A multi-scale approach to InSAR time series analysis

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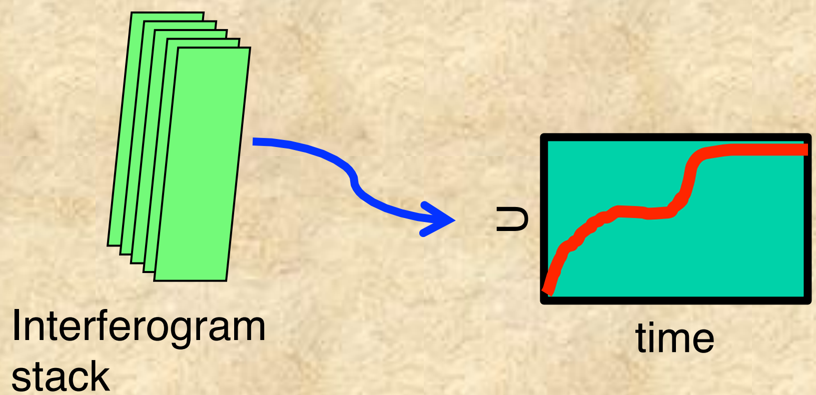

A geophysical perspective on deformation tomography

Examples: Long Valley Caldera, California
Northern Volcanic Zone, Iceland

Seismological Laboratory
California Institute of Technology

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Motivation

Assume that in the future (Sentinel, DESDynI) we will have:

- Frequent repeats (short ΔT)
- Good orbits with small baselines
- Ubiquitous high coherence

Challenge for the future:

- How to deal with $O(10^3)$ interferograms
- How to use C_d - Invert all pixels simultaneously?

  1000 igrams x 1000 x 1000 pixels = 1 billion data
- Computational tractability

Approach (MInTS = Multi-scale InSAR Time Series):

1. Time domain: A generalized physical parameterization (GPS-like)
2. Space domain: Wavelets – use all data simultaneously

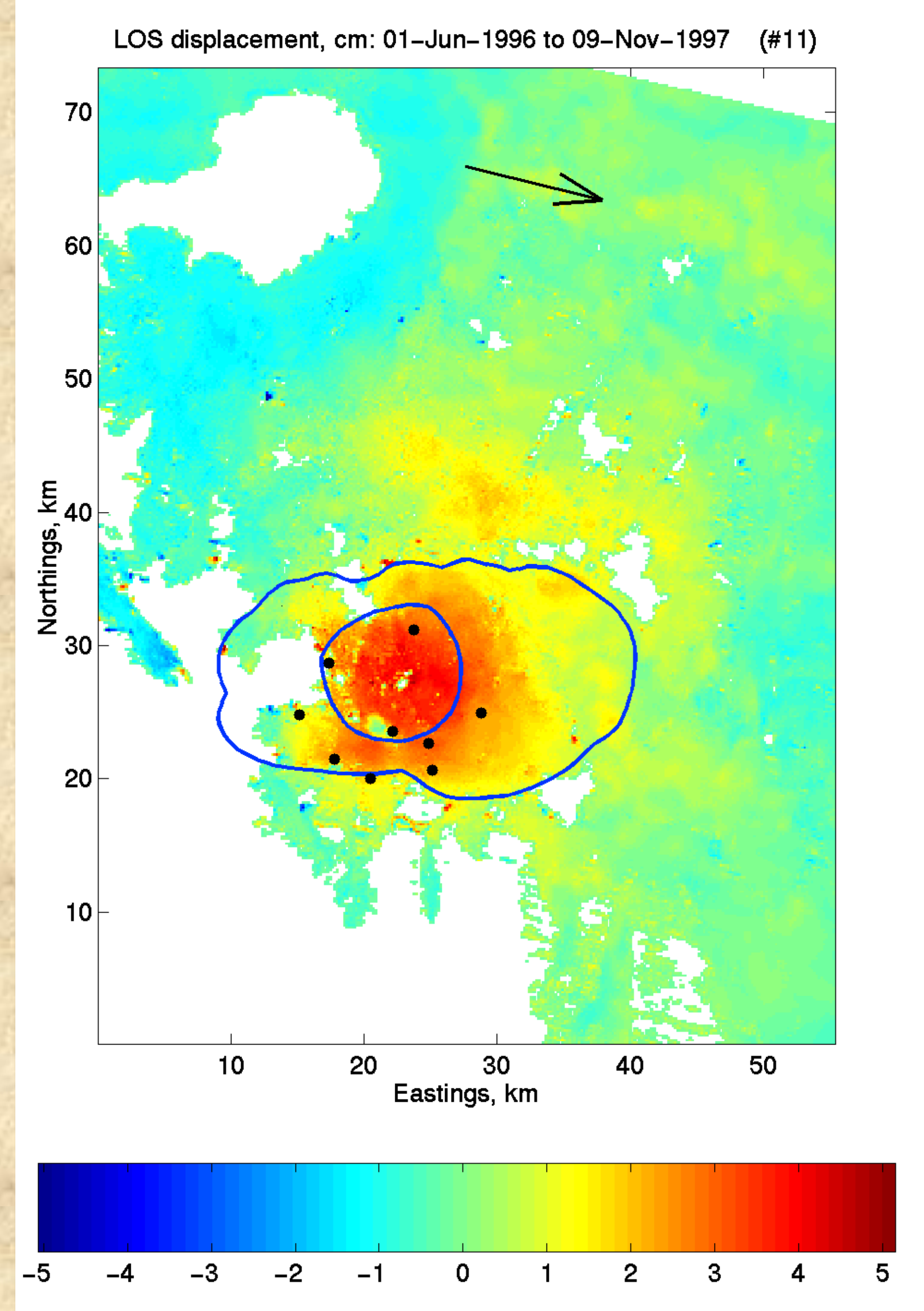
Note: Currently only applied to unwrapped data (we want this to change)

How to deal with holes from decorrelation & unwrapping in individual scenes?

We want to avoid the union of all holes in final time series.

Interpolate in space?
Interpolate in time?

Our approach: Space & time simultaneously (tomography-like)



MInTS Recipe

1. Interpolate unwrapping holes in each interferogram where needed (temporary)

2. Wavelet decomposition of each interferogram

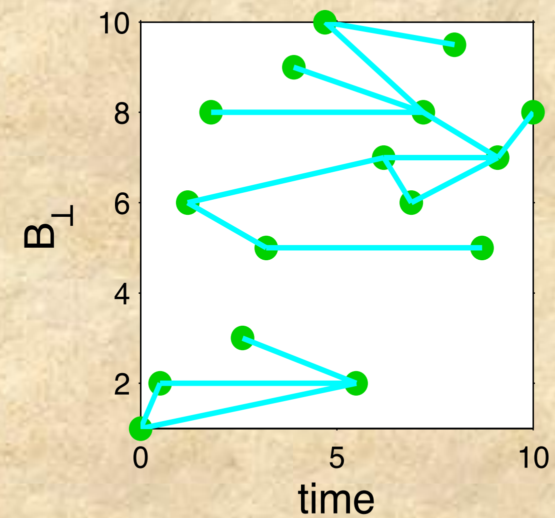
   For later weighting purposes, track relative extent to which each wavelet coefficient is associated with actual data versus interpolated data

