State Estimation of Hybrid Systems

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Outline

- Resiliency Context
- Problem Statement
- Prior Work
- Bounding Conflict Directed A*
- Status
- Future Work

Resiliency Context

- Control Systems short course
  - Feedback control
  - Receding horizon control
- AI short course
  - Planning
  - Model-based Programming

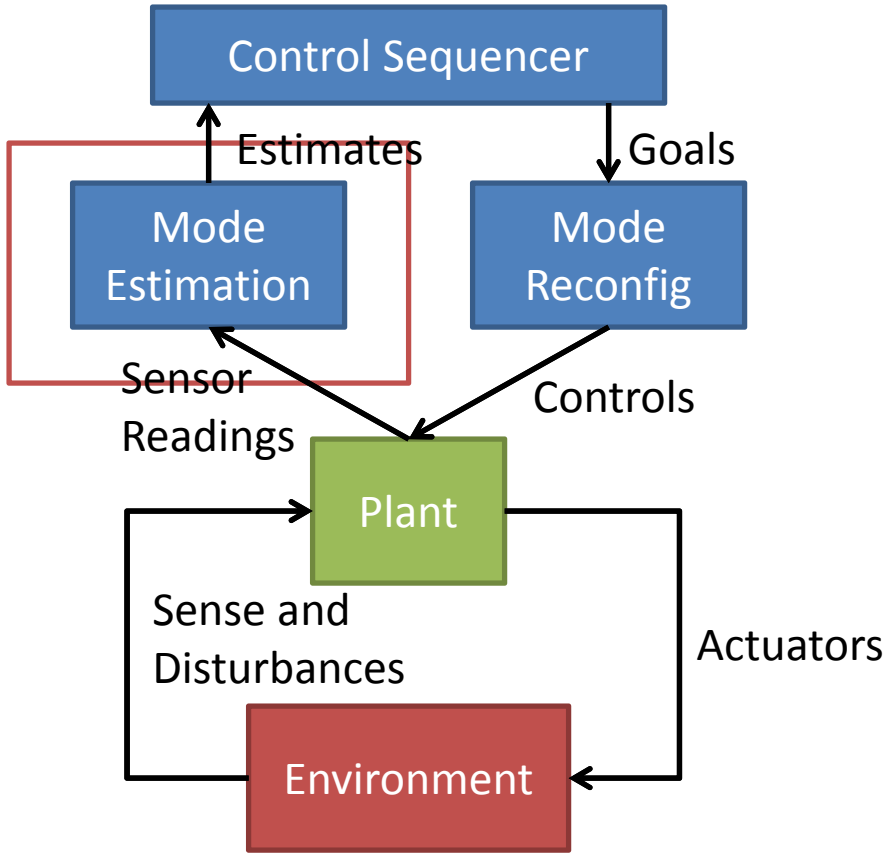
State Estimation

- Critical to know state of the system
  - Low level control loops
  - High level planning algorithms

- Systems of interest have both discrete and continuous variables to estimate
  - Discrete variables = operating modes (open, closed, stuck open, etc.)

Model-based Programming and Execution

- Program conditioned on hidden state of plant
  - Executive automatically estimates state from model
- Program by setting the hidden state of the plant
  - Executive interprets goals and automatically generates action sequence to achieve goal



Problem Statement

- Given a model of a system and continuous observations, estimate the discrete operating modes and continuous values of the system

Probabilistic Hybrid Automata

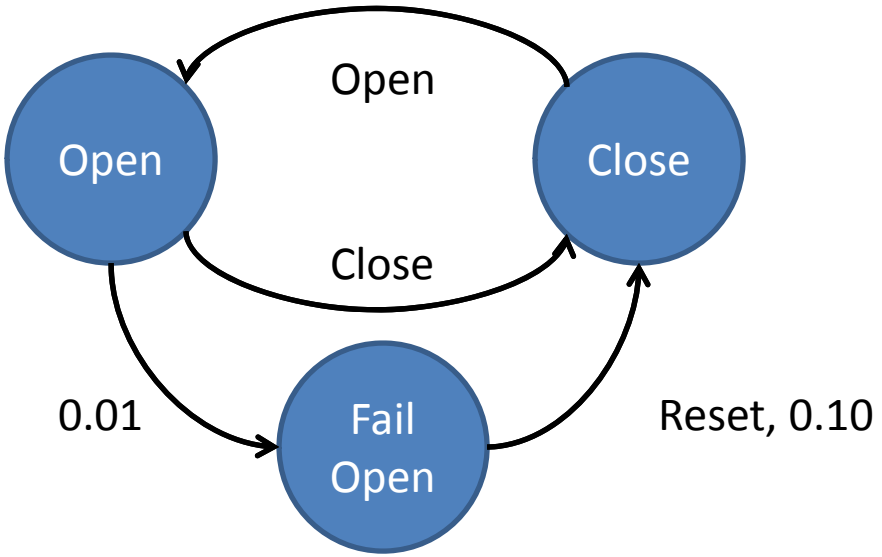
- Description of a component and its continuous and discrete evolutions over time
  - Continuous:

