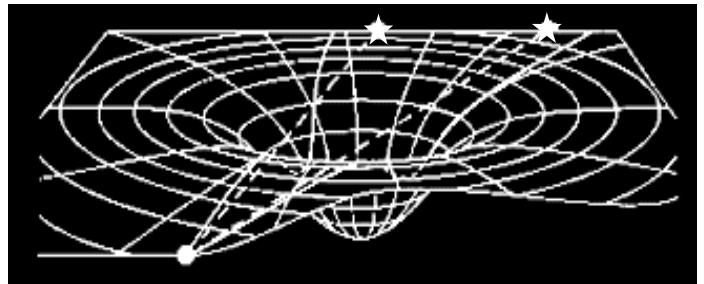
- Anomalous precession of Mercury's perihelion :
  - 43 arcsec/cy can not be explained by Newton's gravity
- Before publishing GR, in 1915, Einstein computed the expected perihelion precession of Mercury
  - When he got out 43 arcsec/cy – a new era just began!!

Almost in one year LeVerrier both confirmed the Newton's theory (Neptune) & cast doubt on it (Mercury's anomaly).

Albert Einstein  
(1879-1955)



# The First Test of General Theory of Relativity



## Gravitational Deflection of Light:

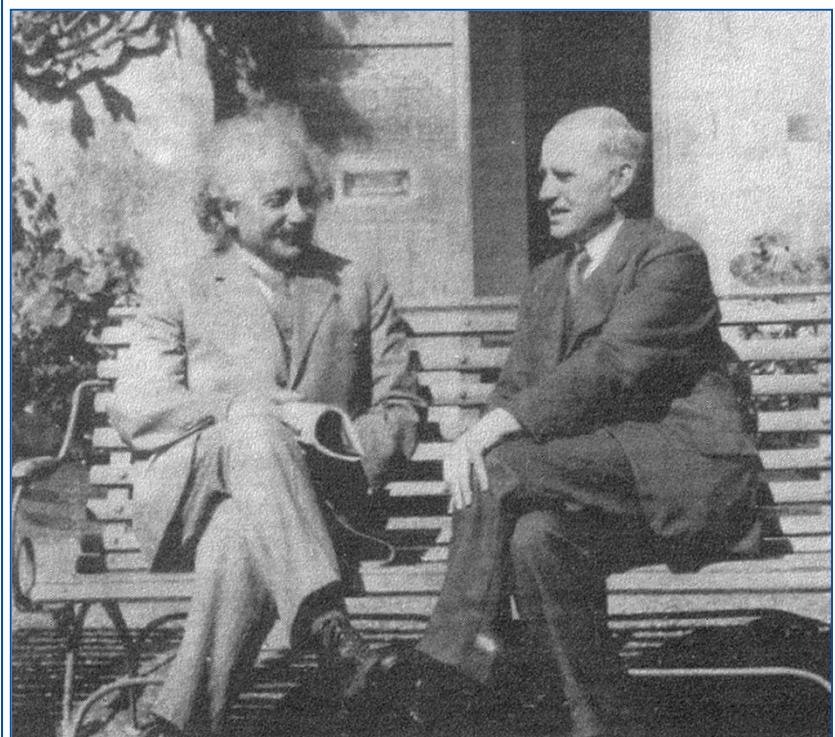
$$\theta_{gr}(b) = \frac{2(1+\gamma)GM_\odot}{bc^2} \simeq 8 \times 10^{-6} \left(\frac{1+\gamma}{2}\right) \left(\frac{R_\odot}{b}\right)$$



Campbell's telegram to Einstein, 1923

Solar Eclipse 1919:  
possible outcomes

Deflection = 0;  
Newton = 0.87 arcsec;  
Einstein = 2 x Newton = 1.75 arcsec

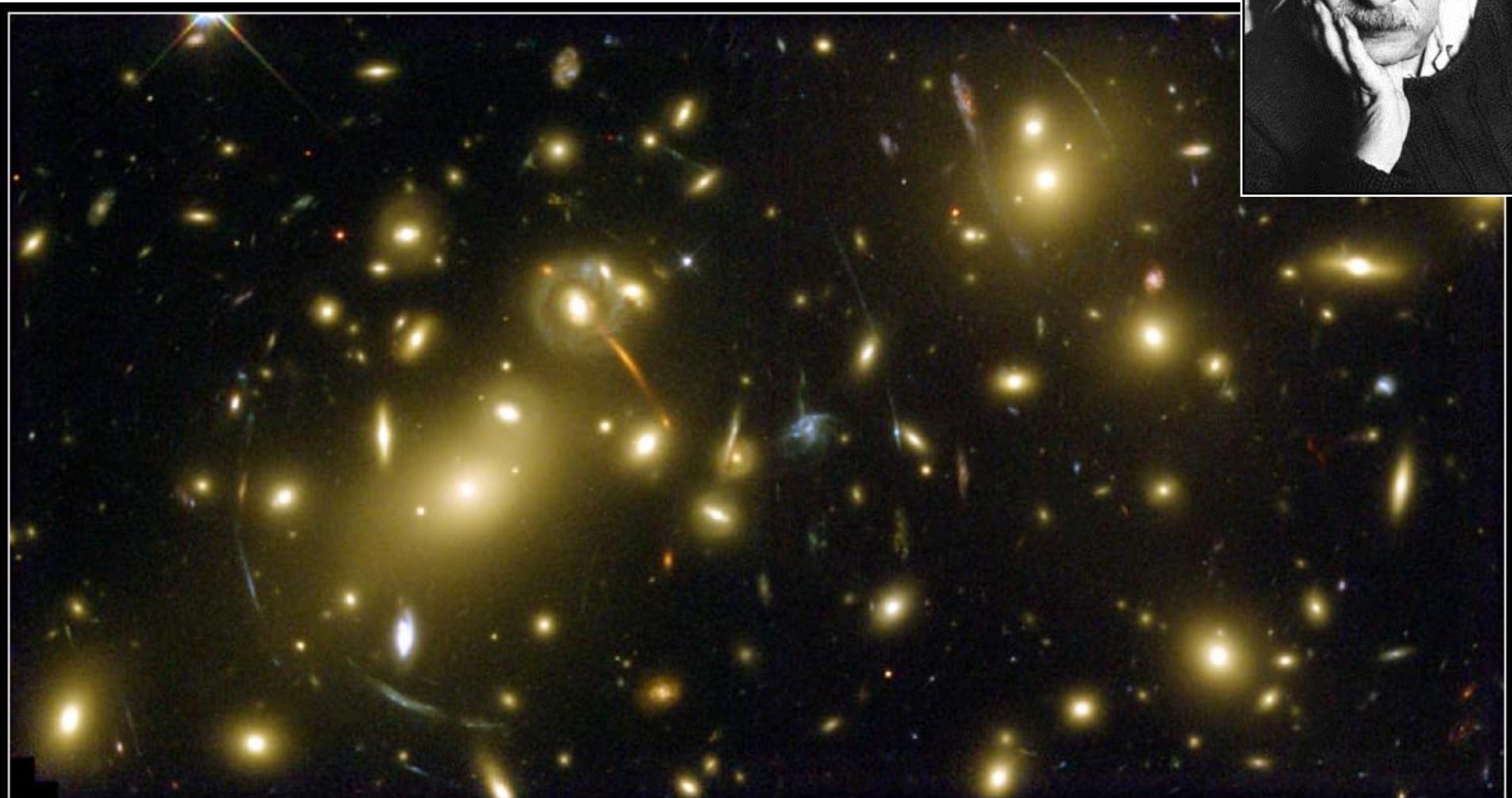
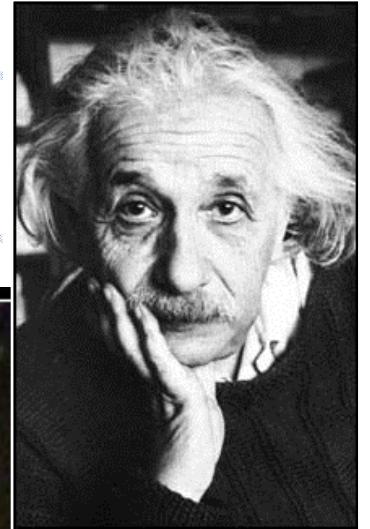


Einstein and Eddington, Cambridge, 1930



TESTS OF RELATIVISTIC GRAVITY IN SPACE

## Gravitational Deflection of Light is a Well-Known Effect Today



**Galaxy Cluster Abell 2218**

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

HST • WFPC2



*not a complete list...*

Newton 1686	Poincaré 1890							
Einstein 1912	Nordstrøm 1912		Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915			
Whitehead 1922	Cartan 1923		Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943			
Milne 1948	Thiry 1948	Papapetrou 1954	Jordan 1955	Littlewood & Bergmann 1956				
Brans & Dicke 1961		Yilmaz 1962	Whitrow & Morduch 1965	Kustaanheimo & Nuotio 1967				
Page & Tupper 1968		Bergmann 1968	Deser & Laurent 1968	Nordtvedt 1970		Wagoner 1970		
Bollini et al. 1970		Rosen 1971	Will & Nordtvedt 1972	Ni 1972	Hellings & Nordtvedt 1972			
Ni 1973	Yilmaz 1973	Lightman & Lee 1973	Lee, Lightman & Ni 1974	Rosen 1975				
Belinfante & Swihart 1975		Lee et al. 1976	Bekenstein 1977	Barker 1978	Rastall 1979			
Coleman 1983	Hehl 1997	Overlooked (20 <sup>th</sup> century)						

**Theory must be:**

- Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
- Some theories are just variations of others
- Some theories were proposed in the 1910s/20s; many theories in the 1960s/70s
- Overlooked: this is not a complete list!
- **Complete:** not a law, but a theory. Derive experimental results from first principles
- **Self-consistent:** get same results no matter which mathematics or models are used
- **Relativistic:** Non-gravitational laws are those of Special Relativity
- **Newtonian:** Reduces to Newton's equation in the limit of low gravity and low velocities



*Theories that fail already*

Newton 1686	Poincaré 1890							
Einstein 1912	Nordstrøm 1912		Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915			
Whitehead 1922	Cartan 1923		Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943			
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Coleman 1983	Hehl 1997	Overlooked (20 <sup>th</sup> century)						

- Newton (1686) - non-relativistic: implicit action at a distance - incompatible with special relativity
- Poincare (1890) and conformally flat theory of Whithrow-Morduch (1965) - incomplete: do not mesh with non-gravitational physics (Maxwell)
- Fierz & Pauli (1939) [ "spin-2 field theory" ] - inconsistent: field equations -> all gravitating bodies move along straight lines, equation of motion -> gravity deflects bodies
- Birkhoff (1943) - not Newtonian: demands *speed of sound = speed of light*.
- Milne (1948) – incomplete - no gravitational red-shift prediction
- Kustaanheimo-Nuotio (1967) – inconsistent – grav. redshift for photons, but not for light waves.