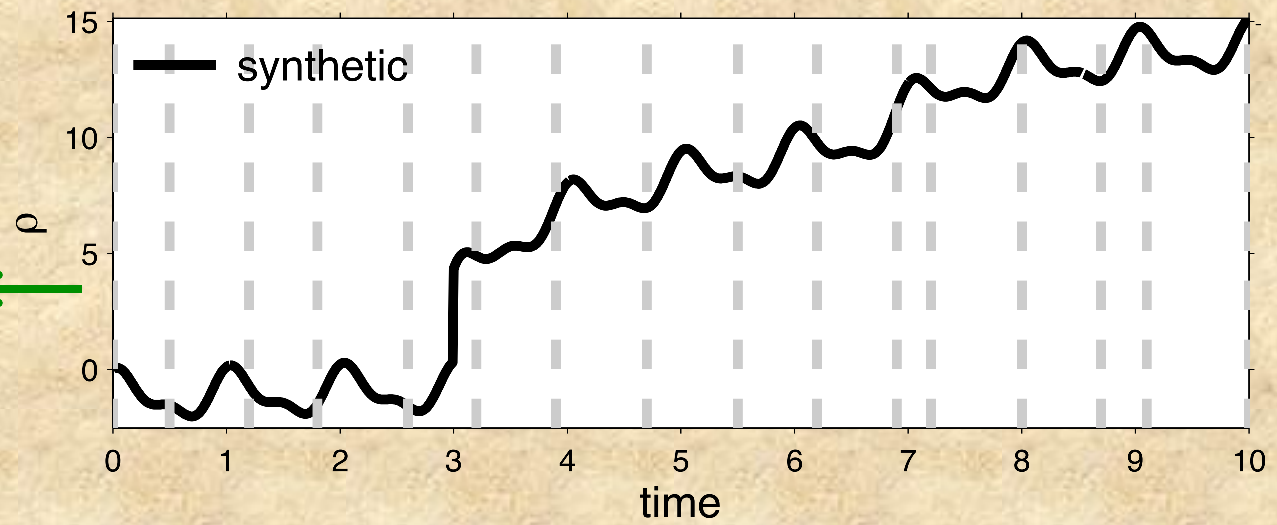
3. Time series analysis on wavelet coefficients

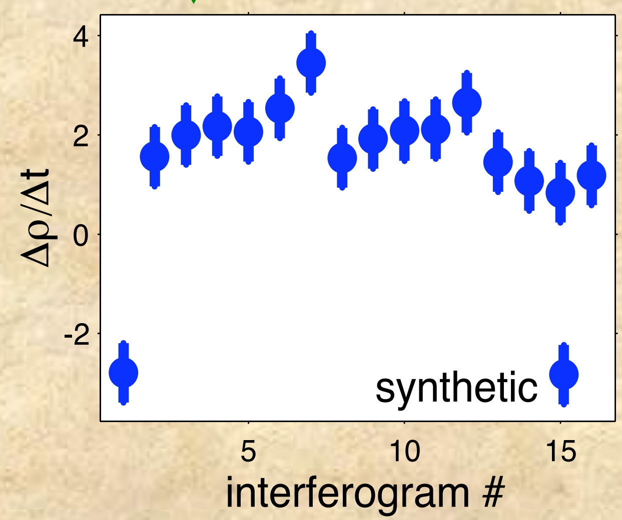
   Physical parameterization + splines for unknown signals - all constrained by weighted wavelet coefficients of observed interferograms

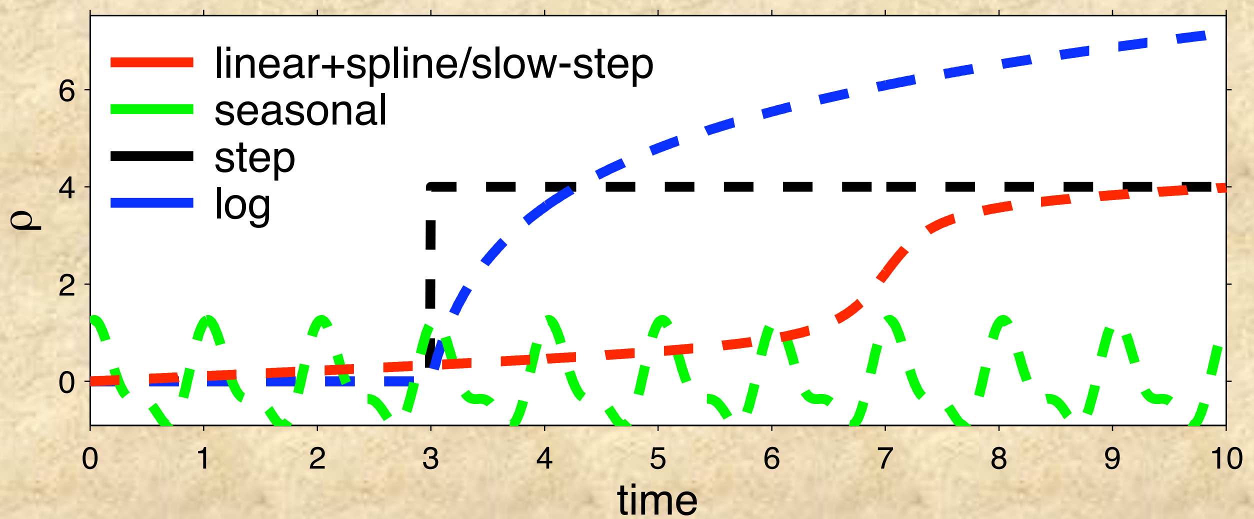
4. Recombine to get total deformation history

Begin with a time domain synthetic for a "single pixel"









InSAR time series: SBAS-pure (2 examples)

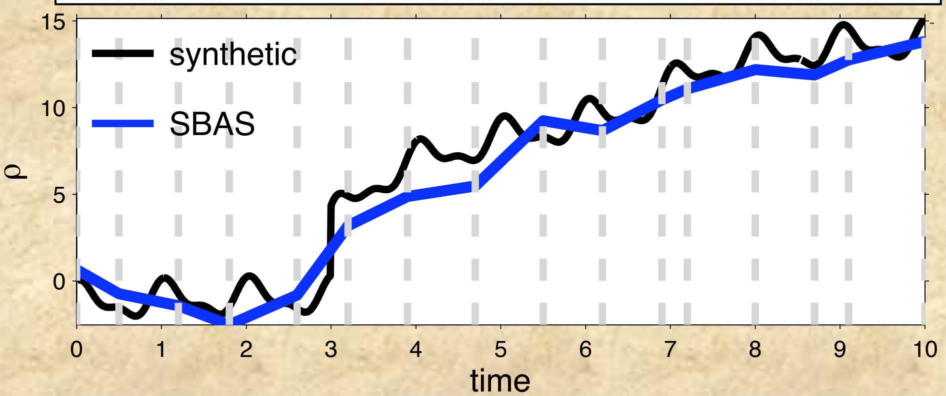
Goal: Continuous displacement record

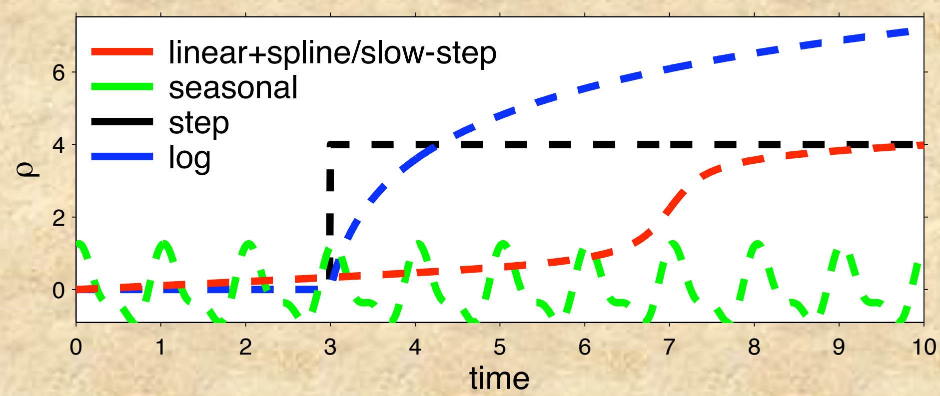
Assumption: Constant V_i between image acquisitions dates