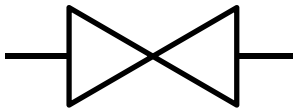
$$x_{c,k} = f_{DE}\left(x_{c,k-1}, u_{c,k-1}, x_{d,k}\right) + \nu_{s,k-1}$$

$$f_{AE}\left(x_{c,k}, u_{c,k}, x_{d,k}\right) \leq 0$$

$$y_{c,k} = g\left(x_{c,k}, u_{c,k}, x_{d,k}\right) + \nu_{o,k}$$

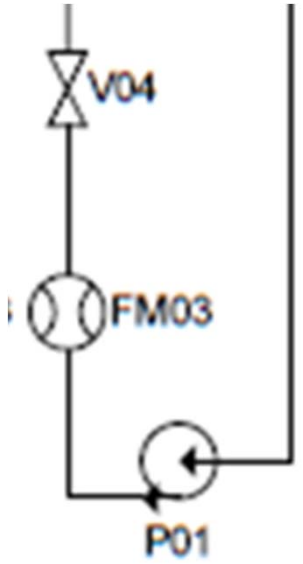
  - Discrete:





Concurrent Probabilistic Hybrid Automata

- Composed of l PHAs
- Connected by shared variables
- Assumption: mode transitions are independent

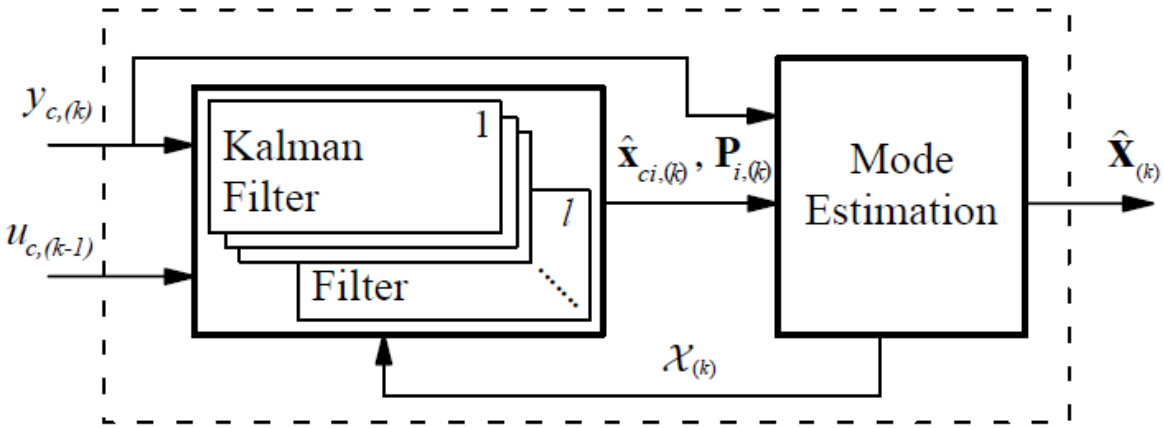


A Previous Approach

- Hybrid Mode Estimation [Hofbaur and Williams, 2004]