Newton 1686 Poincaré 1890

*Theories that violate  
the Einstein's Equivalence Principle*

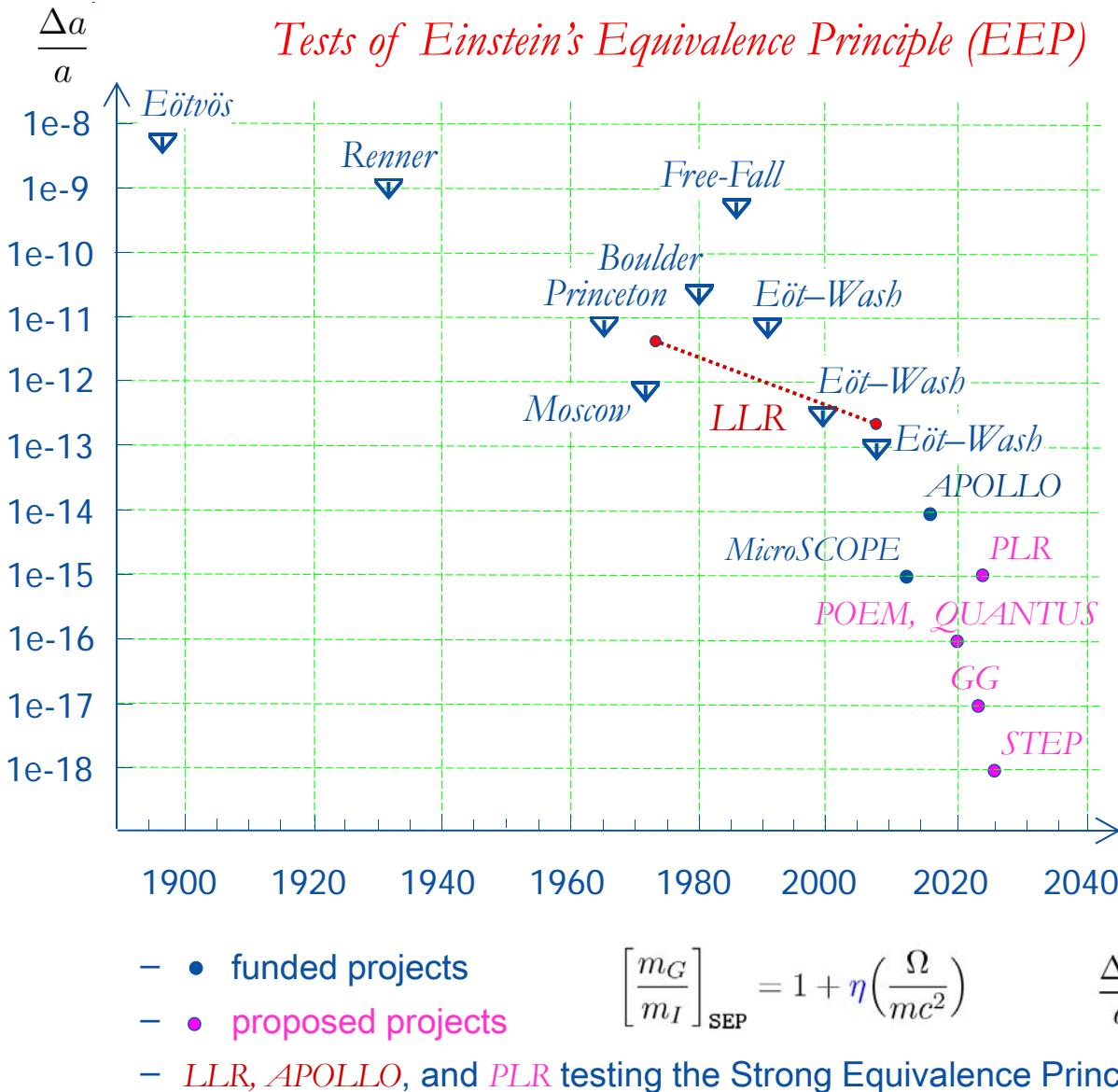
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### Einstein's Equivalence Principle (EEP):

- Uniqueness of Free Fall
- Local Lorentz Invariance
- Local Position Invariance

Only metric theories are viable:

- Belinfante & Swihart (1975): not a metric theory
- Kaluza-Klein (1932): violates EEP
- Still too many theories around...



### Uniqueness of Free Fall

( $\equiv$  Weak Equivalence Principle):

$$\vec{F} = m_I \vec{a} = m_G \vec{g}$$

$$\Rightarrow m_I = m_G$$

All bodies fall with the same acceleration

Define the test parameter that signifies a violation of the **WEP**

$$\frac{\Delta a}{a} = \frac{(a_1 - a_2)}{\frac{1}{2}(a_1 + a_2)} = \left[ \frac{m_G}{m_I} \right]_1 - \left[ \frac{m_G}{m_I} \right]_2$$

Let  $\Omega$  is the gravitational binding energy of a test body, then the test parameter that signifies a violation of the **SEP** is

$$\left[ \frac{m_G}{m_I} \right]_{\text{SEP}} = 1 + \eta \left( \frac{\Omega}{mc^2} \right)$$

$$\frac{\Delta a}{a} = (4\beta - \gamma - 3) \left\{ \left[ \frac{\Omega}{mc^2} \right]_1 - \left[ \frac{\Omega}{mc^2} \right]_2 \right\}$$



*Theories that violate  
Local Lorentz Invariance (LLI)*

Newton 1686 Poincaré 1890

Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
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Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
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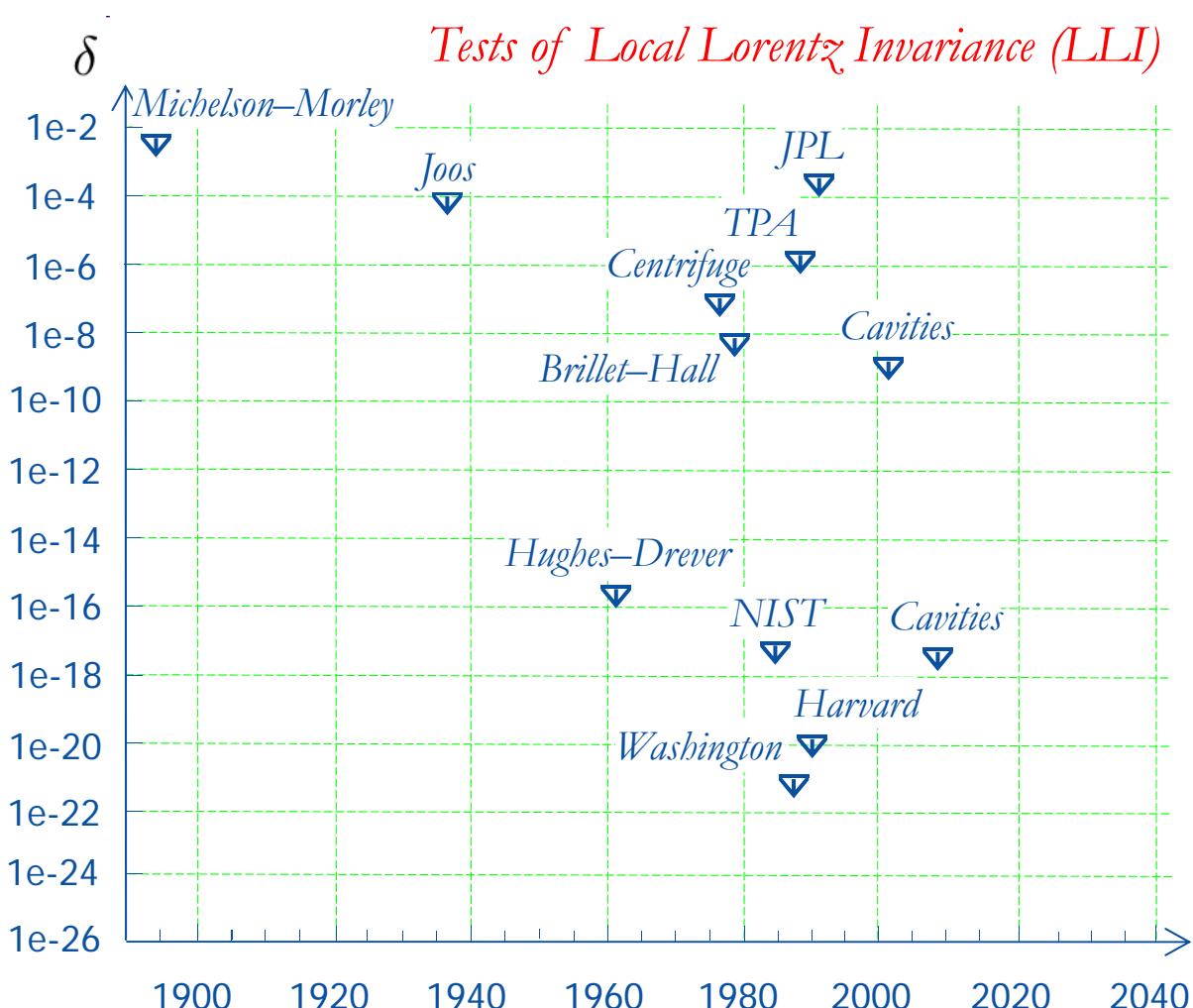
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Coleman 1983	Hehl 1997	Overlooked (20 <sup>th</sup> century)
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Quasi-linear theories:

- Deser & Laurent (1968), Bollini, Giambiagi & Tiomno (1970) both predict existence of a preferred reference frame (i.e.,  $\xi=1$ )
- Whitehead (1922) predicts time-dependence for ocean tides in violation of everyday experience



- Michelson-Morley, Joos, Brillet-Hall: Round-trip propagation
- Centrifuge, TPA, JPL: One-way propagation
- Rest: Hughes-Drever experiments

$$\delta \equiv \frac{c}{c_0} - 1$$

### Local Lorentz Invariance:

- The outcome of a (small-scale) experiment does not depend on the orientation and the velocity of the (inertial) laboratory.
- Frameworks by Kostelecky et al., Jacobson et al.