Approach: Minimize V_i

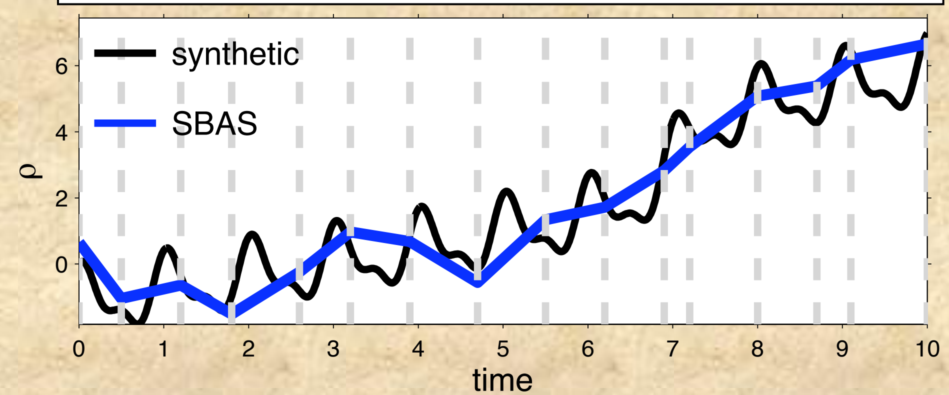
Method: SVD

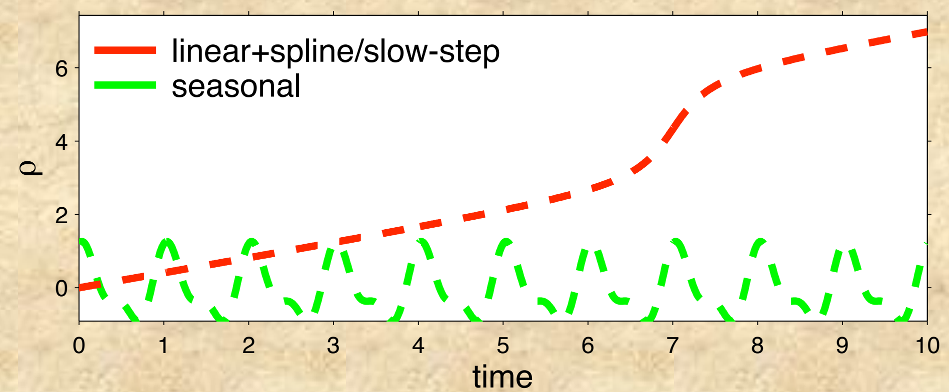
Secular + Seasonal + Transient + Noise + EQ + relaxation





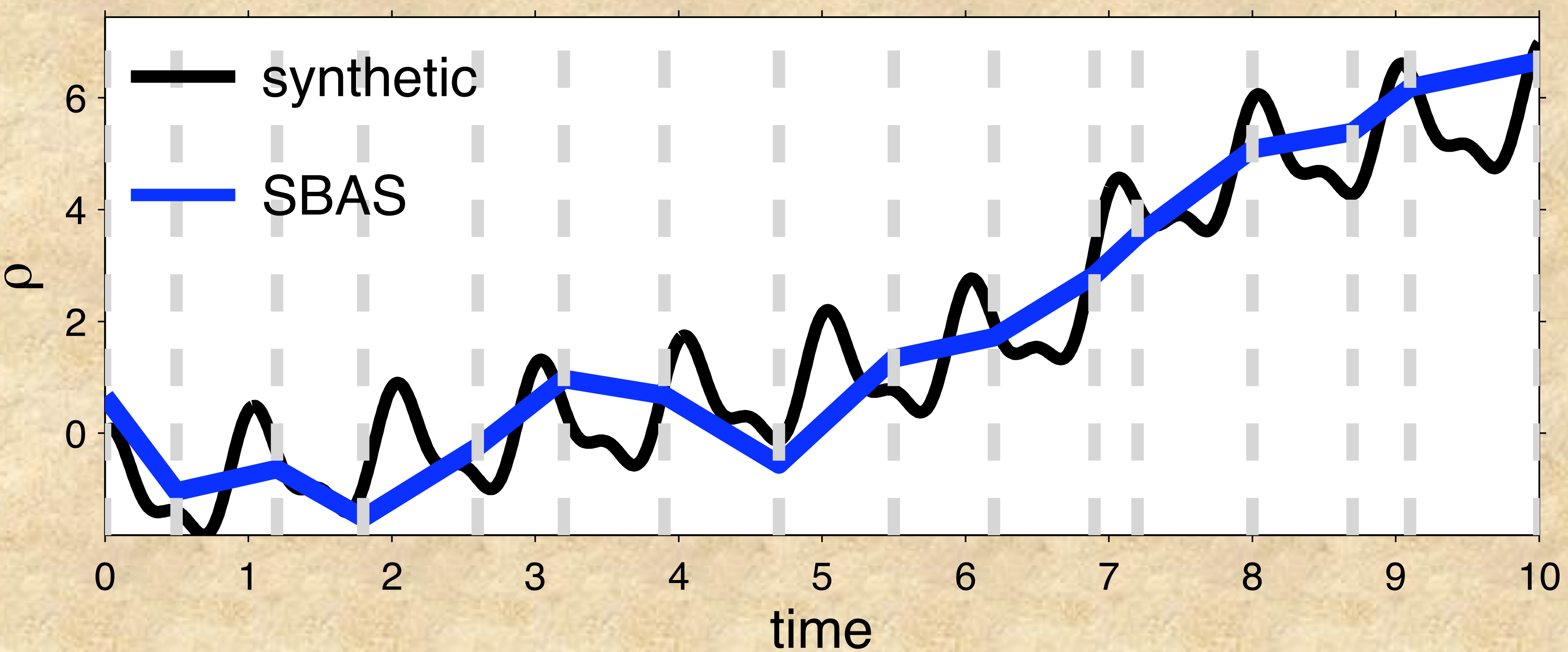
Secular + Seasonal + Transient + Noise





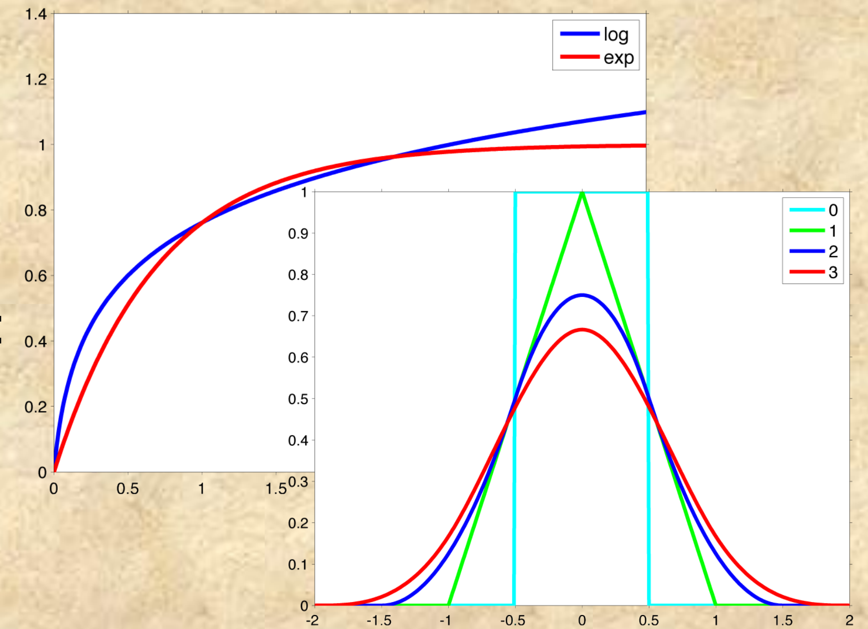
Issues

- Temporal parameterization controlled by dates of image
- No explicit separation of processes contributing to the phase
- Inherently pixel-by-pixel approach (computational restriction)
- Ignores *apriori* information and data/model covariances



MInTS

Generalized temporal description (geophysical intuition)