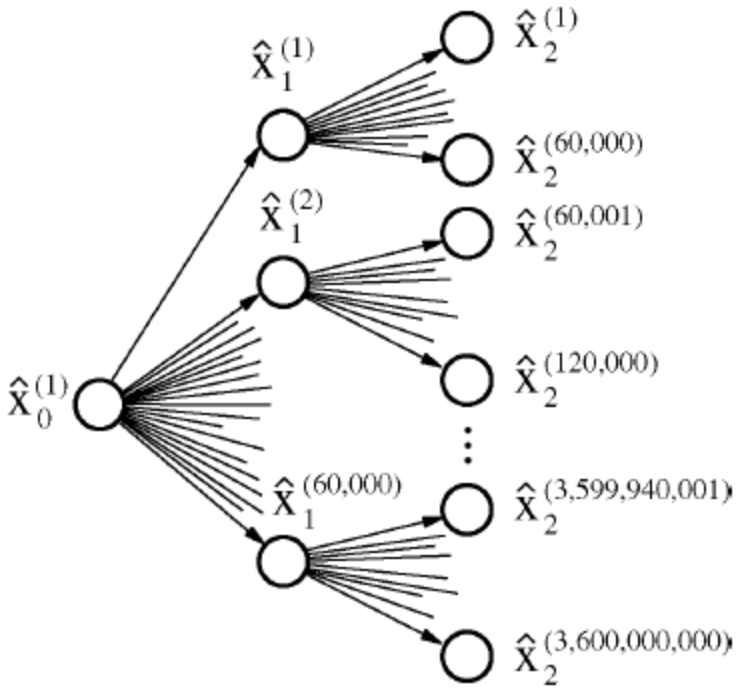
$$b_{k|k-1}^{(j)} = P_T\left(\hat{x}_{d,k}^{(j)}, \hat{x}_{k-1}^{(i)}, u_{d,k-1}\right) b_{k-1}^{(i)}$$

$$b_{(k|k)}^{j} = \frac{P_O\left(y_{c,k}, \hat{x}_{(k|k-1)}^{(j)}, u_{c,k}\right) b_{(k|k-1)}^{(j)}}{\sum_{i=1}^{\lambda_k} P_O\left(y_{c,k}, \hat{x}_{(k|k-1)}^{(i)}, u_{c,k}\right) b_{(k|k-1)}^{(i)}}$$



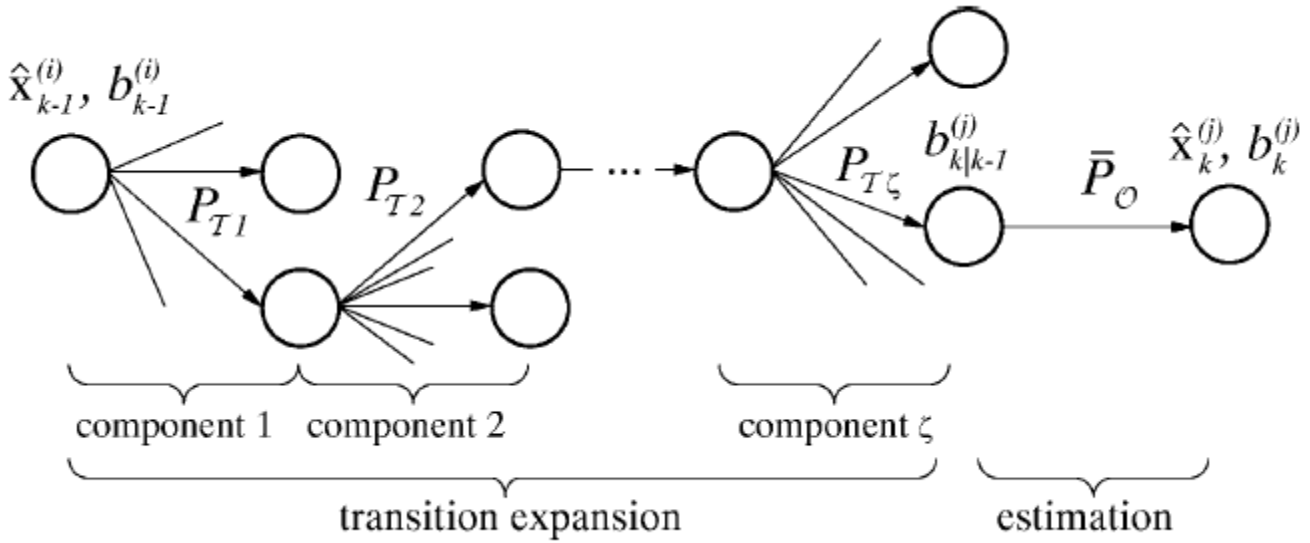
Combinatorics Strike!