### Future experiments:

- Clock comparisons
- Clocks vs microwave cavities
- Time of flight of high energy photons
- Birefringence in vacuum
- Neutrino oscillations
- Threshold effects in particle physics

### Test of one-way speed of light:

- Important to fundamental physics, cosmology, astronomy and astrophysics



## Laboratory tests of Lorentz Invariance: search for preferred-frame effects

frame1 :  $S(T, X)$  e.g. CMB  $v_{sol} \approx 377 \text{ km/s}$

frame2 :  $s(t, x)$  laboratory  $RA, dec = (11.2, -6.4^\circ)$

Mansouri & Sexl, 1977

$$dT = \frac{1}{a}(dt + \frac{v}{c^2}dx) \quad a = 1 + \alpha \frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dX = \frac{1}{b}dx + \frac{v}{a}(dt + \frac{v}{c^2}dx) \quad b = 1 + \beta \frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

$$dY = \frac{1}{d}dy, \quad dZ = \frac{1}{d}dz \quad d = 1 + \delta \frac{v^2}{c^2} + \mathcal{O}(c^{-4})$$

Special Theory of Relativity:  $\alpha = -1/2, \beta = 1/2, \delta = 0$



### Clock comparison experiments:

$$P_{MM} = (\frac{1}{2} - \beta + \delta) \quad \text{Michelson-Morley: orientation dependence}$$

$$P_{KT} = (\beta - \alpha - 1) \quad \text{Kennedy-Thorndike: velocity dependence}$$

$$P_{IS} = |\alpha + \frac{1}{2}| \quad \text{Ives-Stillwell: contraction, dilation}$$

### Precision tests of Lorentz Invariance:

$$P_{MM} = -1.6(\pm 6.1) \times 10^{-12} \quad \text{Eisele et al, PRL 103 (2009) 090401}$$

$$P_{KT} = 3.1(\pm 6.9) \times 10^{-7} \quad \text{Wolf et al, PRL 90 (2003) 060402}$$

$$P_{IS} < 2.2 \times 10^{-7} \quad \text{Saathoff et al, PRL 91 (2003) 190403}$$

### Tests of isotropy of the speed of light:

$$\Delta c_\theta / c \lesssim 1 \times 10^{-17} \quad \text{Herrmann et al, PRD 80 (2009) 105011}$$



*Theories that violate  
Local Position Invariance (LPI)*

Newton 1686 Poincaré 1890

Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
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Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
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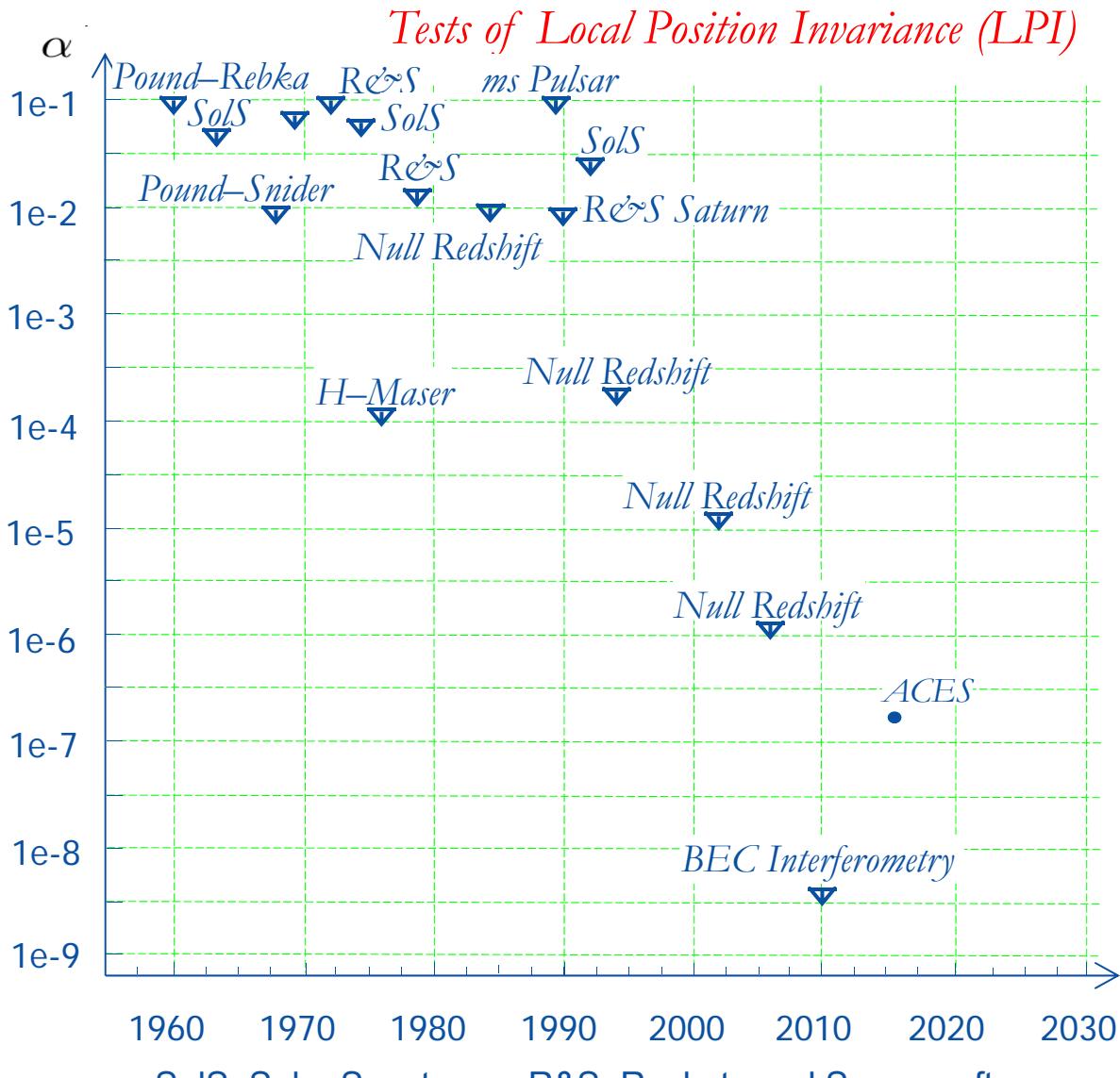
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Coleman 1983	Hehl 1997	Overlooked (20 <sup>th</sup> century)
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Stratified theories with time-orthogonal time slices all predict  $\xi \neq 0$ :

- Einstein (1912), Papapetrou (1954) (actually two theories)
- Yilmaz (1962), Whitrow & Morduch (1965)
- Page & Tupper (1968), Rosen (1971)
- Ni (1972), Coleman (1983)



*Gravitational redshift:*

$$\frac{\Delta\nu}{\nu} = (1 + \alpha) \frac{\Delta U}{c^2}$$

### Local Position Invariance:

- The outcome of any local non-gravitational experiment is independent of where & when in the universe it is performed

Splits into:

- spatial invariance
- temporal invariance
- The best clock result is by Ashby et al., Phys. Rev. Lett. 98, 070802 (2007)

$$|\alpha| < 1.4 \times 10^{-6}$$

- Current best result is by Müller, Peters, and Chu, Nature 463, 926 (2010):

$$|\alpha| < 7 \times 10^{-9}$$



$$\mathcal{S}_G[g_{mn}] = \frac{c^4}{16\pi G_N} \int d^4x \sqrt{-g} R \quad \text{Action of general relativity}$$

$R = g^{mn}R_{mn}$

$R_{mn} = \partial_k \Gamma_{mn}^k - \partial_m \Gamma_{nk}^k + \Gamma_{mn}^k \Gamma_{kl}^l - \Gamma_{ml}^k \Gamma_{nk}^l$

$\Gamma_{mn}^k = \frac{1}{2} g^{kp} (\partial_m g_{pn} + \partial_n g_{pm} - \partial_p g_{mn})$

Ricci scalar, Ricci tensor & Christoffel symbols

$$\mathcal{S}_{SM}[\psi, A_m, H; g_{mn}] = \int d^4x \left[ -\frac{1}{4} \sum \sqrt{-g} g^{mk} g^{nl} F_{mn}^a F_{kl}^a - \sum \sqrt{-g} \bar{\psi} \gamma^m D_m \psi \right.$$

$- \frac{1}{2} \sqrt{-g} g^{mn} \overline{D_m H} D_n H - \sqrt{-g} V(H) - \sum \lambda \sqrt{-g} \bar{\psi} H \psi - \sqrt{-g} \rho_{vac} \right]$

Action of Standard Model

Variational principle:

$$\frac{\delta}{\delta g_{mn}} \otimes [\mathcal{S}_{tot}[\psi, A_m, H; g_{mn}] = \mathcal{S}_G[g_{mn}] + \mathcal{S}_{SM}[\psi, A_m, H; g_{mn}]] \Rightarrow$$

$$R_{mn} - \frac{1}{2} g_{mn} R + \Lambda g_{mn} = \frac{8\pi G_N}{c^4} T_{mn} \quad \Lambda = 8\pi G_N \rho_{vac}/c^4 \quad \rho_{vac} \approx (2.3 \times 10^{-3} eV)^4$$

$$S = \frac{c^3}{4\pi G} \int d^4x \sqrt{-g} \left[ \frac{1}{4} f(\varphi) R - \frac{1}{2} g(\varphi) \partial_\mu \varphi \partial^\mu \varphi + V(\varphi) \right] + \sum_i q_i(\varphi) \mathcal{L}_i$$

$$f(\varphi) = \varphi, \quad g(\varphi) = \frac{\omega}{\varphi}, \quad V(\varphi) = 0.$$

Brans and Dicke (1961)

Scalar-Tensor  
theories of gravity

## Parameterized Post-Newtonian (PPN) formalism

PPN Formalism: Eddington, Fock, Chandrasekhar, Dicke, Nordtvedt, Thorne, Will,...

$$\begin{aligned}
 g_{00} &= 1 - \frac{2}{c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} + \frac{2\beta}{c^4} \left[ \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \right]^2 - \frac{1+2\gamma}{c^4} \sum_{j \neq i} \frac{\mu_j \dot{r}_j^2}{r_{ij}} + \\
 &\quad + \frac{2(2\beta-1)}{c^4} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \sum_{k \neq j} \frac{\mu_k}{r_{jk}} - \frac{1}{c^4} \sum_{j \neq i} \mu_j \frac{\partial^2 r_{ij}}{\partial t^2} + \mathcal{O}(c^{-5}) \\
 g_{0\alpha} &= \frac{2(1+\gamma)}{c^3} \sum_{j \neq i} \frac{\mu_j \dot{r}_j^\alpha}{r_{ij}} + \mathcal{O}(c^{-5}) \\
 g_{\alpha\beta} &= -\delta_{\alpha\beta} \left( 1 + \frac{2\gamma}{c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} + \frac{3\delta}{2c^4} \left[ \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \right]^2 \right) + \mathcal{O}(c^{-5})
 \end{aligned}$$

- Assumption: Local Lorentz Invariance (LLI) and local position invariance (LPI) hold, thus, preferred frame parameters  $\alpha_1, \alpha_2, \alpha_3$  are not included...
- General case, there are 10 PPN parameters:  $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- $\gamma$  are the Eddington's parameterized post-Newtonian (PPN) parameters:

General relativity:  $\gamma = \beta = 1$

Brans-Dicke theory:  $\gamma = \frac{1+\omega}{2+\omega}$ ,  $\beta = 1$

- $\delta$  is the post-PPN parameter – important for next generation of light propagation tests.