Observation equation for LOS deformation:

$$\rho(\Omega, t) = \rho_o(\Omega) + v_\rho(\Omega) \cdot t + F_\rho(\Omega, t)$$

$$F_j(\Omega, t) = \sum_{i \in \mathcal{S}_\Delta} \Delta_{ij}(\Omega)\, \mathcal{H}(t - T_i) +$$

offsets (e.g., eqs., "fast" events)

$$\sum_{i \in \mathcal{L}} \alpha^L_{ij}(\Omega)\, \mathcal{H}(t - T^L_i)\, \ln\left(1 + \frac{t}{\tau^L_i}\right) +$$

$$\sum_{i \in \mathcal{S}_E} \alpha^E_{ij}(\Omega)\, \mathcal{H}(t - T^E_i)\left[1 - \exp\left(\frac{-t}{\tau^E_i}\right)\right] +$$

postseismic-like processes (i.e., one sided)

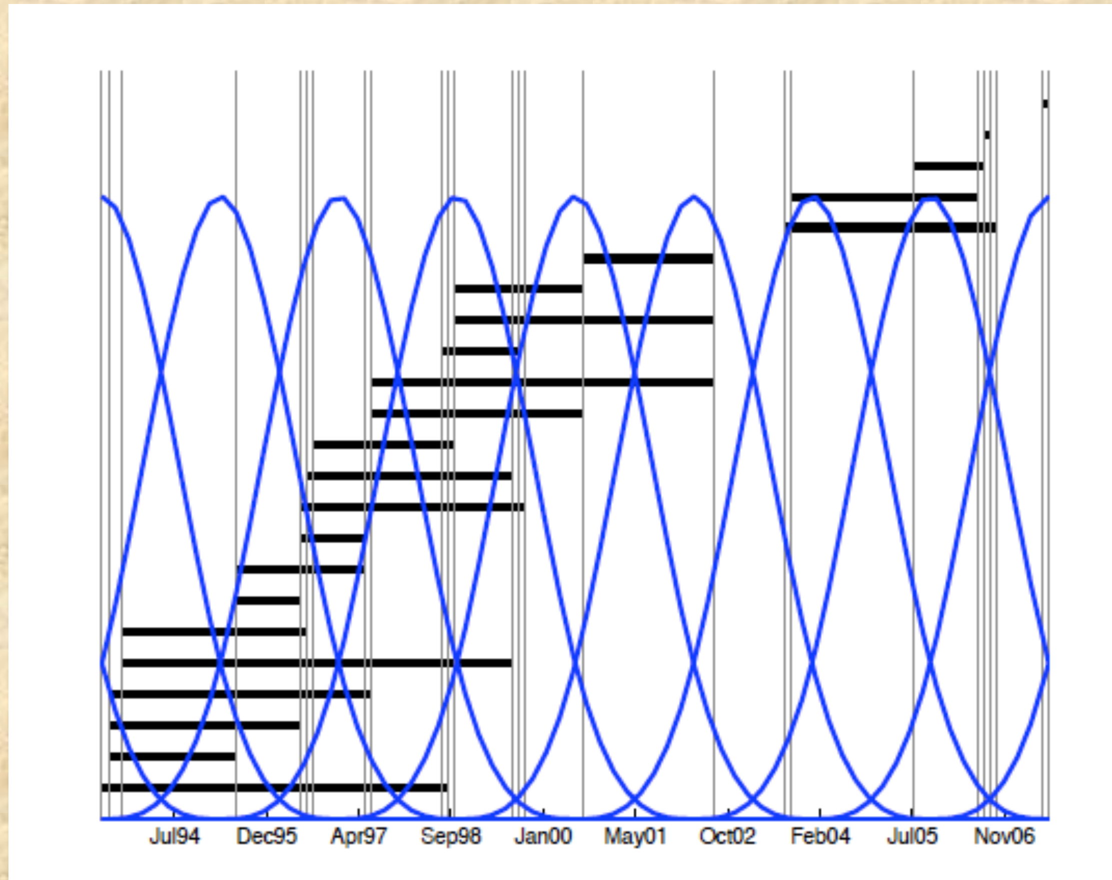
$$\sum_{i \in \mathcal{S}_P} \sigma_{ij}(\Omega) \sin(\omega_i t) + \chi_{ij}(\Omega) \cos(\omega_i t) +$$

periodic deformation (seasonal)

$$\sum_{i \in \mathcal{S}_{UB}} \kappa_{ij}(\Omega) B_n(t - t^\flat_i) + \sum_{i \in \mathcal{S}NB} \kappa'_{ij}(\Omega) B_{ni}(t - t^\sharp_i)$$

generalized functions: B-splines, uniform (symmetric) and non-uniform (asymmetric, localized)

Typical splines (current examples too coarse in time…)



Regularization parameter, λ, on B-splines chosen by cross-validation and is currently assumed constant for all wavelet scales and positions.

Observation equation

$$\Delta\rho_{\alpha\beta}(\Omega) = \rho(\Omega, t_\beta) - \rho(\Omega, t_\alpha) + \mathcal{R}_{\alpha\beta}(\Omega) + \mathcal{N}_{\alpha\beta}(\Omega)$$

interferometric measurement

orbital errors (igram specific)

other "noise" (pixel & igram specific)

$$\rho(\Omega, t_\beta) - \rho(\Omega, t_\alpha) = [t_\beta - t_\alpha]\, v_\rho(\Omega) + [F_\rho(\Omega, t_\beta) - F_\rho(\Omega, t_\alpha)]$$

orbital errors approximated by

$$\mathcal{R}_{\alpha\beta}(x, y) = a_{\alpha\beta} + b_{\alpha\beta}\, x + c_{\alpha\beta}\, y + d_{\alpha\beta}\, xy + \cdots$$

solve via (constrained) least squares – (generalized implementation):

$$||G_{ij}m_j - y_i||^2 + \sum_{k=1}^{p} \lambda_k ||H_{ij}^k m_j^k - h_i^k||^2$$

optional constraints:

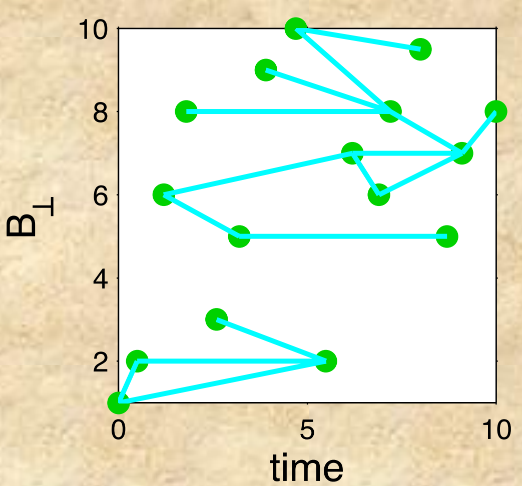
- minimum soln.
- flat soln.
- smooth soln.
- sparse soln.