- Each mode estimate at time t can evolve into an exponential number of mode estimates at time $t+1$
  - Track k-best trajectories
  - But… how do we decide which k of the exponential possibilities to keep?



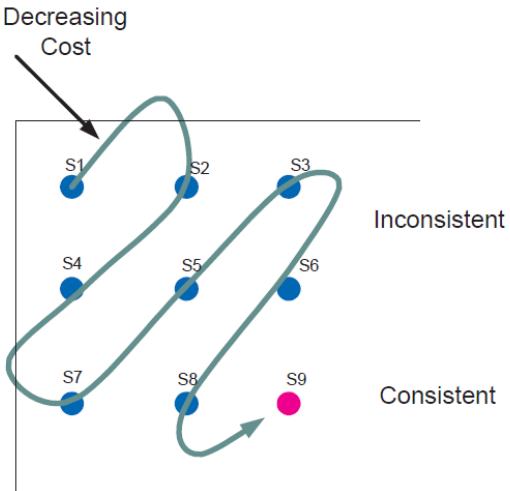
Determining Successors

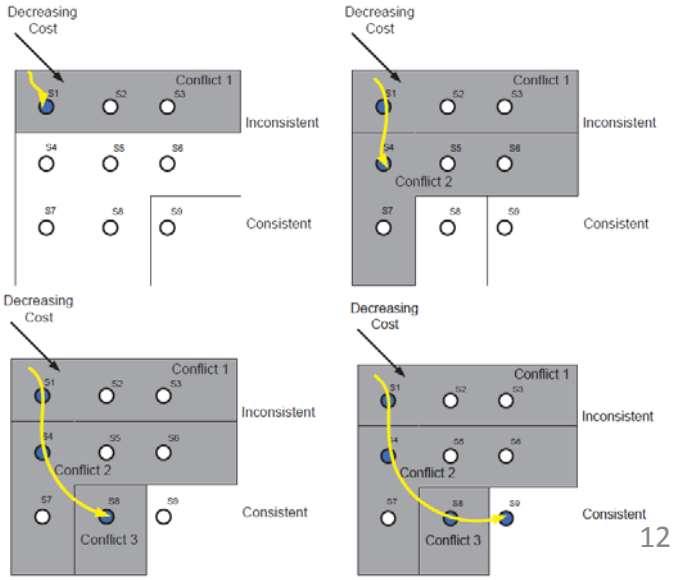
- A* search from estimates at time t to estimates at t+1



Conflict-Directed Search

- Conflict Directed A* [Ragno and Williams, 2003]
  - Solves optimal constraint satisfaction problems
  - Max f(x) s.t. C(x) is satisfied
  - Learns conflicts while searching to jump over areas that look promising





Conflict Directed Hybrid Mode Estimation?

- Can we apply conflict directed search methods to find successors in HME?
- Sort of…
  - Learn conflict when $P_O \leq \Delta$
  - How do we set Δ?
    - Too high: Cut out too much of the search space
    - Too low: No benefit from conflicts

Bounding Conflicts

- Generalize conflicts to the continuous case

- $\gamma = < z, \delta >$ is a bounding conflict if every extension to z has cost greater than δ.

Bounding Conflict Directed A*

- If the cost of a node is much worse than the heuristic predicts, learn a bounding conflict that explains why
- Stratify the search queue based on the bounding conflicts
- Once the best stratum is empty, relax a conflict and start working on the next stratum