## PPN Equations of Motion (a part of the model)

$$\begin{aligned}
 \ddot{\mathbf{r}}_i = & \sum_{j \neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3} \left\{ \left[ \frac{m_G}{m_I} \right]_i - \frac{2(\beta + \gamma)}{c^2} \sum_{l \neq i} \frac{Gm_l}{r_{il}} - \frac{2\beta - 1}{c^2} \sum_{k \neq j} \frac{Gm_k}{r_{jk}} + \right. \\
 & + \gamma \left( \frac{\dot{r}_i}{c} \right)^2 + (1 + \gamma) \left( \frac{\dot{r}_j}{c} \right)^2 - \frac{2(1 + \gamma)}{c^2} \dot{\mathbf{r}}_i \dot{\mathbf{r}}_j + \frac{\dot{G} \cdot t}{G} - \\
 & - \frac{3}{2c^2} \left[ \frac{(\mathbf{r}_i - \mathbf{r}_j)\dot{\mathbf{r}}_j}{r_{ij}} \right]^2 + \frac{1}{2c^2} (\mathbf{r}_j - \mathbf{r}_i) \ddot{\mathbf{r}}_j \Big\} + \\
 & + \frac{1}{c^2} \sum_{j \neq i} \frac{Gm_j}{r_{ij}^3} \left\{ [\mathbf{r}_i - \mathbf{r}_j] \cdot [(2 + 2\gamma)\dot{\mathbf{r}}_i - (1 + 2\gamma)\dot{\mathbf{r}}_j] \right\} (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j) + \\
 & + \frac{3 + 4\gamma}{2c^2} \sum_{j \neq i} \frac{Gm_j \ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m=1}^3 \frac{Gm_m(\mathbf{r}_m - \mathbf{r}_i)}{r_{im}^3} + \sum_{c,s,m} \mathbf{F}_{\text{asteroids}}
 \end{aligned}$$

Possible EP violation

Possible temporal dependence of  $G$

$\left[ \frac{m_G}{m_I} \right]_{\text{SEP}} = 1 + \eta \left( \frac{\Omega}{mc^2} \right)$

$\eta = 4\beta - \gamma - 3$

$\Omega_i = -\frac{G}{2} \int_i d^3x \rho_i U_i = -\frac{G}{2} \int_i d^3x d^3x' \frac{\rho_i(\mathbf{r}) \rho_i(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$

- In general theory of relativity  $\gamma = \beta = 1$ , thus  $\eta = 0$  (this is not the case for scalar-tensor theories of gravity, for instance, where these parameters can have different values).

$$t_2 - t_1 = \frac{r_{12}}{c} + (1 + \gamma) \sum_i \frac{\mu_i}{c^3} \ln \left[ \frac{r_1^i + r_2^i + r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}}{r_1^i + r_2^i - r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}} \right] + \mathcal{O}(c^{-5})$$



Cassini (2003):  $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$

*Theories that predict  $\gamma = 0$  or  $-1$  fail*

Newton 1686 Poincaré 1890

Einstein 1912	Nordstrøm 1912	Nordstrøm 1913	Einstein & Fokker 1914	Einstein 1915
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Whitehead 1922	Cartan 1923	Kaluza & Klein 1932	Fierz & Pauli 1939	Birkhoff 1943
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Coleman 1983	Hehl 1997	Overlooked (20 <sup>th</sup> century)
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### Parameterized Post-Newtonian Formalism (PPN):

- Solar system is the main arena to test weak gravity:
- Expand the metrics; identify various potentials
- They have 10 PPN parameters in front  
 $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- Calculate those parameters & Compare with experiments

[2011: A need for Cosmological “PPN formalism”?]

Conformally-flat theories fail test of time delay and deflection of light:

- Nordstrom (1912)
- Nordstrom (1913)
- Einstein & Fokker (1914)
- Littlewood & Bergmann (1956)
- Ni (1972)

## List of PPN Parameters for Competing Theories

<i>Competing theories of Gravity</i>	$\gamma$	$\beta$	$\xi$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\zeta_1$	$\zeta_2$	$\zeta_3$	$\zeta_4$
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
<i>Scalar Field theories</i>	– Note: in Page-Tupper (1968): parameter $d$ is defined as $\Delta = 1 - \gamma$									
Einstein (1912) [not GR]	0	0	–	-4	0	-2	0	-1	0	0*
Whitrow-Morduch (1965)	0	-1	–	-4	0	0	0	-3	0	0*
Rosen (1971)	$\lambda$	$\frac{3}{4} + \frac{\lambda}{4}$	–	$-4(1-\lambda)$	0	-4	0	-1	0	0
Papapetrou (1954a, 1954b)	1	1	–	-8	-4	0	0	2	0	0
Ni (1972) (stratified)	1	1	–	-8	0	0	0	2	0	0
Yilmaz (1958, 1962)	1	1	–	-8	0	-4	0	-2	0	-1*
Page-Tupper (1968)	$\gamma$	$\beta$	–	$-4\Delta$	0	$-2\Delta$	0	$\zeta_2$	0	$\zeta_4$
Nordström (1912, 1913)	-1	$\frac{1}{2}$	–	0	0	0	0	0	0	0*
Einstein-Fokker (1914)	-1	$\frac{1}{2}$	–	0	0	0	0	0	0	0
Ni (1972) (flat)	-1	$1-q$	–	0	0	0	0	$\zeta_2$	0	0*
Whitrow-Morduch (1960)	-1	$1-q$	–	0	0	0	0	$q$	0	0*
Littlewood (1953), Bergman (1956)	-1	$\frac{1}{2}$	–	0	0	0	0	-1	0	0*

– Note: \* The theory is incomplete, and  $\zeta_4$  can take one of two values. The value closest to zero is listed.



## List of PPN Parameters for Competing Theories

<i>Competing theories of Gravity</i>	$\gamma$	$\beta$	$\xi$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\zeta_1$	$\zeta_2$	$\zeta_3$	$\zeta_4$
Einstein (1915) GR	1	1	0	0	0	0	0	0	0	0
<i>Scalar-Tensor theories</i>										
Bergmann (1968), Wagoner (1970)	$\frac{1+\omega}{2+\omega}$	$\beta$	0	0	0	0	0	0	0	0
Nordtvedt (1970), Bekenstein (1977)	$\frac{1+\omega}{2+\omega}$	$\beta$	0	0	0	0	0	0	0	0
Brans-Dicke (1961)	$\frac{1+\omega}{2+\omega}$	1	0	0	0	0	0	0	0	0
<i>Vector-Tensor theories</i>										
Hellings-Nordtvedt (1973)	$\gamma$	$\beta$	0	$\alpha_1$	$\alpha_2$	0	0	0	0	0
Will-Nordtvedt (1972)	1	1	0	0	$\alpha_2$	0	0	0	0	0
<i>Bimetric theories</i> – Note: in Rosen (1975): parameter $k_2$ is defined as $k_2 = (c_0/c_1) - 1$										
Rosen (1975)	1	1	0	0	$k_2$	0	0	0	0	0
Rastall (1979)	1	1	0	0	$\alpha_2$	0	0	0	0	0
Lightman-Lee (1973)	$\gamma$	$\beta$	0	$\alpha_1$	$\alpha_2$	0	0	0	0	0
<i>Stratified theories</i>										
Lee-Lightman-Ni (1974)	$ac_0/c_1$	$\beta$	$\xi$	$\alpha_1$	$\alpha_2$	0	0	0	0	0
Ni (1973)	$ac_0/c_1$	$bc_0$	0	$\alpha_1$	$\alpha_2$	0	0	0	0	0



### *Unlikely Scalar-Tensor Theories*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 Einstein 1915

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 Will & Nordtvedt 1972 Ni 1972 Hellings & Nordtvedt 1972

Ni 1973 Yilmaz 1973 Lightman & Lee 1973 Lee, Lightman & Ni 1974 Rosen 1975

Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20<sup>th</sup> century)

Scalar-Tensor theories are extremely constrained by Viking (1976) and Cassini (2003) results on  $\gamma$ :

- Thiry (1948), Jordan 1955
- Brans & Dicke (1961):  $\omega > 6500$  (Viking, 1976),  $\omega > 40,000$  (Cassini, 2003)
- Bergmann (1968), Nordtvedt (1970)
- Wagoner (1970), Bekenstein (1977)
- Barker (1978)



### *Mercury's Perihelion: Theories that fail*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**

**Ni 1973** **Yilmaz 1973** **Lightman & Lee 1973** **Lee, Lightman & Ni 1974** **Rosen 1975**

Belinfante & Swihart 1975 **Lee et al. 1976** Bekenstein 1977 Barker 1978 **Rastall 1979**

Coleman 1983 **Hehl 1997** **Overlooked (20<sup>th</sup> century)**

Stratified theories predict preferred frame effects on perihelion shift:

- Ni (1973)
- Lee, Lightman & Ni (1974)

$$\dot{\pi} = (2 + 2\gamma - \beta) \frac{GM_{\odot}n_M}{c^2a_M(1 - e_M^2)} + \frac{3}{4} \left(\frac{R_{\odot}}{a_M}\right)^2 \frac{J_{2\odot}n_M}{(1 - e_M^2)^2} (3 \cos^2 i_M - 1), \quad "/cy$$

$$\dot{\pi} = 42''.98 \left[ \frac{1}{3}(2 + 2\gamma - \beta) + 0.296 \cdot J_{2\odot} \times 10^4 \right], \quad "/cy$$

$J_{2\odot} \simeq 2 \times 10^{-7}$  from helioseismology; confirmed by Konopliv et al., 2010



## *GW & Binary Pulsar: Theories that fail*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 **Cartan 1923** Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 **Will & Nordtvedt 1972** Ni 1972 **Hellings & Nordtvedt 1972**