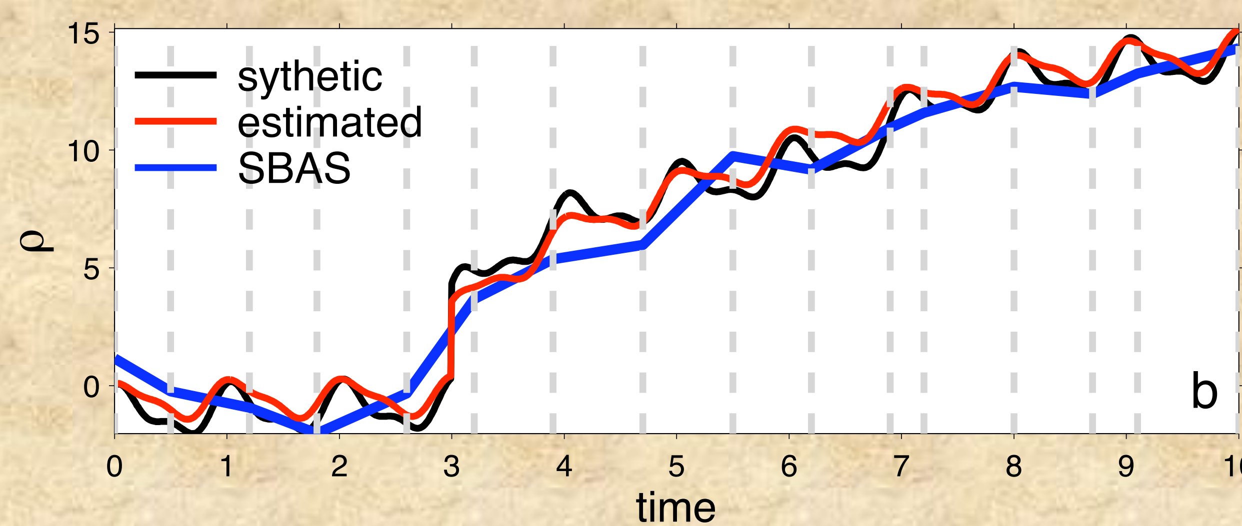
applied individually on each model parameter, or function

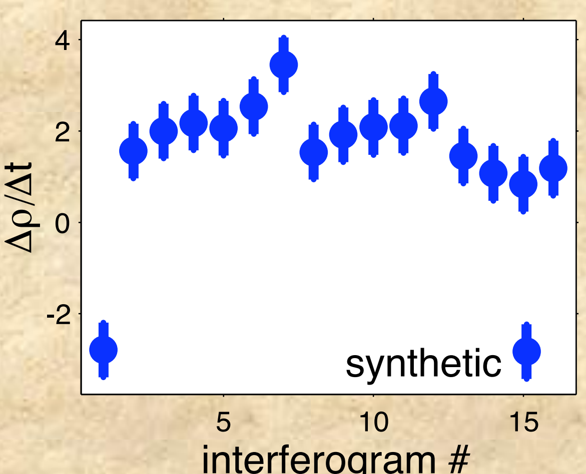
Use cross-validation to determine λ's

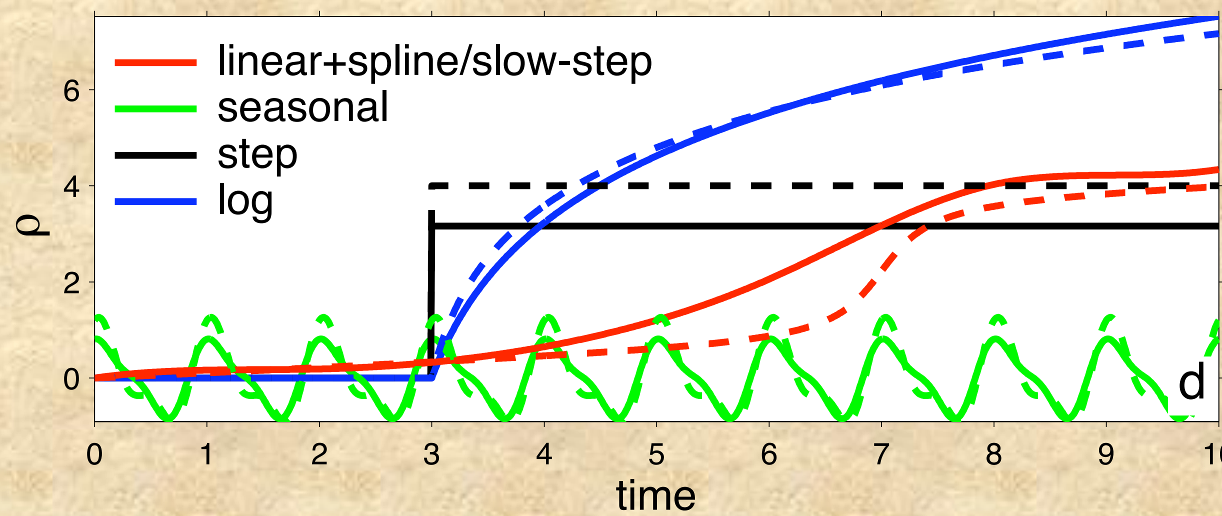
Demo time series – single pixel:

Time series is parameterized physically - not with image acquisition times - disconnected sets do not require any additional parameters