bCDA* Example

min $\quad x_1 + x_2 + 4k(y_1) + 3k(y_2) + 2k(y_3) + k(y_4)$

s.t. $\quad \neg y_1 \qquad\qquad \rightarrow x_1 + x_2 \geq 15$

$\neg y_3 \wedge \neg y_4 \quad \rightarrow x_2 \geq 10$

$x_1, x_2 \geq 0$

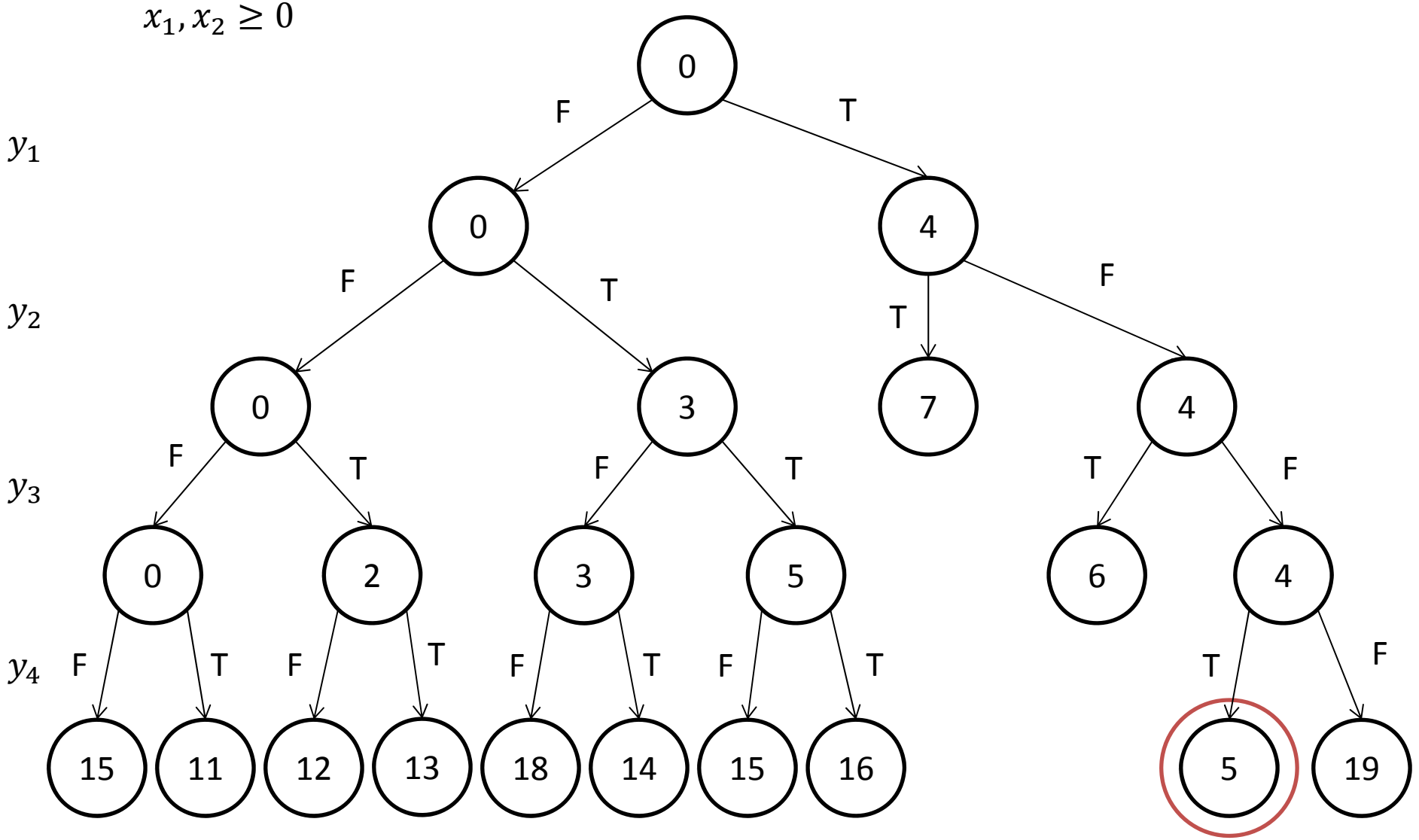
$$\min x_1 + x_2 + 4k(y_1) + 3k(y_2) + 2k(y_3) + k(y_4)$$