Ni 1973 **Yilmaz 1973** **Lightman & Lee 1973** Lee, Lightman & Ni 1974 **Rosen 1975**

Belinfante & Swihart 1975 **Lee et al. 1976** Bekenstein 1977 Barker 1978 **Rastall 1979**

Coleman 1983 **Hehl 1997** **Overlooked (20<sup>th</sup> century)**

Bi-metric Theories predict a dipole radiation. Can't be...:

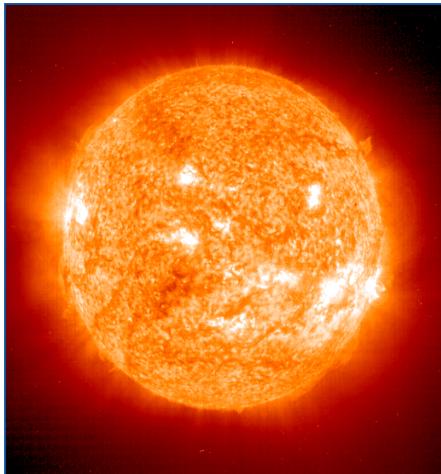
- Rosen (1975)
- Lee et al. (1976)
- Rastall (1979)
- Lightman & Lee (1973)

## The Current Values of the PPN Parameters



Parameter	What is measured relative to General Relativity?	Current value	Effects	Experiments
$\gamma-1$	Measure of space curvature produced by unit mass	$2.3 \times 10^{-5}$	Time delay, light deflection	Cassini tracking
$\beta-1$	Measure of non-linearity in gravitational superposition	$1.1 \times 10^{-4}$	Nordtvedt effect, perihelion shift	Lunar laser ranging
$\xi$	Measure of existence of preferred location effects	$1 \times 10^{-3}$	Earth tides	Gravimeter data
$\alpha_1$	Measure the existence of preferred frame effects	$1 \times 10^{-4}$	Orbit polarization	Lunar laser ranging
$\alpha_2$		$4 \times 10^{-7}$	Spin precession	Sun axis' alignment w/ ecliptic
$\alpha_3$		$4 \times 10^{-20}$	Self-acceleration	Pulsar spin-down statistics
$\zeta_1$	Measure (plus $\alpha_3$ ) of the failure of conservation laws of energy, momentum and angular momentum	$2 \times 10^{-2}$	–	Combined PPN bounds
$\zeta_2$		$4 \times 10^{-5}$	Binary pulsar acceleration	Pulsar: PSR 1913+16
$\zeta_3$		$1 \times 10^{-8}$	Newton's 3rd law	Lunar acceleration
$\zeta_4$		$6 \times 10^{-3}$	–	Kreuzer experiment

# Laboratory for Relativistic Gravity Experiments: Our Solar System

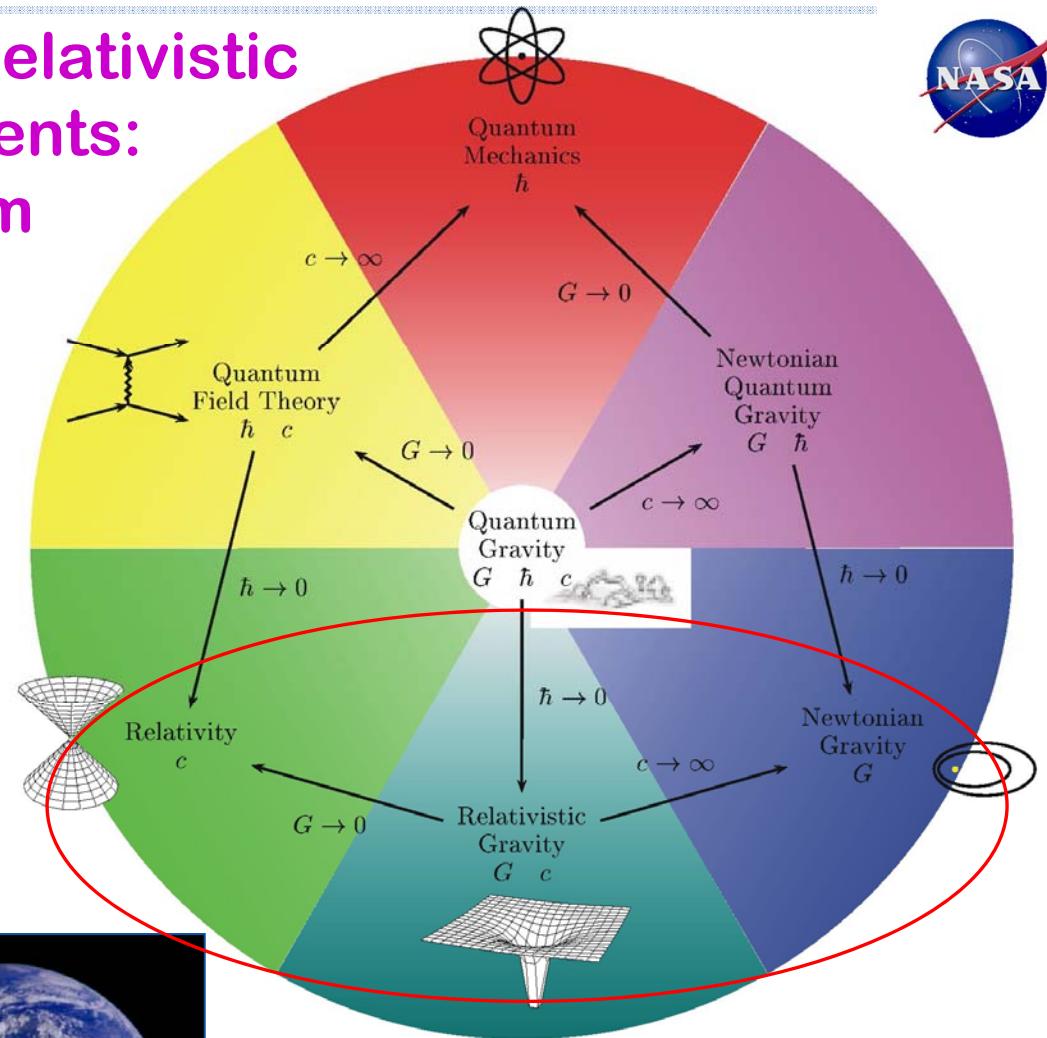
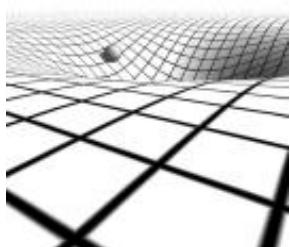


Strongest gravity potential

$$\frac{GM_{Sun}}{c^2 R_{Sun}} \sim 10^{-6}$$



$$\frac{GM_{\oplus}}{c^2 R_{\oplus}} \sim 10^{-9}$$



Most accessible region for gravity tests in space:

- ISS, LLR, SLR, free-fliers

Technology is available to conduct tests in the immediate solar proximity

# 40 Years of Solar System Gravity Tests



## Techniques for Gravity Tests:

### Radar Ranging:

- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

### Laser:

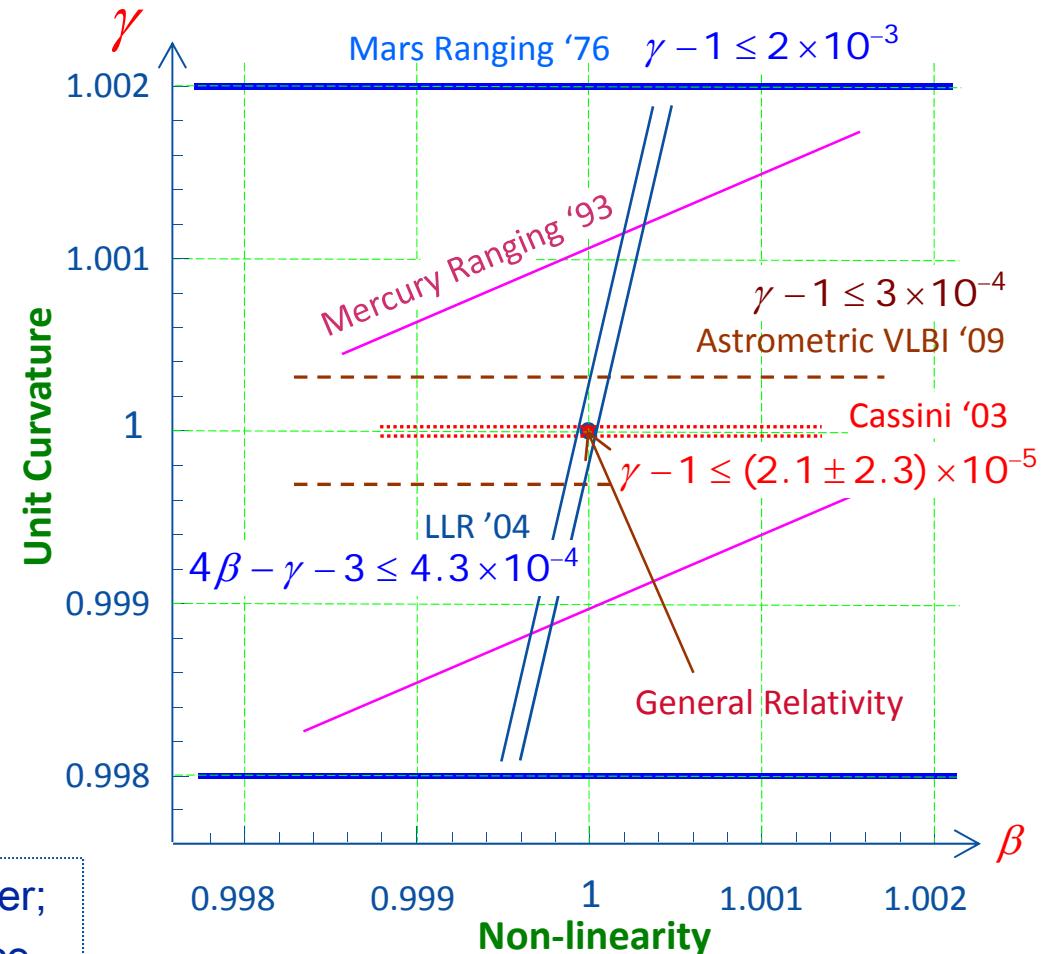
- SLR, LLR, interplanetary, etc.

## Dedicated Gravity Missions:

- **LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76,'92; GP-B, '04; LARES, '12; LISA, 2025+(?)