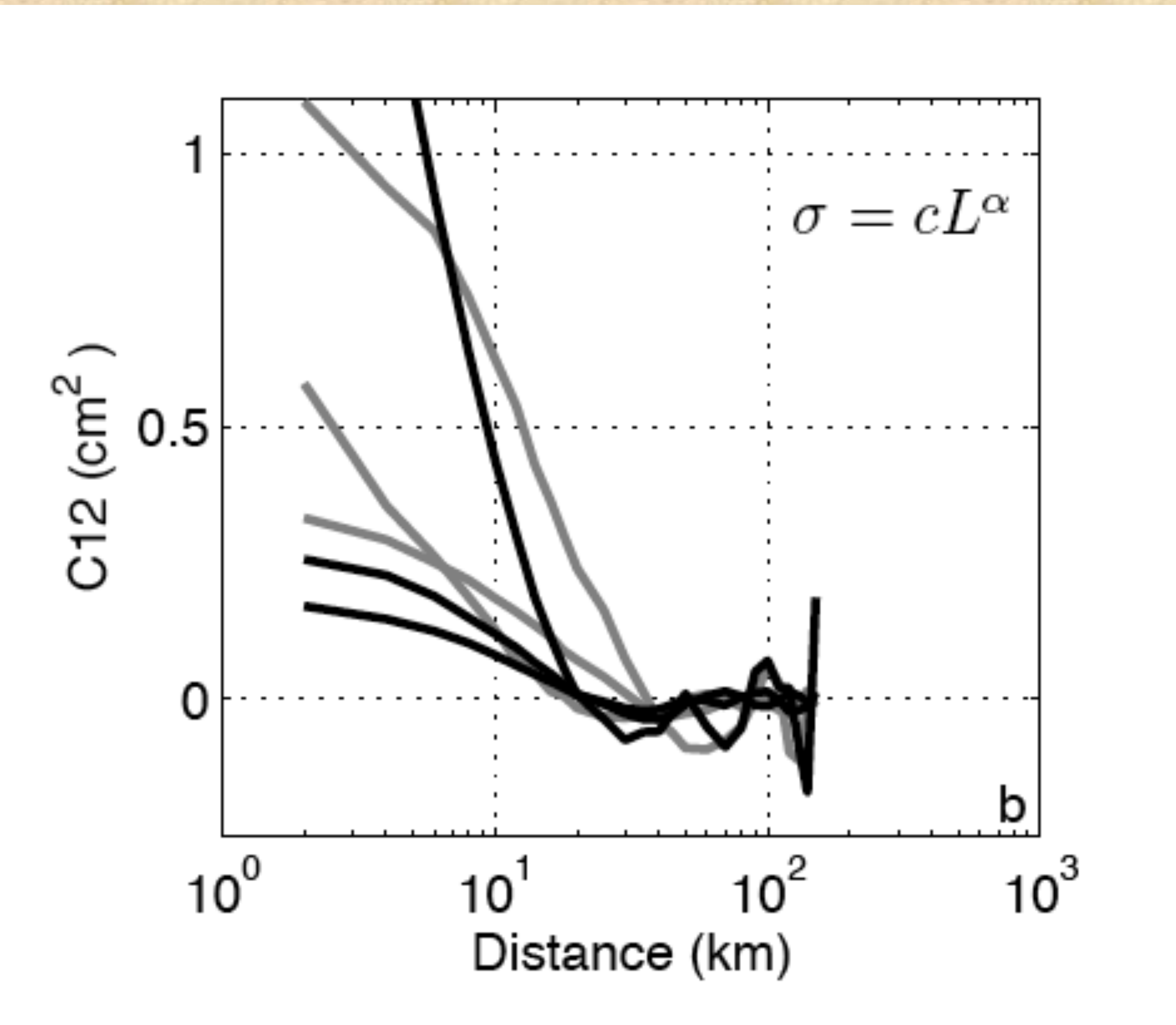






Now the spatial problem

Neighboring pixels are highly correlated



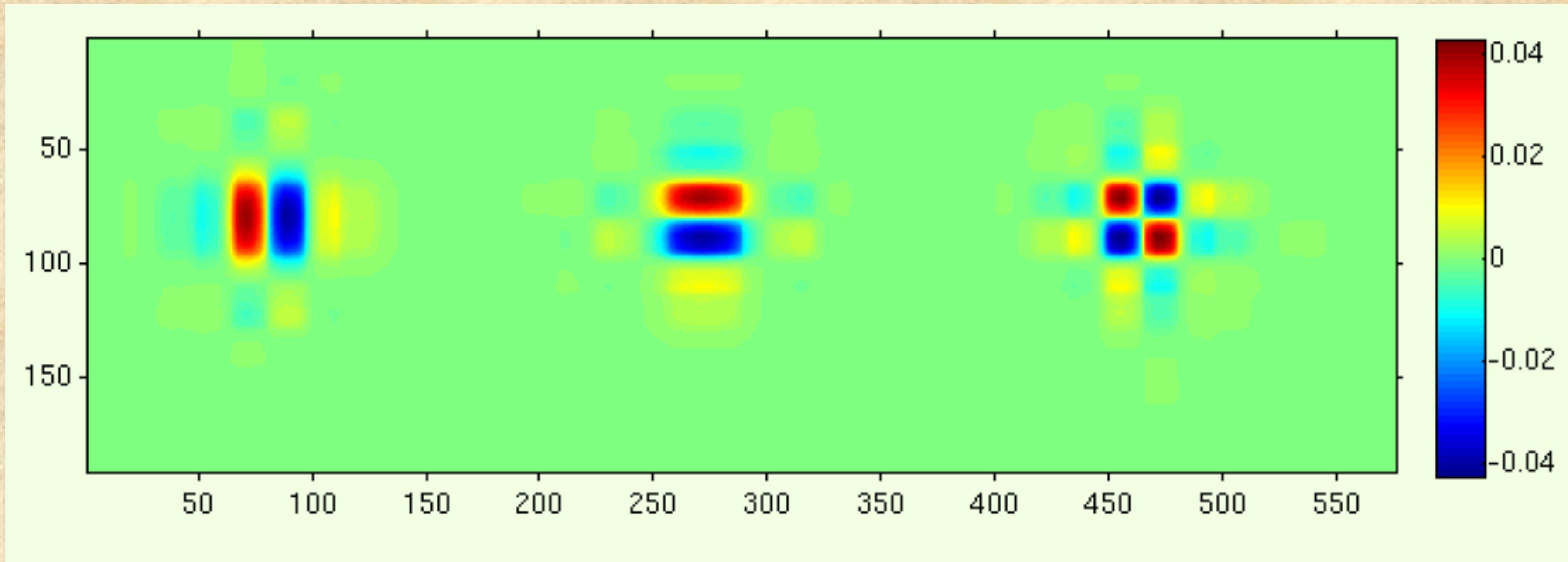
Hanssen, 2001
Emardson et al., 2003
Lohman & Simons, 2005

Challenge: Coupling all pixels together is expensive

MInTS: Spatial parameterization

Farras or Meyer 2D Wavelets

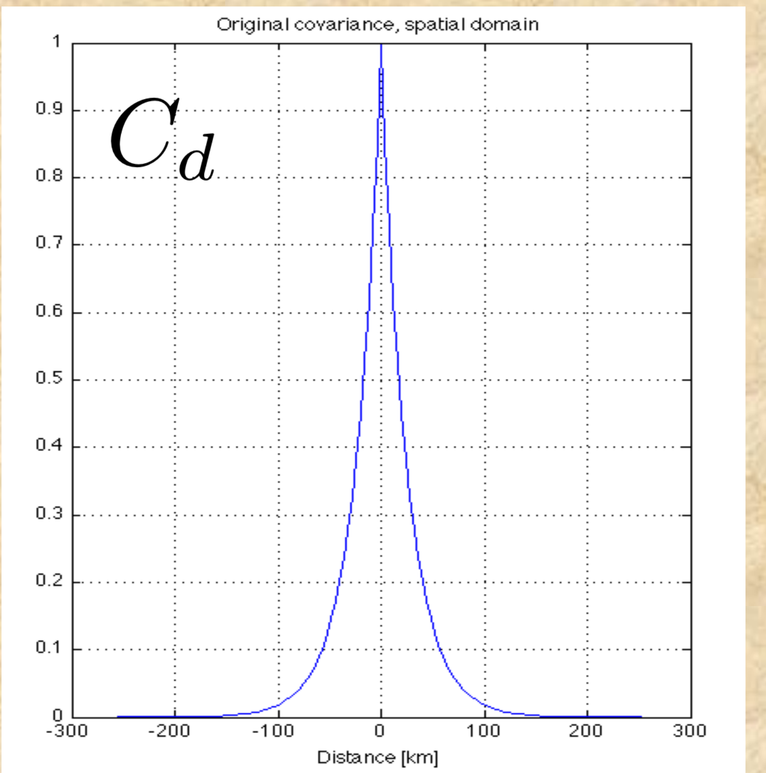
3 wavelets per scale (lossless)



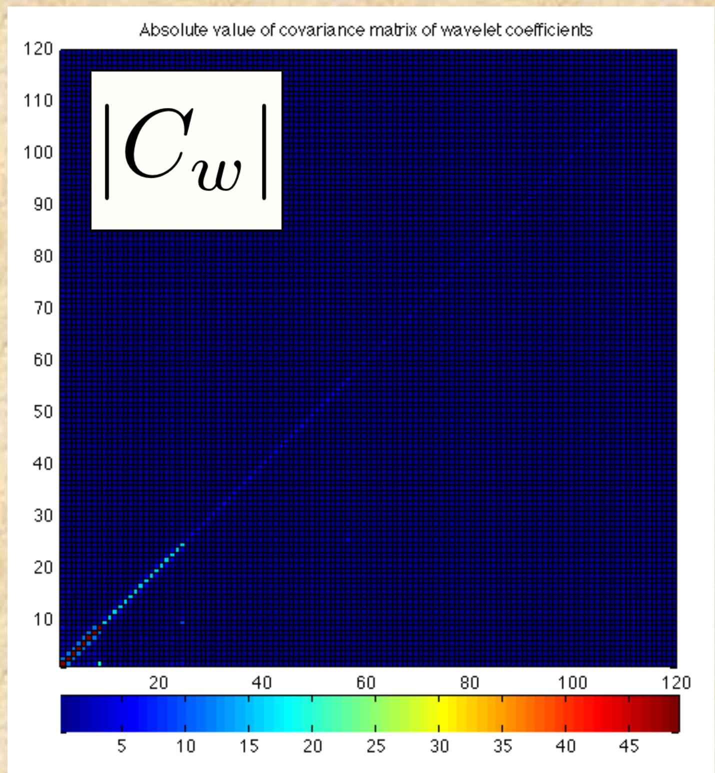
Wavelet parameterization permits implicit inclusion of C_d