s.t. $\neg y_1 \rightarrow x_1 + x_2 \geq 10$

$\neg y_3 \wedge \neg y_4 \rightarrow x_2 \geq 15$

$x_1, x_2 \geq 0$

A*



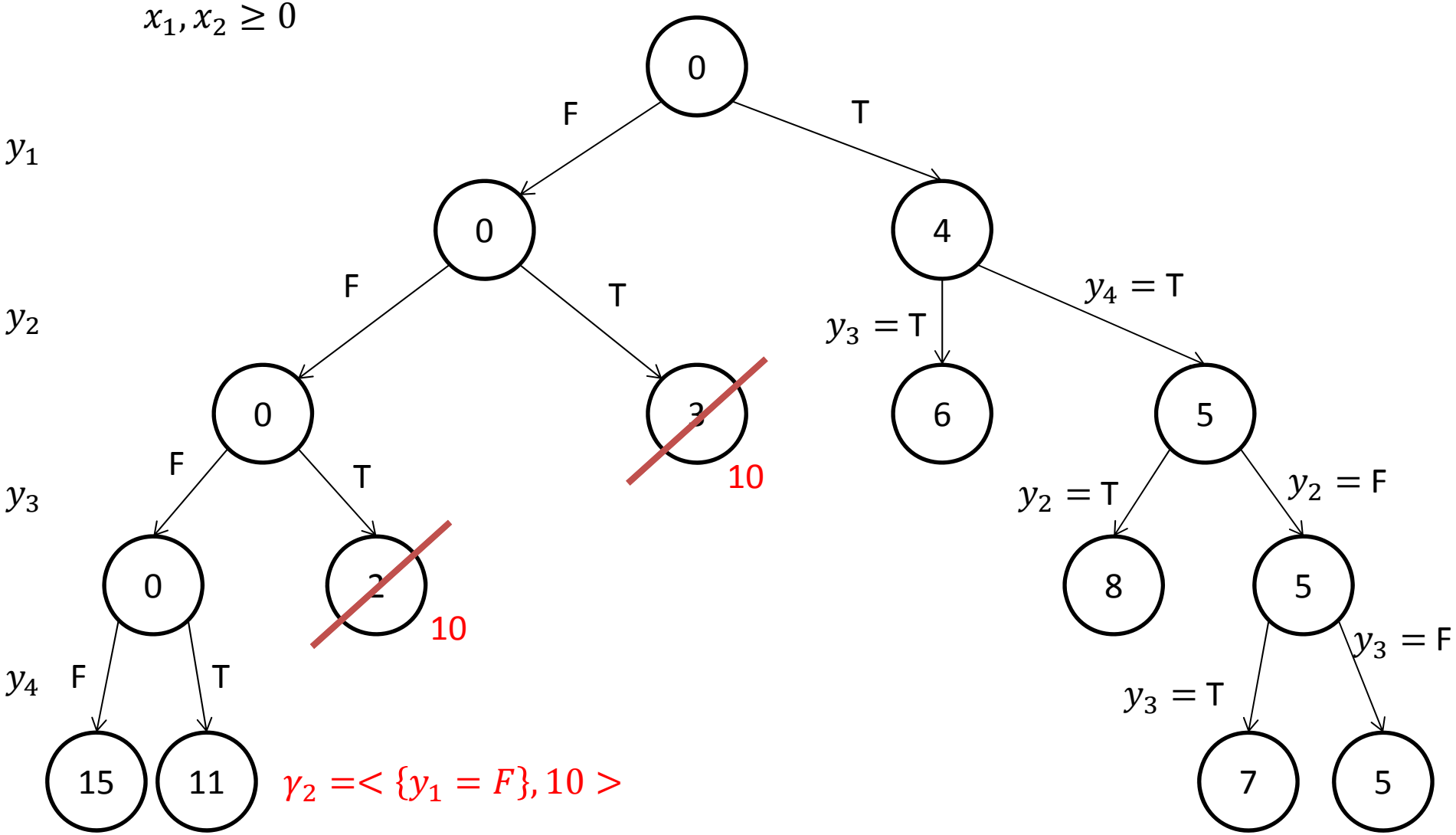
$$\min x_1 + x_2 + 4k(y_1) + 3k(y_2) + 2k(y_3) + k(y_4)$$

$$\text{s.t.} \quad \neg y_1 \rightarrow x_1 + x_2 \geq 10$$

$$\neg y_3 \wedge \neg y_4 \rightarrow x_2 \geq 15$$

$$x_1, x_2 \geq 0$$

bCDA*



$\gamma_2 =< \{y_1 = F\}, 10 >$

$\gamma_1 =< \{y_3 = F, y_4 = F\}, 15 >$

Status

- Applied to fluid system with algebraic constraints, no difference constraints.

Future Work

- Characterize performance on fluid system with simulated sensors
- Add in dynamics (Kalman filter propagation)
- Benchmark bCDA* with other search algorithms on random MLLPs

Take-aways

- State estimation is a key enabler for resilient systems
  - Generative planning
  - Intuitive model-based programming

- Hybrid estimation schemes are subject to exponential blow-up

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