## New Engineering Discipline – Applied General Relativity:

- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation & astrometry (SIM, Gaia,....).



A factor of 100 in 40 years is impressive, but is not enough for the near future!



*Some Theories resist to fail*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 Einstein 1915

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 Will & Nordtvedt 1972 Ni 1972 Hellings & Nordtvedt 1972

Ni 1973 Yilmaz 1973 Lightman & Lee 1973 Lee, Lightman & Ni 1974 Rosen 1975

Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20<sup>th</sup> century)

- Will & Nordtvedt (1972) and Hellings & Nordtvedt (1972) are vector-tensor theories. Deviations can only be significant in high energy regime (e.g. Planck-scale energy)
- Yilmaz (1973) was mathematically inconsistent, but now is fixed. Does not predict black holes
- Cartan (1923), Hehl (1997) introduces matter spin



## *“Aesthetics-Based” Conclusion for 20<sup>th</sup> Century*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

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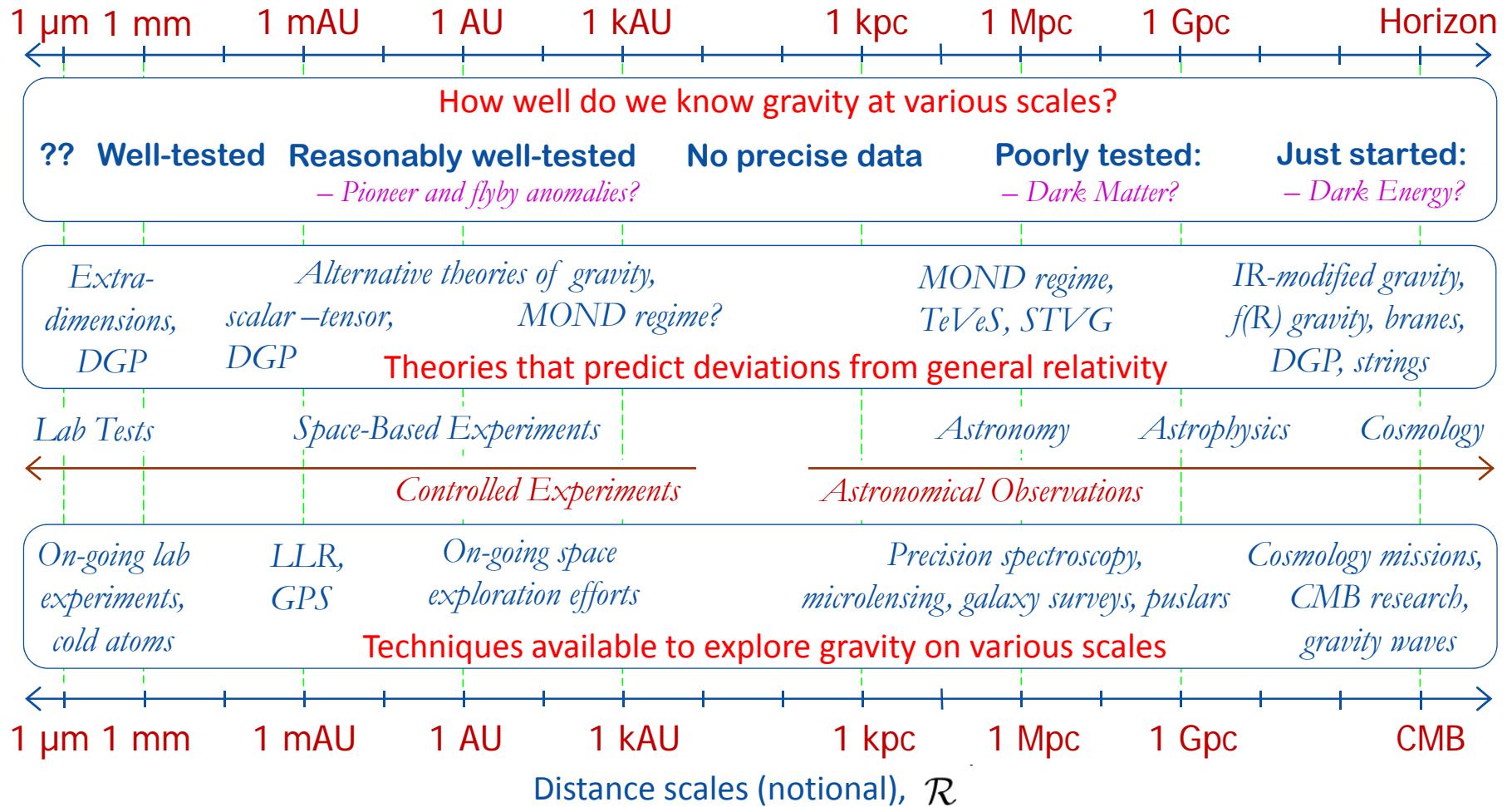
Bollini et al. 1970 Rosen 1971 Will & Nordtvedt 1972 Ni 1972 Hellings & Nordtvedt 1972

Ni 1973 Yilmaz 1973 Lightman & Lee 1973 Lee, Lightman & Ni 1974 Rosen 1975

Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20<sup>th</sup> century)

- “Among all bodies of physical law none has ever been found that is simpler and more beautiful than Einstein's geometric theory of gravity”
  - Misner, Thorne and Wheeler, 1973
- “[...] Unfortunately, any finite number of effects can be fitted by a sufficiently complicated theory. [...] Aesthetic or philosophical motives will therefore continue to play a part in the widespread faith in Einstein's theory, even if all tests verify its predictions.”
  - Malcolm MacCallum, 1976



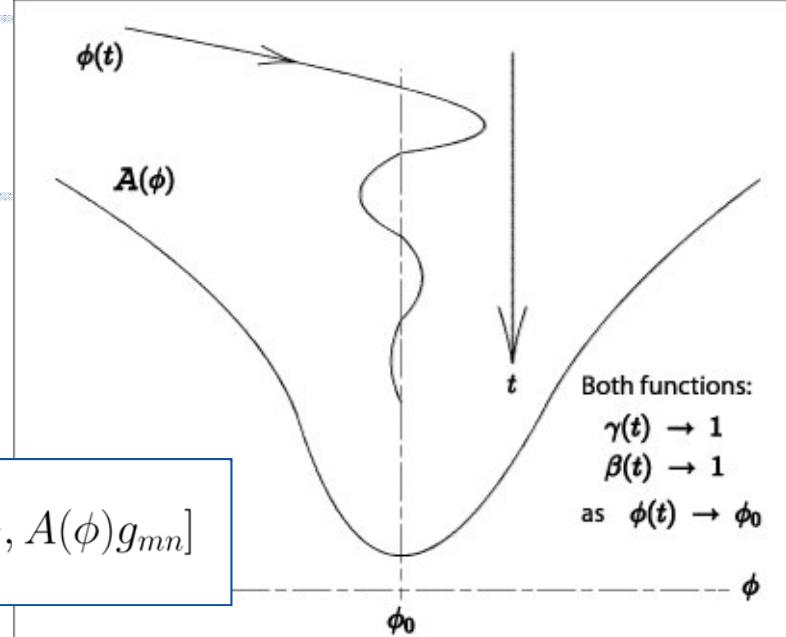
Solar system experiments allow for improvements in our knowledge of gravity

# Theoretical Motivation for New Gravity Tests

## Long-range massless [or low-mass] scalar:

The low-energy limit of the String Theory in ‘Einstein Frame’ (Damour-Nordtvedt-Polyakov 1993) suggests:

$$S = -\frac{1}{16\pi G} \int dx^4 \sqrt{-g} \left( R - 2g^{mn} \nabla_m \phi \nabla_n \phi \right) + S_M[\psi_M, A(\phi) g_{mn}]$$



Expansion  $A(\phi)$  around background value  $\phi_0$  of the scalar leads:

$$\ln A(\varphi) = \ln A(\varphi_0) + \alpha_0(\varphi - \varphi_0) + \frac{1}{2}k_0(\varphi - \varphi_0)^2 + \mathcal{O}(\Delta\varphi^3)$$

Slope  $\alpha_0$  measures the coupling strength of interaction between matter and the scalar.

$$\gamma - 1 = \frac{-2\alpha_0^2}{1 + \alpha_0^2} \simeq -2\alpha_0^2$$

$$\beta - 1 = \frac{1}{2} \frac{\alpha_0^2 k_0}{(1 + \alpha_0^2)^2} \simeq \frac{1}{2} \alpha_0^2 k_0 \simeq \frac{1}{4}(1 - \gamma)k_0$$

Scenario for cosmological evolution of the scalar (Damour, Piazza & Veneziano 2002):

$$\gamma - 1 \sim 7.3 \times 10^{-7} \left( \frac{H_0}{\Omega_0^3} \right)^{\frac{1}{2}}$$

$\Rightarrow$

$$\gamma - 1 \sim 10^{-5} - 10^{-7}$$

The unit curvature PPN parameter  $\gamma$  is the most important quantity to test



## Modifications of Einstein Gravity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_m[g_{\mu\nu}, \psi]$$

Carroll et al, PRD 70 (2004) 043528

...

## Modification of PPN Gravity

$$\gamma - 1 = - \frac{f''(R)^2}{f'(R) + 2f''(R)^2},$$

$$\beta - 1 = \frac{1}{4} \frac{f'(R) \cdot f''(R)}{2f'(R) + 3f''(R)^2} \frac{d\gamma}{dR}.$$

Analogy between scalar-tensor and higher-order gravity

Constraints on ...  $f(R)$  from solar system experiments...