$C_d = A \exp(L/L_c)$

$L_c = 25$ km



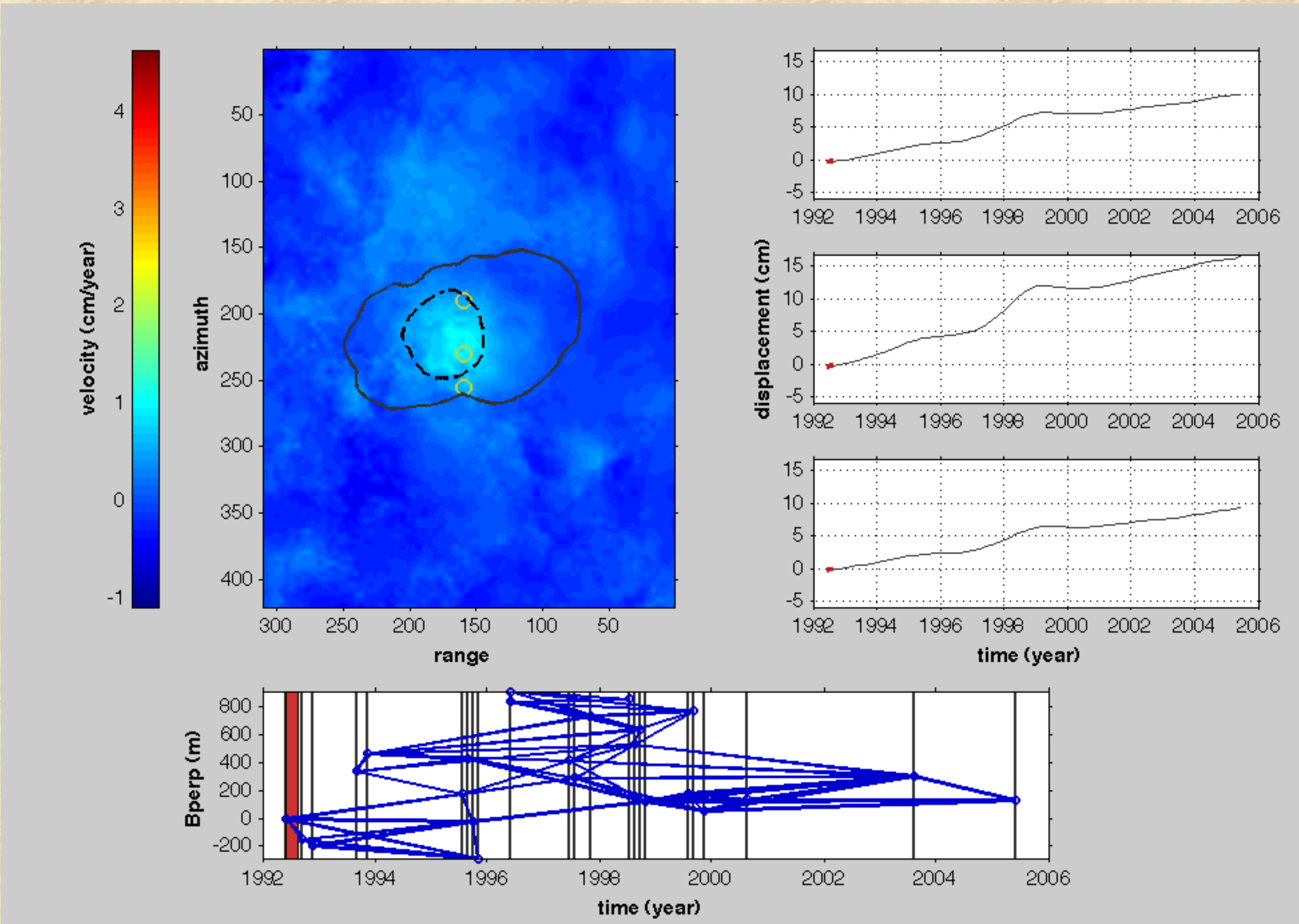
Corresponding wavelets coefficients nearly uncorrelated



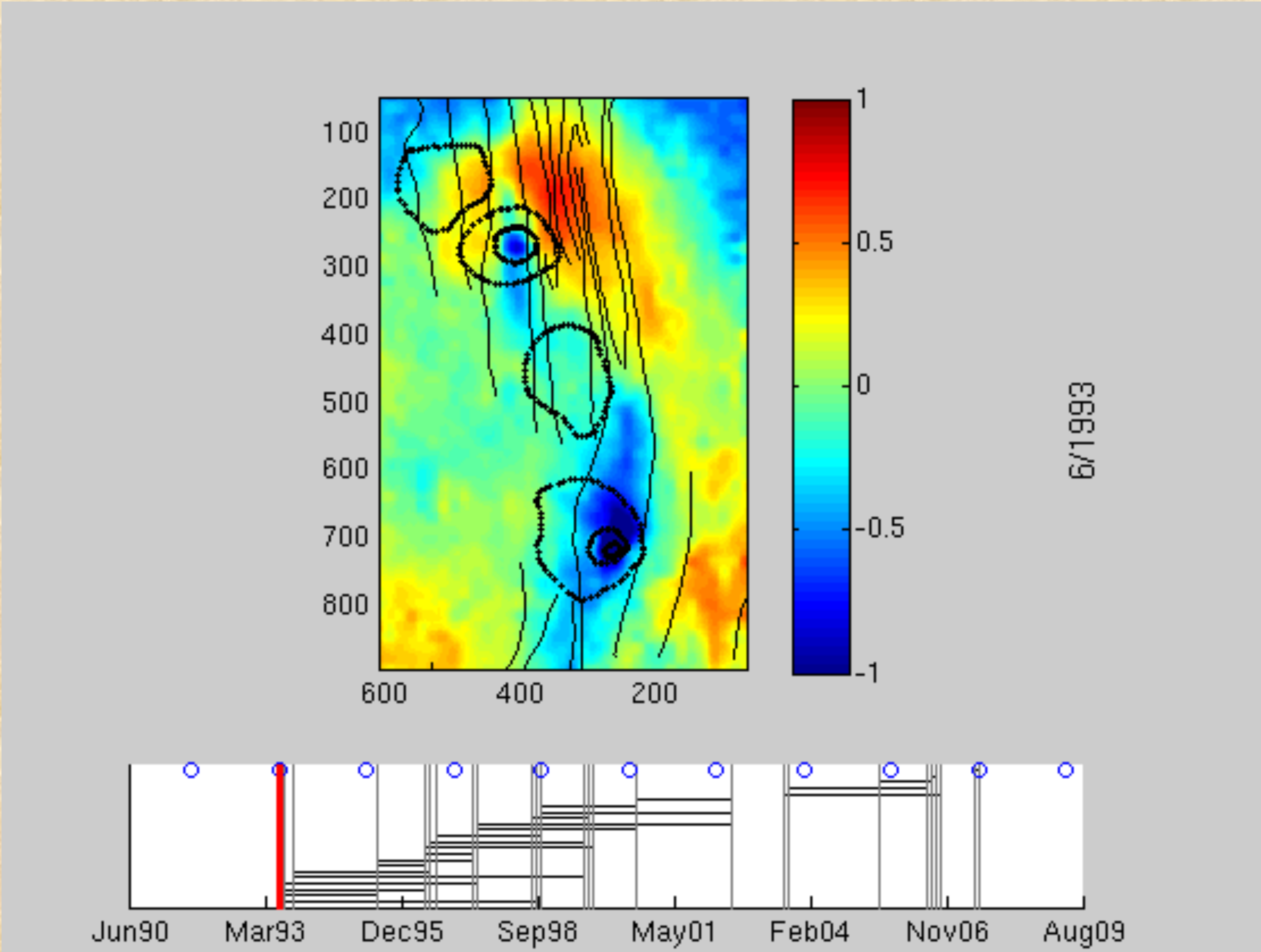
$$C_w = W C_d W' \text{ where } W \text{ is the wavelet transform matrix}$$

Note: wavelet approach has no master pixel – inherently relative displacements at a given scale