...tight restrictions on the form of the gravitational Lagrangian

Need for cosmological “PPN formalism”

Capozziello, Stabile, Trosi, gr-qc/0603071

# Modified Gravity: f(R) theories

- A broad class of alternative theories

$$\mathcal{L}_\phi = -\frac{M^2}{2}\omega(\phi)(\partial\phi)^2 - V(\phi)$$

$$S = \int d^n x \sqrt{-g} \left[ \frac{1}{2}f(R, \phi) + \mathcal{L}_\phi(g_{\mu\nu}, \phi, \partial\phi) + \mathcal{L}_m(g_{\mu\nu}, \Psi) \right]$$

$$(\partial\phi)^2 = \nabla_\mu\phi\nabla^\mu\phi$$

$$F(R, \phi) = \partial f(R, \phi)/\partial R$$

Generalized gravity	$\frac{1}{2}f(R, \phi)$	$\mathcal{L}_\phi(\phi, \partial\phi)$	$p(R, \phi)$	$\varphi$	$\hat{V}(\varphi)$
Nonlinear gravity	$\frac{1}{2}f(R)$	$\omega = 0, V = 0$	$p = F(R)$	$\sqrt{\frac{3}{2}} \ln F$	$\frac{FR-f}{2F^2}$
$R^2$ -gravity	$\frac{1}{2}(R + \alpha R^2)$	$\omega = 0, V = 0$	$p = 1 + 2\alpha R$	$\sqrt{\frac{3}{2}} \ln F$	$\frac{FR-f}{2F^2}$
$1/R$ -gravity	$\frac{1}{2}(R - \mu^4/R)$	$\omega = 0, V = 0$	$p = 1 + \mu^4/R^2$	$\sqrt{\frac{3}{2}} \ln F$	$\frac{FR-f}{2F^2}$
Scalar-tensor theory	$\frac{1}{2}F(\phi)R$	$\omega(\phi), V(\phi)$	$p = F(\phi)$	$\int \sqrt{\frac{\omega}{F} + \frac{3}{2} \frac{F'^2}{F^2}} d\phi$	$\frac{V}{F^2}$
Brans-Dicke theory	$\phi R$	$\omega(\phi) = 2\frac{\omega}{\phi}, V = 0$	$p = \phi$	$\int \sqrt{\frac{\omega}{F} + \frac{3}{2} \frac{F'^2}{F^2}} d\phi$	0
Dilaton	$\frac{1}{2}e^{-\phi}R$	$\omega(\phi) = e^{-\phi}, V = 0$	$p = e^{-\phi}$	$\frac{5}{2}\phi$	0
NMC scalar	$\frac{1}{2}(1 + \xi\phi^2)R$	$\omega = 1, V(\phi)$	$p = 1 + \xi\phi^2$	$\int \frac{\sqrt{1 + \xi(6\xi - 1)\phi^2}}{1 - \xi\phi^2} d\phi$	$\frac{V}{1 - \xi\phi^2}$
CC ( $\xi = \frac{1}{6}$ )	$\frac{1}{2}(1 + \frac{1}{6}\phi^2)R$	$\omega = 1, V(\phi)$	$p = 1 + \frac{1}{6}\phi^2$	$\sqrt{6} \tanh^{-1} \frac{\phi}{\sqrt{6}}$	$\frac{V}{1 - \frac{1}{6}\phi^2}$
Induced Gravity	$\frac{1}{2}\epsilon\phi^2R$	$\omega = 1, V(\phi)$	$p = \epsilon\phi^2$	$\sqrt{6 + \frac{1}{\epsilon}} \ln \phi$	$\frac{V}{\epsilon\phi^2}$
GR with a scalar	$\frac{1}{2}R$	$\omega = 1, V(\phi)$	$p = 1$	$\phi$	$V$

$\psi$  – are matter fields;  $\phi$  – is a scalar field

too many references...



*First decade of 21'st century... they are back!*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 Einstein 1915

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

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Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20<sup>th</sup> century) Scalar-Tensor Theories

Arkani-Hamed, Dimopoulos & Dvali 2000 Dvali, Gabadadze & Poratti 2003 Strings theory?

Bekenstein 2004 Moffat 2005 Multiple f(R) models 2003-10 Bi-Metric Theories

### Need for new theory of gravity:

- Classical GR description breaks down in regimes with large curvature
- If gravity is to be quantized, GR will have to be modified or extended

### Other challenges:

- Dark Matter
- Dark Energy
- Neutrino velocity

### Motivations for new tests of GR:

- GR is a fundamental theory
- Alternative theories & models
- New ideas & techniques require comprehensive investigations

## Cassini 2002 Conjunction Experiment



### Results of Cassini Conjunction Experiment:

- Spacecraft—Earth separation > 1 billion km
- Doppler/Range: X~7.14GHz & Ka~34.1GHz
- Result:  $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$

### Gravitational deflection of light (Einstein, 1915):

$$\theta_{\text{gr}}^{\odot}(b) = \frac{2(1+\gamma)GM_{\odot}}{bc^2} \simeq 8 \times 10^{-6} \left( \frac{1+\gamma}{2} \right) \left( \frac{R_{\odot}}{b} \right)$$

### Gravitational delay of light signals (Shapiro, 1976):

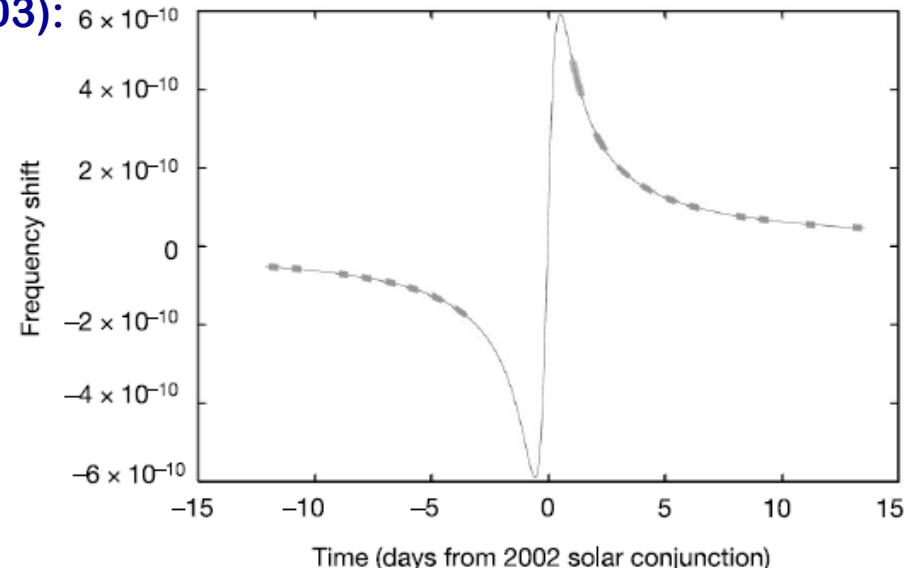
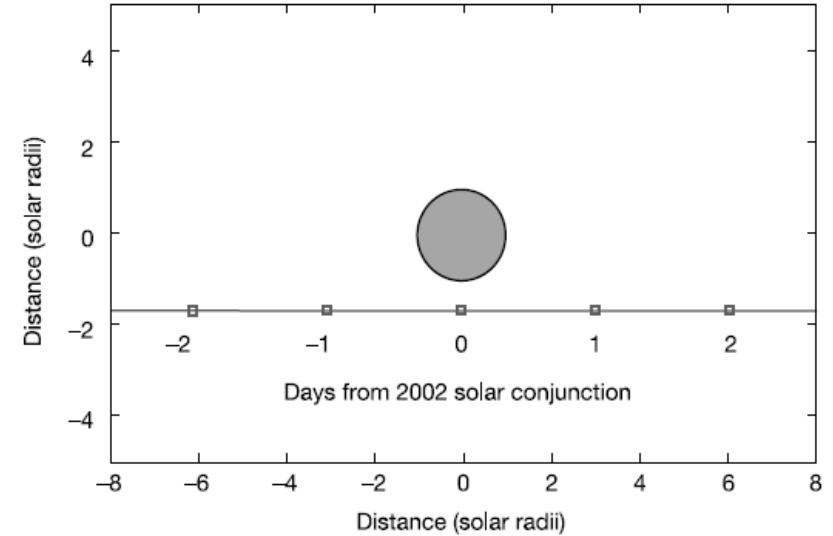
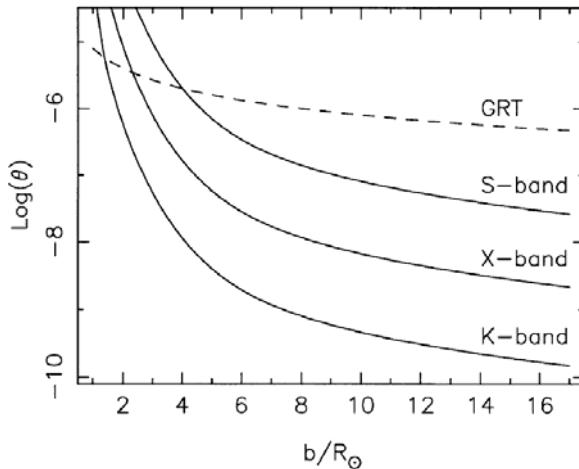
$$\Delta t = (1+\gamma) \frac{GM_{\odot}}{c^3} \ln \left[ \frac{r_1 + r_2 + r_{12}}{r_1 + r_2 - r_{12}} \right] \simeq (1+\gamma) \frac{GM_{\odot}}{c^3} \ln \left[ \frac{2r_1 r_2}{b^2} \right]$$

### Gravitational frequency drift (Bertotti et al, 2003):

$$y_{\text{gr}}(b) \simeq -\frac{v}{c} \theta_{\text{gr}}^{\odot}(b) = \frac{2(1+\gamma)GM_{\odot}}{bc^2} \frac{v}{c} \simeq 8 \times 10^{-10} \left( \frac{1+\gamma}{2} \right) \left( \frac{R_{\odot}}{b} \right)$$

$$y = \frac{\Delta\nu}{\nu}$$

Modern-day DSN  
frequency stability  
 $y \sim 10^{-15}$

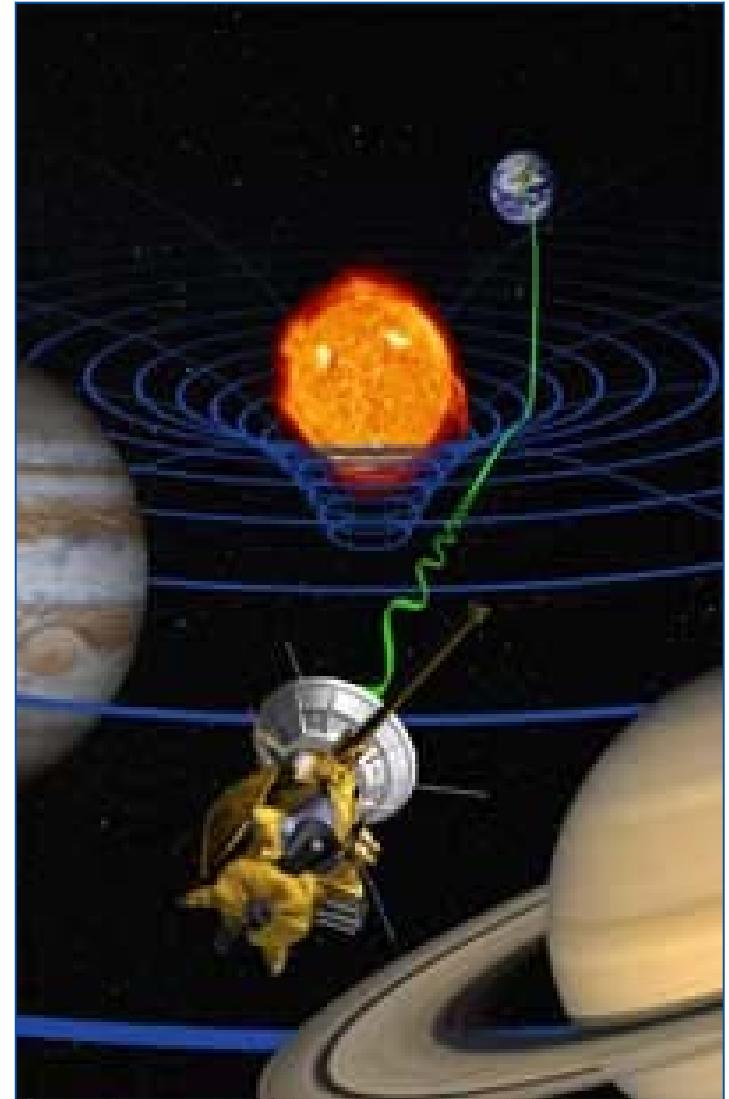


## Cassini Conjunction Experiment:

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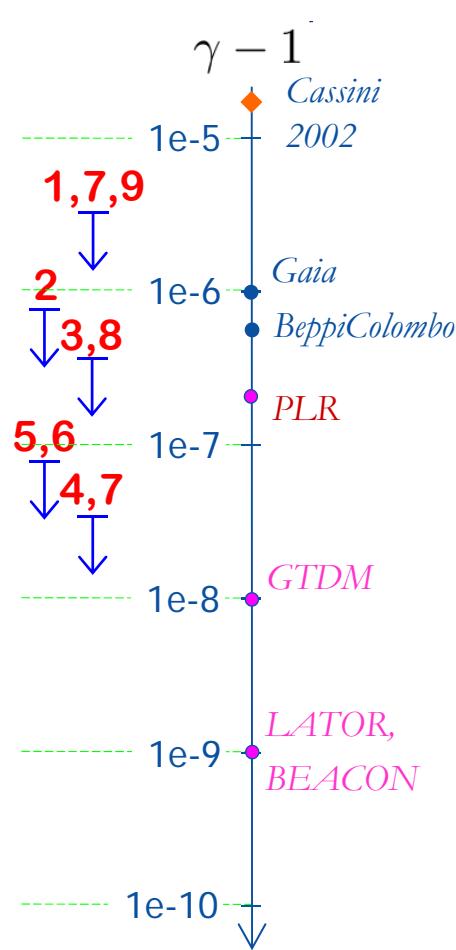
## Possible with Existing Technologies?!

- VLBI [current  $\gamma = 3 \times 10^{-4}$ ]: limited to  $\sim 1 \times 10^{-4}$ :
  - uncertainty in the radio source coordinates
- LLR [current  $\eta = 4 \times 10^{-4}$ ]: in 5 years  $\sim 3 \times 10^{-5}$ :
  - mm accuracies [APOLLO] & modeling efforts
- $\mu$ -wave ranging to a lander on Mars  $\sim 6 \times 10^{-6}$
- tracking of BepiColombo s/c at Mercury  $\sim 2 \times 10^{-6}$
- Optical astrometry [current  $\gamma = 3 \times 10^{-3}$ ]:  
Gaia mission  $\sim 1 \times 10^{-6}$  (2016/18?)



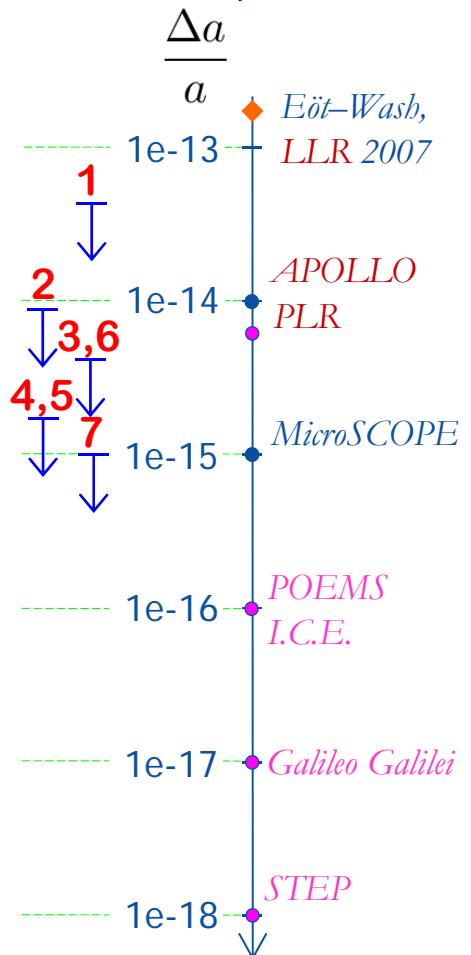
One needs a dedicated mission to explore accuracies better than  $10^{-6}$  for both PPN parameters  $\gamma$  (and  $\beta$ ). Interplanetary laser ranging is a possibility.

# Theoretical Landscape of the 21th Century: Confrontation Between Theory and Experiment

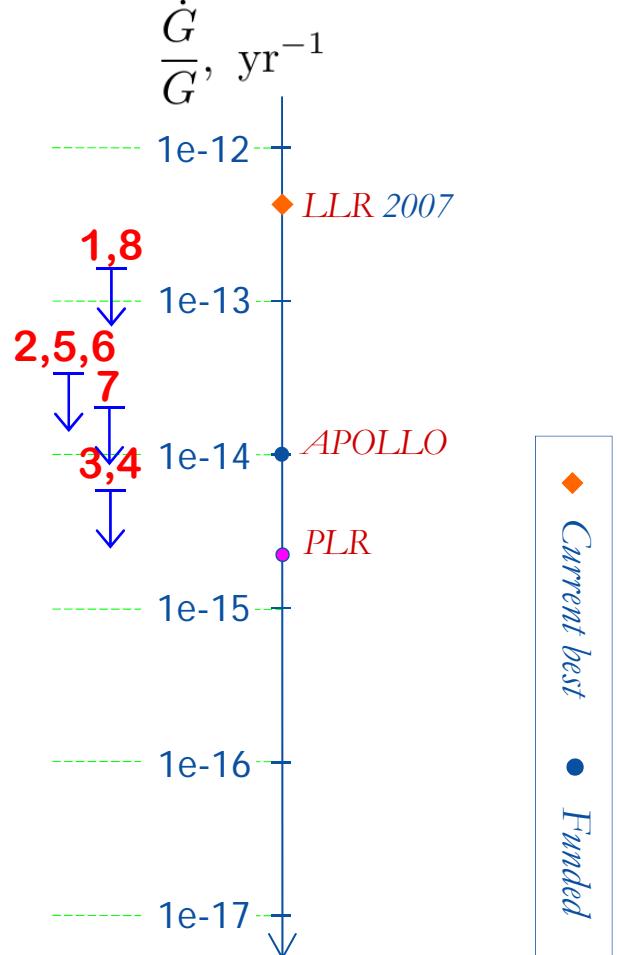


## New Theories & Future Tests

- 1 Damour-Polyakov-Nordtvedt 1993
- 2 Damour-Esposito-Farese 1996
- 3 Damour-Piazza-Veneziano 2002



- 4 Arkani-Dimopoulos-Dvali 2000
- 5 Dvali-Gabadadze-Poratti 2003
- 6 F(R) gravity models 2003-07



- 7 Bekenstein 2004
- 8 Moffat 2005
- 9 Jaekel-Reynaud 2006

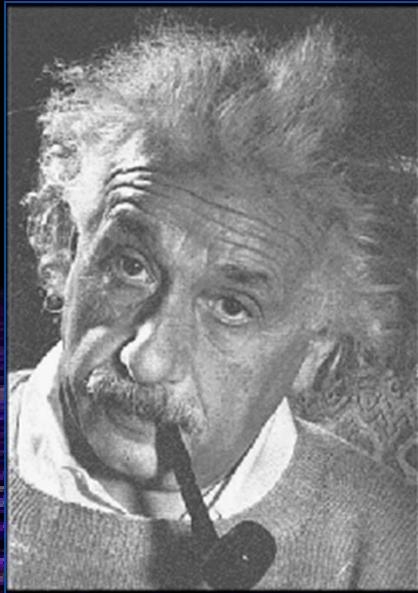


- Frequency Standards:
  - $\mu$ -wave atomic clocks:
    - NIST F-1 Cs fountain  $\mu$ -wave clock:  $5 \times 10^{-16}$  accuracy (1 day)
  - $\mu$ -wave atomic clocks on a chip:
    - NIST Rb chip-scale  $\mu$ -wave clock:  $6 \times 10^{-12}$  accuracy (1 sec)
  - Optical atomic clocks
    - NIST Hg+ clock now  $< 3 \times 10^{-17}$  ( $10^4$  sec)
- Matter-wave interferometers:
  - Gravity gradiometer:
    - Demonstrated differential acceleration sensitivity:  $3 \times 10^{-9} \text{ g/Hz}^{1/2}$
  - Gyroscope
    - Achieved stability  $2 \times 10^{-6} \text{ deg/hr}^{1/2}$  ARW (angle random walk)
  - Accelerometer
    - Demonstrated performance  $2.3 \times 10^{-9} \text{ g/Hz}^{1/2}$

## Conclusions



- Recent technological progress: [arXiv:0902.3004 \[gr-qc\]](https://arxiv.org/abs/0902.3004)
  - Resulted in new instruments with unique performance
  - Could lead to major improvements in the tests of relativistic gravity
  - Already led to a number of recently proposed gravitational experiments
- Challenges for solar system tests of gravity:
  - Dedicated space-based experiments are very expensive – the science must worth the cost... – *EP, G-dot and PPN  $\gamma$  tests are most relevant.*
  - Motivation for the tests in a weak gravity field is a challenge: there is no strong expectation to see deviations from GR in the solar system (we are looking for anomalies...) – *access to strong(er) gravity regime is needed!*
  - GR is very hard to modify, embed, extend or augment (whatever your favorite verb is...) – *thus, perhaps, those anomalies are important...*
  - PPN formalism becomes less relevant for modern gravity research...
  - Looking to Cosmos for help? There is none: Little or no correspondence between cosmological tests and physical principles in the foundation of tests of PPN gravity – *EP, LLI, LPI, energy-momentum conservation, etc...*



Thank You!