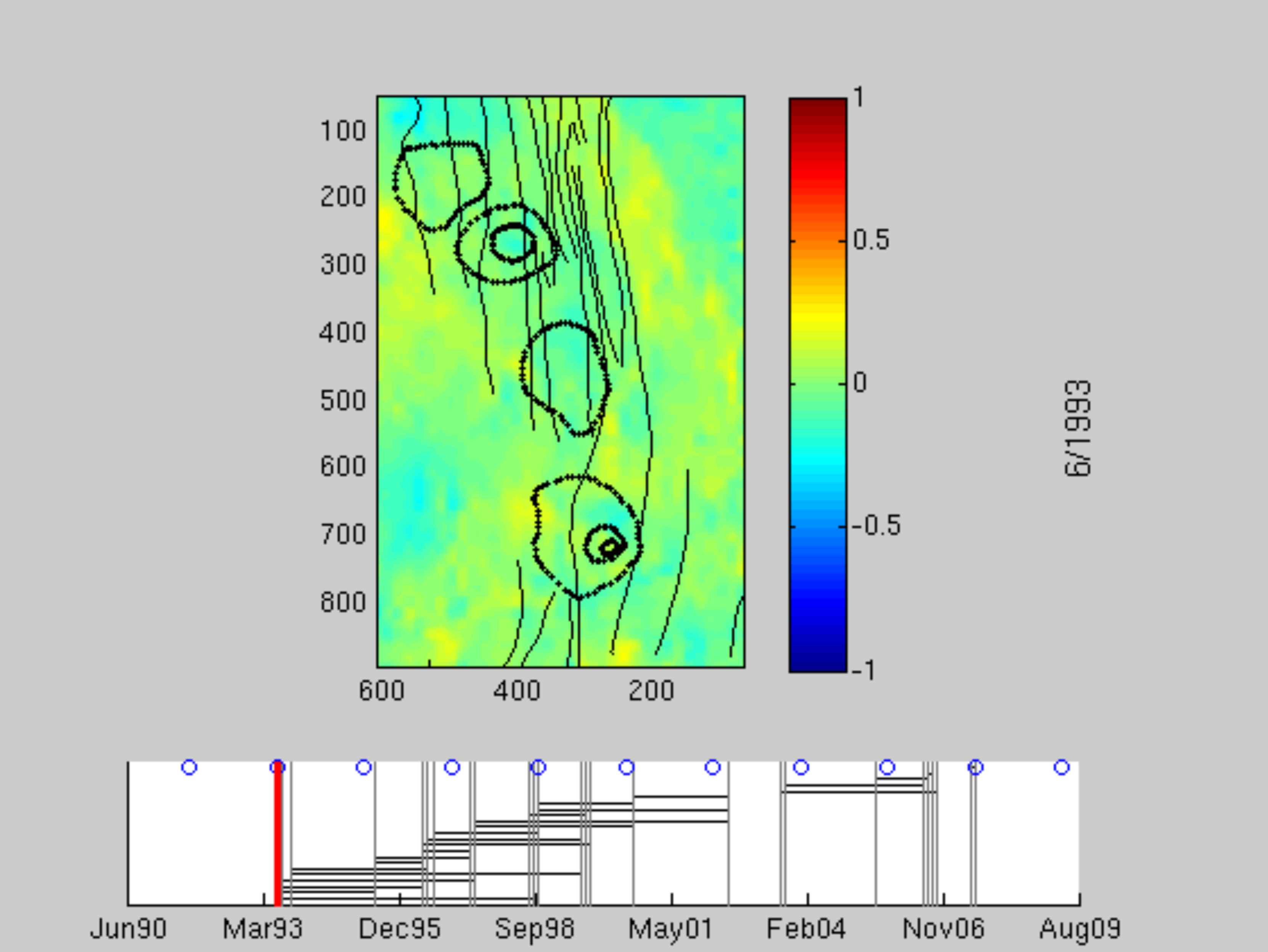
Long Valley: (ERS/Envisat in geographic coordinates) – Velocity



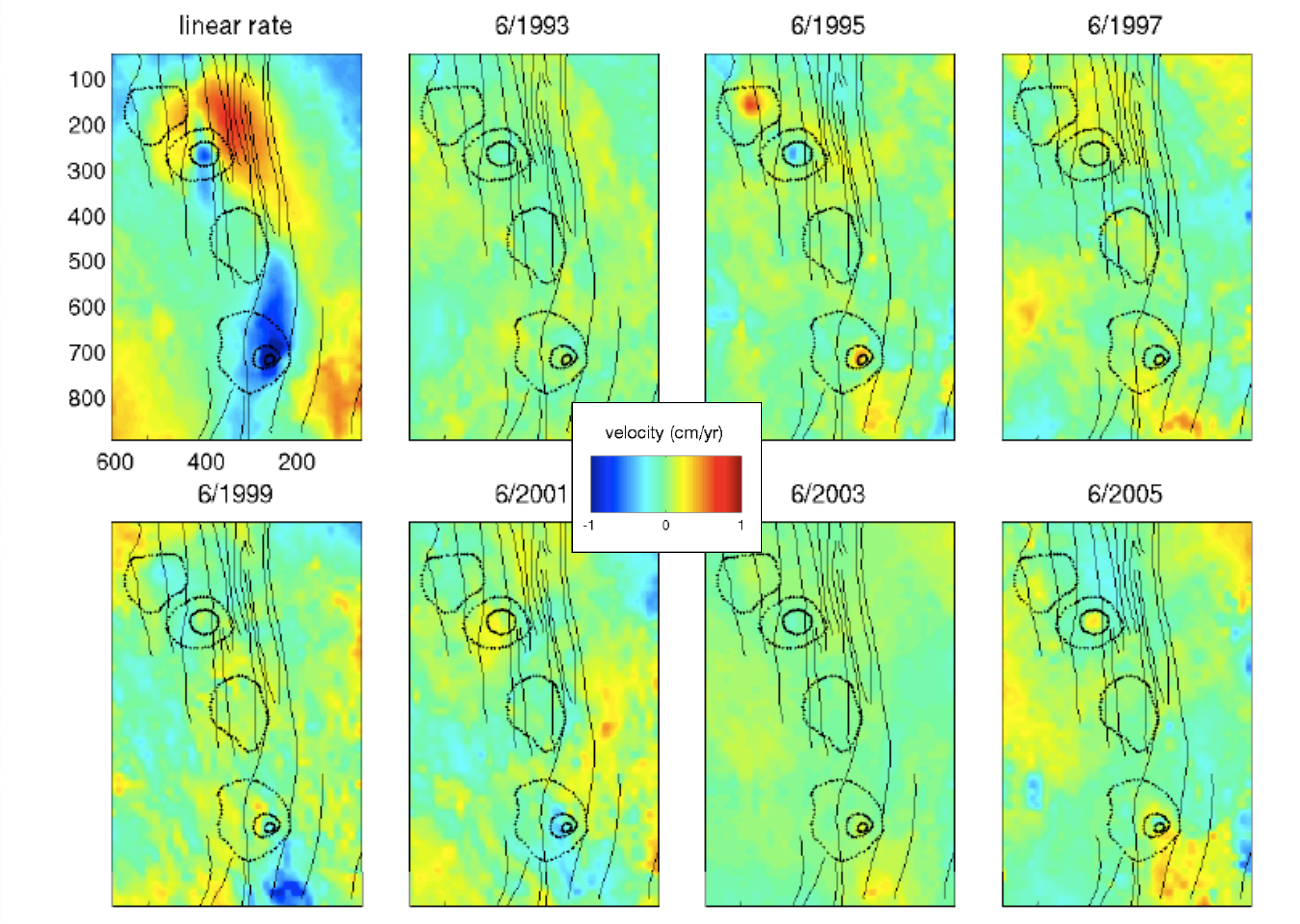
Example: Iceland Northern Volcanic Zone – Instantaneous Velocity



Example: Iceland Northern Volcanic Zone – Instantaneous Velocity (nonlinear)



Example: Iceland Northern Volcanic Zone – Instantaneous Velocity



MInTS

• Allows all scenes to be used even when isolated holes are present

• Interpolates holes in time & space – "deformation tomography"

• Implicitly includes expected data covariance

• Physically parameterized but with ability to "discover" transients

• Flexible choice of regularization on different components of the parameterization (smoothness, sparsity,…)

• Computationally efficient

• Potential for incorporation of other data (e.g., GPS) and N,E,U parameterization given enough LOS diversity.

• Potential for integrated phase unwrapping